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Research Article

A TOPSIS-Integrated Approach for Fixed Charge Multi-Objective Transportation Problems with Interval Goals

Vishwas Deep Josh[1], Priya Agarwal[1], Lenka Cepova[2], Huda Alsaud[3], B. Swarna[4], Ajay Kumar[5*]

[1]Department of Mathematics, School of Sciences, JECRC University, Jaipur, Rajasthan, 303905, India
[2]Department of Machining, Assembly and Engineering Metrology, Faculty of Mechanical Engineering, VSB-Technical University of Ostrava, Ostrava, 70800, Czech Republic
[3]Department of Mathematics, College of Science, King Saud University, P.O. Box 22452, Riyadh, 11495, Saudi Arabia
[4]Department of Biosciences, Saveetha School of Engineering, Saveetha Institute of Medical and Technical Sciences, Chennai, 602105, India
[5]Department of Mechanical Engineering, School of Engineering, Faculty of Science, Technology & Architecture, Manipal University Jaipur, Jaipur, Rajasthan, 303007, India
E-mail: ajay.kumar1@jaipur.manipal.edu, ajay.kumar30886@gmail.com

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Abstract: This study addresses the complexities of Fixed Charge Multi-Objective Transportation Problems (FCMOTPs) where both fixed and variable transportation costs must be minimized while balancing multiple conflicting objectives under uncertainty. The proposed model enhances optimization planning by managing uncertainties and aligning solutions with Decision Maker (DM) priorities. And weight assignment technique is introduced, allowing DMs to prioritize objectives based on their preferences, leading to diverse Pareto-optimal solutions. Utilizing the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) methodology, the proposed research demonstrates that DMs derive significant benefits from the proposed model. A novel numerical example is developed to demonstrate the applicability of the proposed model, which is solved using LINGO 18 optimization software.

Keywords: Multi-Objective Transportation Problem (MOTP), uncertainty management, decision-maker preferences, interval goal, weight assignment, fixed charge optimization, pareto-optimal solution, ranking methodology

MSC: 90B06, 90C29, 90C70, 90C11

Abbreviation

TP	Transportation Problem
MOTP	Multi-Objective Transportation Problem
FC	Fixed Charge
FCTP	Fixed Charge Transportation Problem
FCMOTP	Fixed Charge Multi-Objective Transportation Problem
DM	Decision Maker
TOPSIS	Technique for Order Preference by Similarity to Ideal Solution

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1. Introduction

There is a lot of history and many uses for the Transportation Problem (TP), which is a classic optimization problem. It is a kind of linear programming problem that is very important in operations research. The TP's main goal is to find the best way to move goods from many suppliers to many customers while keeping costs as low as possible and meeting supply and demand needs. They help companies make their transportation networks more efficient, cut costs, and speed up deliveries. In the early 1900s, the issue got more formal attention as part of efforts to make better use of resources and lower transportation costs in supply chains. Kantorovich [1] created the idea of linear programming to help with problems with resource allocation. He also created a mathematical framework and gave solutions to make resource allocation, including transportation costs, as efficient as possible. Hitchcock [2] and Koopmans [3] described a way to solve the TP. Hitchcock [2] talked about how to get products from many places to many places. Koopmans [3] found the best way to use transportation systems. This method involved finding the best transportation plan that would cost the least. The TP is now a major issue in operations research that is used in many fields, including manufacturing, logistics, supply chain management, and the military. A standard TP has one goal, which is usually to lower the total cost of transportation. But in the real world, there may be more than one goal that needs to be met. Das et al. [4] discussed Multi-Objective Transportation Problems (MOTPs) incorporating interval costs, sources, and destination parameters. Maity and Roy [5] employed the utility function methodology to address MOTPs characterized by interval targets. Nomani et al. [6] introduced an innovative approach for solving MOTPs. Uddin et al. [7] employed fuzzy linear membership functions in their goal programming technique for MOTPs. Joshi et al. [8] focused on fuzzy transportation planning to handle uncertainty and choose options with multiple goals. Joshi et al. [9] created a goal programming method to solve linear transportation problems with more than one goal. Joshi et al. [10] advocate for multimodal route planning in smart cities, highlighting the importance of intelligent infrastructure and sustainability within urban transportation networks. Joshi et al. [11] present a neutrosophic model for fractional solid transportation, addressing uncertain logistics data to facilitate environmentally sustainable decision-making. Joshi et al. [12] propose a healthcare transportation model that tackles uncertainty through mixed constraints to ensure efficient and dependable service delivery. The Fixed Charge Transportation Problems (FCTPs) is a more advanced version of the classical TP that includes both fixed and variable costs in the objective function. Balinsky [13] looked into fixed cost transportation problems and talked about how fixed costs make things more complicated. In an FCTP, there is a set cost for opening or using a transport route, and this cost goes up as more people use that route. The goal of this problem is to find the best shipping plan that meets supply and demand needs while keeping transportation costs as low as possible. This problem is more complicated because the model now includes binary variables and the choice of whether or not to use a certain route. Hirsch and Dantzig [14] elaborate on the fixed charge problem, offering insights into the optimization of systems characterized by fixed charges. The Fixed Charge Multi-Objective Transportation Problems (FCMOTPs) merge the ideas of the FCTPs with the multi-objective framework of the MOTPs. Chevali et al. [15] present the application of Gray's state decision-making theory to FCMOTPs. Shivani and Rani [16] proposed an enhanced Vogel's approximation method specifically designed to expedite the resolution of Fermatean fuzzy solid transportation problems. Yang and Yang [17] put forward a sequential reduction technique in the same year to solve big fixed-charge network flow problems. Revathi and Mohanaselvi [18] develop a neutrosophic fixed charge model incorporating carbon emission and budgetary constraints within multi-dimensional transportation systems. Tomar and Kumar [19] assess intuitionistic fuzzy Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) in multi-objective transportation issues, emphasizing its advantages and drawbacks for decision-making assistance. FCMOTPs tries to lower both fixed and variable transportation costs while also taking into account multiple conflicting goals and making sure that supply and demand needs are met. In FCMOTPs, goals include lowering fixed and variable transportation costs, as well as other important goals like lowering transportation time, raising service levels, or lowering environmental impact. Saini et al. [20] developed rough programming for linear fractional transportation problems in Fixed Charge Multi-Objective (FCMO). Zhang et al. [21] investigated fixed charge solid transport problems in uncertain contexts and formulated pertinent algorithms. Zhu et al. [22] examine the pure fixed charge transportation problem, providing theoretical insights and optimization strategies for logistics cost structures. The TOPSIS method, developed by Hwang and Yoon [23] and improved by Chakraborty [24], ranks transportation solutions using the formula:

$$\text{Rank} = \frac{\mathfrak{S}^-}{(\mathfrak{S}^- + \mathfrak{S}^+)}.$$

Where $\mathfrak{S}^-$ represents the distance from the negative-ideal solution (maximum value) and $\mathfrak{S}^+$ represents the distance from the positive-ideal solution (minimum value). This ranking system helps decision makers identify the most favourable transportation solution, facilitating efficient decision making.

A comprehensive methodology for TP is included in the model for FCMOTPs that the authors have suggested. The decision-maker is able to establish a hierarchy of objectives according to their importance by assigning weight to each objective. As a result, it is easier to make trade-offs between the transportation duration, fixed and variable expenses, and other factors. The model is able to adapt to changing conditions as a result of the integration of uncertainty and interval objectives, which acknowledge the inherent unpredictability that exists in transportation planning. The major importance of this issue to supply chain operations and logistics in the actual world is the incentive for selecting it. When making decisions, decision-makers are required to take into consideration both the variable transportation costs and the fixed prices that are connected with certain routes or resources. In addition, the actual applications that are implemented on a regular basis sometimes involve a number of conflicting goals. These include the minimization of costs, the minimization of the duration of transportation, and the minimization of the potential for injury. Despite the fact that there is a large amount of literature that focuses on traditional transportation and multi-objective optimization challenges, there are very few integrated models that simultaneously address fixed costs and a variety of criteria (Table 1). The proposed method that utilizes a distinctive ranking system allows decision-makers to choose balanced options that are close to the ideal, hence improving the quality and use of transportation planning in situations that are characterized by uncertainty and multiple criteria. This work makes the following major contributions to the field of FCMOTPs:

• **Uncertainty Handling with Interval-Based Representation:** Unlike traditional models that use crisp values, the authors represent transportation costs, supply, and demand as interval values, effectively capturing real-world uncertainties.

• **Weight Assignment for Objective Prioritization:** Authors introduce a systematic weight assignment technique that allows Decision Makers (DMs) to prioritize conflicting objectives based on their preferences, ensuring flexibility in decision-making.

• **Pareto-Optimal Solutions for Multi-Objective Optimization:** The proposed approach generates a diverse set of Pareto-optimal solutions, allowing DMs to balance trade-offs among multiple objectives efficiently.

• **Application of TOPSIS for Decision Support:** Authors integrate the TOPSIS to rank and evaluate optimal solutions, enhancing decision-making under uncertainty.

• **Practical Applicability to Real-World Logistics Problems:** The proposed model applies to various domains such as supply chain management, waste disposal, and humanitarian logistics, where fixed charges and uncertainty significantly impact decision-making.

By addressing both uncertainty and multi-objective optimization, the proposed methodology provides a structured and adaptable framework for improving transportation planning in complex logistics scenarios. This paper addresses the FCMOTP, a practical and complex extension of the classical transportation model. The motivation behind selecting this problem lies in its strong relevance to real-world logistics and supply chain operations where DM must consider not only variable transportation costs but also fixed charges incurred when utilizing specific routes or resources. Additionally, real applications often involve multiple conflicting objectives such as minimizing cost, transportation time, and potential damage. Despite the extensive literature on classical transportation and multi-objective optimization problems, integrated models that simultaneously handle fixed costs and multiple criteria are limited. This paper aims to fill that gap by developing a unified framework that reflects the complexities encountered in practical scenarios. A novel ranking-based solution approach is also proposed, allowing DM to identify balanced solutions that are closer to the ideal, thereby improving the quality and applicability of transportation planning in uncertain and multi-criteria environments. Section 2: Introduces concepts and notation. Section 3: Formulates the model and outlines the steps of the proposed model solution.

Section 4: Provides numerical examples and solutions. Section 5: Interprets results and make decisions. Section 6: Conclusions.

Table 1. Comprehensive research data table

Author's name	Year	Objective function	FCTPs	FCMOTPs	Uncertainty (objective, supply and demand)	Weight assign to each objective	Ranking
Balinski [13]	1961	1	✓	×	×	×	×
Hirsch and Dantzig [14]	1968	1	✓	×	×	×	×
Murty [25]	1968	1	✓	×	×	×	✓
Sadagopan and Ravindran [26]	1982	3	×	✓	×	×	✓
Adlakha and Kowalski [27]	1999	1	✓	×	×	×	×
Das et al. [4]	1999	3	×	×	×	×	×
Nomani et al. [6]	2016	3	×	×	×	✓	×
Zhang et al. [21]	2016	1	✓	×	✓	×	×
Chevli et al. [15]	2019	3	×	✓	×	×	×
Saini et al. [20]	2021	3	×	✓	×	×	×
Uddin et al. [7]	2021	3	×	×	✓	×	×
Joshi et al. [8, 9, 11]	2023	3	×	×	✓	✓	✓
This paper	This year	4	✓	✓	✓	✓	✓

2. Preliminaries

This section includes some basic fundamental definitions:

Definition 2.1 Compromise optimal solution [28]: A compromise optimal solution can be defined as a feasible solution that strikes a balance between conflicting objectives or criteria while getting as close as possible to the ideal or optimal solution. This type of solution involves making concessions or trade-offs to accommodate competing interests or constraints, with the ultimate goal of achieving the best achievable outcome within given limits. An optimal solution is a feasible solution where the objective function reaches its maximum or minimum value (shown in Figure 1).

Definition 2.2 Ideal Solution [28]: It refers to the best possible outcome or point in the objective space (shown in Figure 1). It represents the solution that optimizes all objectives simultaneously, achieving maximum benefit in each criterion without any compromise. In practice, the ideal solution is often unattainable due to conflicting objectives or resource limitations. Assume that $f_i(x)$ is an objective function $\forall\ i = 1, 2, \ldots, n$ and X is a feasible set. Ideal Solution x_{ideal}:

$$f_{ideal} = \left(\min_{x \in X} f_1(x), \min_{x \in X} f_2(x), \ldots, \min_{x \in X} f_n(x) \right).$$

Definition 2.3 Anti-ideal solution [28]: The anti-ideal solution represents the worst possible outcome or point in the objective space (show in Figure 1) . This is in contrast to the ideal solution, where each objective is minimized or maximized to its worst possible value. The anti-ideal solution provides a reference point for evaluating the performance of other solutions in the objective space. Assume that $f_i(x)$ is an objective function $\forall\ i = 1, 2, \ldots, n$ and X is a feasible set. Anti-Ideal Solution $x_{anti-ideal}$:

$$f_{anti-ideal} = \left(\max_{x \in X} f_1(x), \max_{x \in X} f_2(x), \ldots, \max_{x \in X} f_n(x) \right).$$

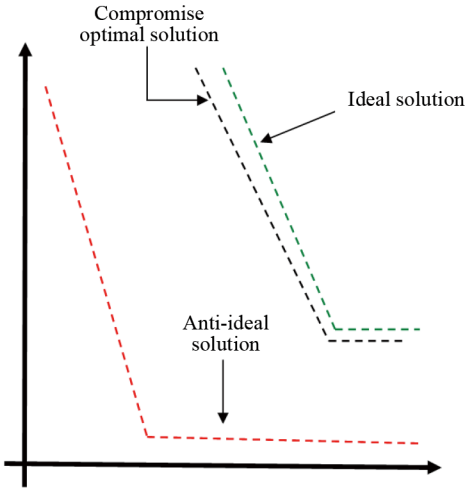


Figure 1. Visualization of solution space

Definition 2.4 Pareto Dominance: Assume that $f_i(x)$ is an objective function $\forall\ i = 1, 2, \ldots, n$ and X is a feasible set. A solution $x_1 \in X$ is said to be dominate another solution $x_2 \in X$ ($denoted\ x_1 < x_2$) if and only if $\forall i \in \{1, 2, \ldots, n\}$, $f_i(x_1) \leq f_i(x_2)$ and $\exists j \in \{1, 2, \ldots, n\}$, $f_j(x_1) < f_j(x_2)$ this means x_1 is no worse than x_2 in all objectives and strictly better in at least one objective (show in Figure 2).

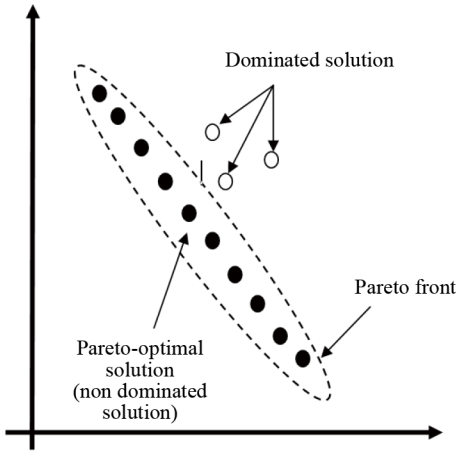


Figure 2. Pareto optimality analysis

Definition 2.5 Pareto-optimal solution [28]: "Pareto-optimal solution" refers to the situation where no further improvements can be made to one criterion without negatively impacting at least one other criterion. In other words, it is a solution where any change to improve one aspect will worsen the other aspect. A solution $x^* \in X$ is Pareto optimal if there is no other solution $x \in X$ that dominates x^* is a Pareto optimal solution if $\nexists\ x \in X$ such that $x \leq x^*$. In other words,

it represents a trade-off between competing goals where no improvement in one aspect is possible without sacrificing the other (shown in Figure 2).

Definition 2.6 Pareto-front [28]: A Pareto front is a set of solutions within a search space where no solution in one criterion can be improved without sacrificing performance in the other criterion. It is defined as: $F_{Pareto} = \{f(x)|x \in X_{Pareto}\}$ where: $X_{Pareto} = \{x^* \in X|\nexists\, x \in X$ such that $x < x^*\}$. These solutions represent trade-offs between conflicting objectives in MOTPS (show in Figure 2).

Definition 2.7 Uncertain variable [29, 30]: A variable called τ is defined as having an associated uncertainty distribution represented by the symbol Q. Any real variable can be assigned the value $Q(x)$ by this distribution, which is represented by the function $Q(x)$. The uncertainty measure function Υ, which represents the degree of uncertainty associated with the variable g, yields the result $Q(x) = \Upsilon(\tau \leq x)$.

When a variable is marked with $\tau \leq x$, it means that its precise value is unknown or cannot be determined. It also means that the uncertain variable τ is thought to have less uncertainty than x. It is imperative to acknowledge that the attributes of the uncertain variable τ and the uncertainty measurement function Υ may vary contingent upon the particular circumstance or the author's methodology.

Definition 2.8 Uncertain parameters [29–31]: An uncertain parameter ψ is defined as normal if it is defined as a function $\psi(x) = \left[1 + exp\left(\frac{\pi(e-x)}{\sqrt{3}\sigma}\right)\right]^{-1}$ is represented by the distribution, where e and σ are real numbers with $\sigma > 0$. This distribution is denoted by $N(e, \sigma)$, which represents a normal distribution with mean e and standard deviation σ.

In the context of TP, when there may be unpredictability in costs, supply, and demand, the MOTPS uses uncertainty parameters to address and navigate these uncertainties. This model uses special techniques to analyze uncertain data and gain valuable insights, which is similar to using a special compass to search and find the optimal solution in an environment of uncertainty (shown in Figure 3).

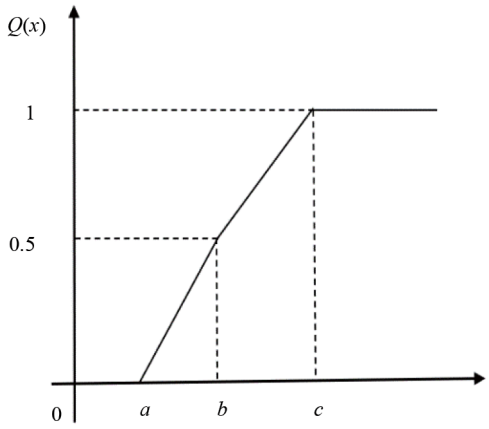


Figure 3. Visual representation of zigzag uncertainty distribution

Theorem 2.1 Measurement Reversal Theorem [32, 33]: By virtue of the Inverse Measure Theorem, crisp values can be recovered from the uncertain domain for any given uncertainty measure Υ and the corresponding cumulative distribution function Q. The theorem states that there is an inverse mapping Q^{-1} such that $Q^{-1}(x)$ retrieves the original probabilistic information from the uncertainty measure. Since $Q(x) = \Upsilon(\tau \leq x)$ then $\Upsilon(\tau \geq x) = 1 - Q(x)$.

Definition 2.9 Uncertain distribution [33, 34]: The inverse uncertainty distribution of τ is characterised by the uncertainty distribution Q and the inverse function $Q^{-1}(x)$. This function may be expanded to cover the limitations $Q^{-1}(0)$ and $Q^{-1}(1)$. It is well-defined on the interval $(0, 1)$. The function that increases monotonically with the normal uncertain variable $N(e, \sigma)$ on the interval $[0, 1]$ it is known as the inverse normal uncertainty distribution. It may be expressed as follows (shown in Figure 3):

$$Q^{-1}(x) = e + \frac{\sigma\sqrt{3}}{\pi} \ln\left(\frac{\phi}{1-\phi}\right).$$

Chosen uncertainty paradigm and formalization (Uncertainty theory—Liu).

To avoid mixed paradigms and to keep the paper self-contained, this study adopts Uncertainty Theory in the sense of Liu [29, 31–34] as the exclusive framework for modeling imprecision. The authors do not interpret uncertain quantities as random variables and we avoid the term "confidence" (often interpreted probabilistically). Instead, the author use Liu's terminology and denote a satisfaction level by $\varphi \in (0, 1)$.

An uncertain parameter $\tilde{p}$ is an uncertain variable with an uncertainty distribution $\Phi_{\tilde{p}}(t) = \mathcal{M}\{\tilde{p} \leq t\}$. The authors use the normal uncertain variable $\mathcal{N}(e, \sigma)$ (Liu) when a symmetric uncertainty shape is appropriate; its inverse uncertainty distribution $\Phi_{\tilde{p}}^{-1}(\varphi)$ is well-defined and monotone in φ. For an uncertain normal $\tilde{p} \sim \mathcal{N}(e, \sigma)$ the decision maker's deterministic representative at satisfaction level φ is

$$p^{(\varphi)} = \Phi_{\tilde{p}}^{-1}(\varphi),$$

which we call the Φ-representative (not "probability quantile"). Using this mapping determinizes uncertain inputs and yields a deterministic MIP to be solved for a chosen φ. All references to "confidence" in the earlier draft are replaced by "satisfaction level φ".

Assumption

The following notations (Tables 2-4) are used to complete the model formulation:

Table 2. Description index notation

Set	Description
i	Origin of goods ($i = 1, 2 \ldots m$)
j	Destination of goods ($j = 1, 2 \ldots n$)
k	Objective function

Table 3. Parameter notations

Parameter	Description
a_i	Supply parameter
b_j	Demand parameter
S_i	The supply available at source i
D_j	The demand at destination j
c_{ij}	The factor parameter (cost, time, rate, ...) represents the unit price for transportation from origin i to destination j corresponding to objective Z_k
g_{ij}	Fixed charge parameter corresponding to objective Z_k
τ	Uncertain variable
Q	Uncertain distribution
ψ	Normal uncertain distribution
γ	Uncertain measure function

Table 3. (cont.)

Parameter	Description
e	Expected value of the parameter
σ	Standard deviation
ϕ	Confidence level
N	Satisfaction level of the decision maker
$\wp$	The general deviational variable for all objectives
Z_k	Objective value of the k^{th} goal
Z^*	The ideal objective of Z_k
w_k	The weight assigned to the k^{th} objective
$Q^{-1}(x)$	Inverse uncertain normal distribution
$(\overline{\Pi} - \underline{\Pi})$	Interval goals of costs
$\overline{\Pi}$	Upper value of interval goals
$\underline{\Pi}$	Lower value of interval goals
S^-	Negative ideal solution
S^+	Positive ideal solution

Table 4. Description of decision variable notation

Decision variable	Description
x_{ij}	The amount of goods transported from the origin to destination j
ξ_{ij}	Binary value (0 or 1) Binary variable (0 or 1) where $\xi_{ij} = \begin{cases} 0, & \text{if } x_{ij} = 0 \\ 1, & \text{if } x_{ij} > 0 \end{cases}$

3. Mathematical model

In this study, the authors aim to provide DM with a comprehensive framework for optimizing four important objectives in logistics and supply chain management.

• The first objective focuses on minimizing total transportation costs, including both variable costs and fixed costs.

• The second objective aims to minimize damage costs, considering both variable costs and fixed costs.

• The third objective is to reduce transportation time, ensuring faster delivery of goods.

• Ultimately, the authors want to reduce the deterioration rate of goods while maintaining their quality during transit.

By addressing these four goals together, decision makers can increase the efficiency, cost-effectiveness, and reliability of their supply chain operations, thereby improving overall performance and customer satisfaction.

Model I: Multi-Objective Transportation Problem.

MOTPs consider multiple, often conflicting, objectives in transportation and logistics (shown in Figure 4). Here, the authors take an in-depth look at the types of objectives, advanced solution methods, real-world applications, and a more comprehensive example. DM involvement in the MOTPs is important to ensure that the chosen solution is consistent with strategic goals and practical constraints. By carefully defining objectives, setting priorities, and analysing trade-offs, the decision-maker can effectively navigate the complexities of multi-objective optimization to make informed and balanced decisions. This model and its solutions provide a framework for balancing multiple criteria in transportation logistics, allowing DM to evaluate trade-offs and choose the most appropriate transportation plans. Focuses solely on optimizing

multiple conflicting objectives, such as minimizing transportation cost, time, or environmental impact. The cost function considers only variable costs based on transportation volume. The decision variable x_{ij} represents the quantity of goods transported from the source i to destination j.

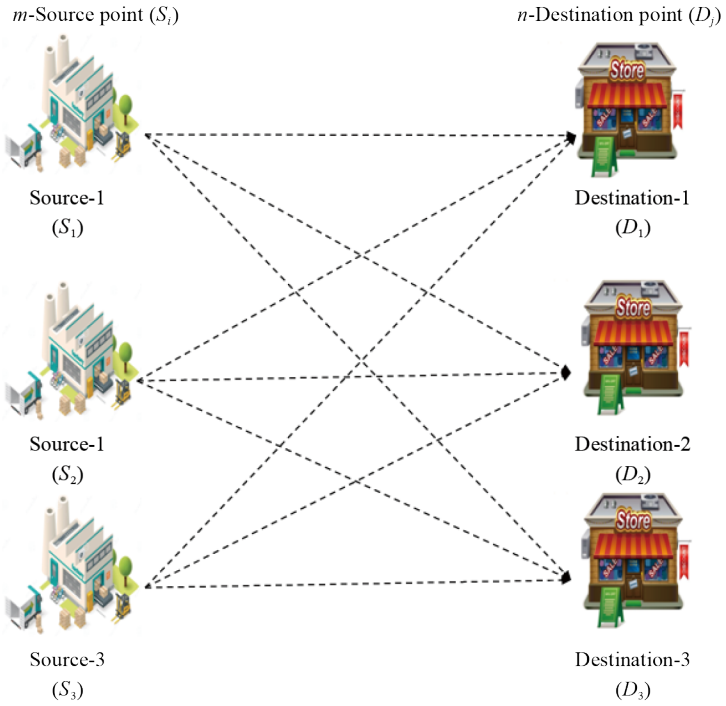


Figure 4. Visualization MOTP

Minimize/Maximize

$$Z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{k} x_{ij}. \tag{1}$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} = a_i. \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} = b_j. \tag{3}$$

$$x_{ij} \geq 0 \ \ \forall\, i = 1, 2 \ldots m, \ \ j = 1, 2 \ldots n.$$

The feasibility condition is $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. Minimization of transportation cost

• Z_1 $(Minimize\ Z_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{(1)} x_{ij})$.

• Damage costs Z_2 $(Minimize\ Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{(2)} x_{ij})$.

• Transportation time Z_3 $(Minimize\ Z_3 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{(3)} x_{ij})$.

• Deterioration rate Z_4 $(Minimze\ Z_4 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{(4)} x_{ij})$.

MOTPs are an extension of the classic TP where multiple, often contradictory, objectives need to be optimized simultaneously. This type of problem arises in various fields such as logistics, supply chain management, and resource allocation. Here x_{ij} represents the quantity of product and c_{ij} is the factor coefficient corresponding to the objective Z_k for the transportation of an item of commodities travelling from point i to j. Furthermore, the supply, i.e., available at source i and the necessary demand at the destination j are indicated by the variable a_i and b_j, respectively.

Model II: The Fixed Charge Multi-Objective Transportation Problem (shown in Figure 5).

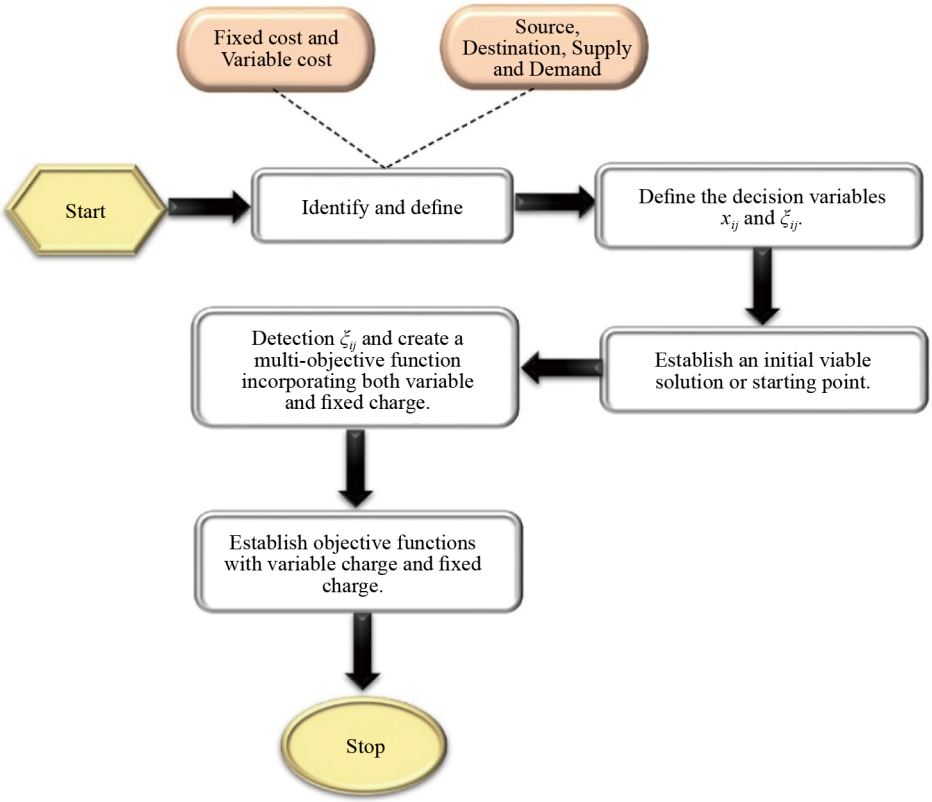


Figure 5. Flowchart: Solving FCMOTPs

The FCT challenge is a complicated distribution situation that requires effective connections between a network of m providers and n consumer. The ability for each supplier to ship to any customer exists, but each shipping route has two costs:

• A fixed cost g_{ij} for opening the route.

• A variable cost c_{ij} per unit of product transported from the supplier i to customer j.

The objective is to determine the optimal combination of open shipping routes and the corresponding shipment quantities along those routes, ensuring that the overall cost is minimized. This involves balancing the fixed and variable costs while meeting all supply and demand requirements.

FCTP formulation:

Minimize

$$Z = Z_{Var\backslash cost} + Z_{Fixed\backslash cost} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + g_{ij}\xi_{ij}. \tag{4}$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} = a_i\ , \quad \sum_{i=1}^{m} x_{ij} = b_j, \quad x_{ij} \geq 0 \quad \forall\, i = 1, 2 \ldots m, \quad j = 1, 2 \ldots n. \tag{5}$$

$$\begin{cases} \xi_{ij} = 0, & \text{if } x_{ij} = 0 \\ \\ \xi_{ij} = 1, & \text{if } x_{ij} > 0 \end{cases}.$$

$$x_{ij} \leq M_{ij}\, \xi_{ij}, \quad x_{ij} \geq \varepsilon\, \xi_{ij}, \quad \xi_{ij} \in \{0, 1\}\,, \quad (x_{ij} > 0).$$

Binary-continuous linking (big-M) and choice of M_{ij}:

To ensure correct fixed-charge modelling, the authors explicitly enforce the standard linking constraints between continuous flow variables x_{ij} and route-opening binaries ξ_{ij}:

$$x_{ij} \leq M_{ij}\, \xi_{ij} \forall i,\, j$$

and (optional, numerical stability)

$$x_{ij} \geq \varepsilon\, \xi_{ij} \forall i,\, j,$$

with $\xi_{ij} \in \{0, 1\}$.

Here $\varepsilon > 0$ is a small tolerance (e.g. 10^{-6}) used only if the solver requires a strict link. The authors choose a tight value for M_{ij} from supply/demand bounds to reduce relaxation weakness:

$$M_{ij} = \min\{a_i, b_j\},$$

where a_i is supply at origin i and b_j is demand at destination j. Using this choice ensures x_{ij} cannot exceed physical supply/demand and reduces big-M induced numerical/relaxation issues. The authors added the constraints above to Model II and to the deterministic MIP instances solved.

The feasibility condition is $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. Balinski's [13] observation on the FCTPs is an important insight for solving the problem. The observation indicates that there is an optimal solution for the relaxed version of FCTPs, where the integer restrictions on ξ_{ij} are relaxed with a specific property:

$$\xi_{ij} = \frac{X_{ij}}{\min(a_i, b_j)}.$$

FCMOTPs extend the MOTPs by including a fixed charge associated with using each route in addition to variable costs dependent on the volume of transportation. Here x_{ij} represents the quantity of product, c_{ij} is the factor coefficient corresponding to the objective Z_k for transportation of a unit of product and g_{ij} fixed charge corresponding to the objective Z_k associated with using the route, vehicle, transportation cost, damage cost arriving to destination j from source i. the supply i.e., available at source i and the needed demand at destination j are represented, respectively, by a_i and b_j. This problem involves several objectives, such as minimizing transportation cost and time, and is formulated with continuous decision variables representing transportation volume and ξ_{ij} binary variables (0 or 1) indicating route usage. Where ξ_{ij} is defined as a function by $\xi_{ij} = 1$ if $x_{ij} > 0$ and otherwise $\xi_{ij} = 0$.

Minimize/Maximize

$$Z_k = Z_{Var/cost} + Z_{Fixed/cost} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij} + g_{ij}\xi_{ij}. \tag{6}$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} = a_i\ , \quad \sum_{i=1}^{m} x_{ij} = b_j, \quad x_{ij} \geq 0 \quad \forall\, i = 1, 2 \ldots m, \quad j = 1, 2 \ldots n. \tag{7}$$

$$\begin{cases} \xi_{ij} = 0, \text{ if } x_{ij} = 0 \\ \\ \xi_{ij} = 1, \text{ if } x_{ij} > 0 \end{cases}.$$

The feasibility condition is $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

Extends MOTP by incorporating fixed costs (e.g., setup costs for using a transportation route, vehicle or facility). The objective function includes both variable costs (proportional to transported quantity) and fixed costs (incurred when a route is used). Introduces an additional binary decision variable ξ_{ij} where

$$\begin{cases} \xi_{ij} = 0, & \text{if } x_{ij} = 0 \\ \\ \xi_{ij} = 1, & \text{if } x_{ij} > 0 \end{cases},$$

$\xi_{ij} = 1$ if a route is used $(x_{ij} > 0)$ and $\xi_{ij} = 0$ otherwise. The solution involves balancing multiple criteria in transportation logistics, allowing decision makers to evaluate trade-offs and choose the most appropriate transportation plans, considering both variable and fixed costs.

Model III: Proposed Method.

The model assigns weights to each objective, allowing prioritization based on relative importance. Objectives include reducing fixed charges, variable costs, and potentially other factors such as transportation time. To handle uncertainty, models incorporate robust or stochastic optimization techniques, ensuring that decisions are flexible to changes in parameters such as demand and cost. By balancing these elements, the model aims to find solutions that are close to optimal, adaptable to changing conditions, and capable of meeting diverse transportation objectives. This mathematical model for FCMOTPs with uncertainty and interval goals allows for a balanced and flexible approach to transportation optimization. By incorporating fixed charge and variable costs, weighting objectives, and adjusting uncertainty and lag goals, the model provides robust solutions that are close to optimal in complex, real-world scenarios.

Maximize

$$\frac{N}{\sqrt{\wp(1-w_k)}}. \tag{8}$$

Subject to:

$$N \leq \frac{\acute{\Pi} - \left(\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij} + g_{ij}\xi_{ij}\right)}{\left(\acute{\Pi} - \grave{\sqcup}\right)}. \tag{9}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k}x_{ij} + g_{ij}\xi_{ij} \leq Z_k^* + \sqrt{\wp(1-w_k)}. \tag{10}$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad \sum_{i=1}^{m} x_{ij} = b_j, \quad \sum_{k=1}^{K} w_k = 1, \quad 0 \leq w_k \leq 1 \quad \forall\, k = 1, 2 \ldots K. \tag{11}$$

$$0 \leq \mathfrak{N} \leq 1, \quad \wp > 0, \quad x_{ij} \geq 0 \quad \forall\, i = 1, 2 \ldots m, \quad j = 1, 2 \ldots n.$$

$$\begin{cases} \xi_{ij} = 0, & \text{if } x_{ij} = 0 \\ \\ \xi_{ij} = 1, & \text{if } x_{ij} > 0 \end{cases}.$$

The feasibility condition is $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. The proposed model for addressing FCMOTPs with uncertainty and interval goals integrates several key elements to optimize TP.

In this study, uncertainty is represented through interval targets rather than probability or Liu's uncertainty theory formalism. Parameters such as transportation costs, supply, and demand are expressed as interval values to capture the inaccuracy of real-world data. The term confidence level (ϕ) It is used as a DM satisfaction index to determine the degree of optimism or conservatism in interval assessment, and not as a probability measure. Therefore, the proposed model follows an interval-based MOTP framework under uncertainty.

3.1 *Steps to solve the proposed model (shown in Figure 6)*

Step 1: Start.

Step 2: Identify Primary Objectives—Define the primary objectives for the transportation problem: reduce fixed costs and variable costs, reduce damage costs, minimize transportation time and deterioration rate.

Step 3: The normal distribution (N), expected value (e), standard deviation (σ), and confidence level (ϕ) selected by the DM may all be used to determine particular values for indeterminate parameters.

Step 4: Create a MOTPs use the precise data that were acquired in Step 3—Develop a MOTPS with the determined crisp values.

Step 5: Develop the Model II—formulate the second model (details of Model II will be specified in Section 3).

Step 6: Find the optimal value of each objective Z_k^*: Determine the optimal value for each defined objective.

Step 7: Assign weight to each objective—Assign specific weight to each objective based on its importance and alignment with DM priorities, then apply Model III.

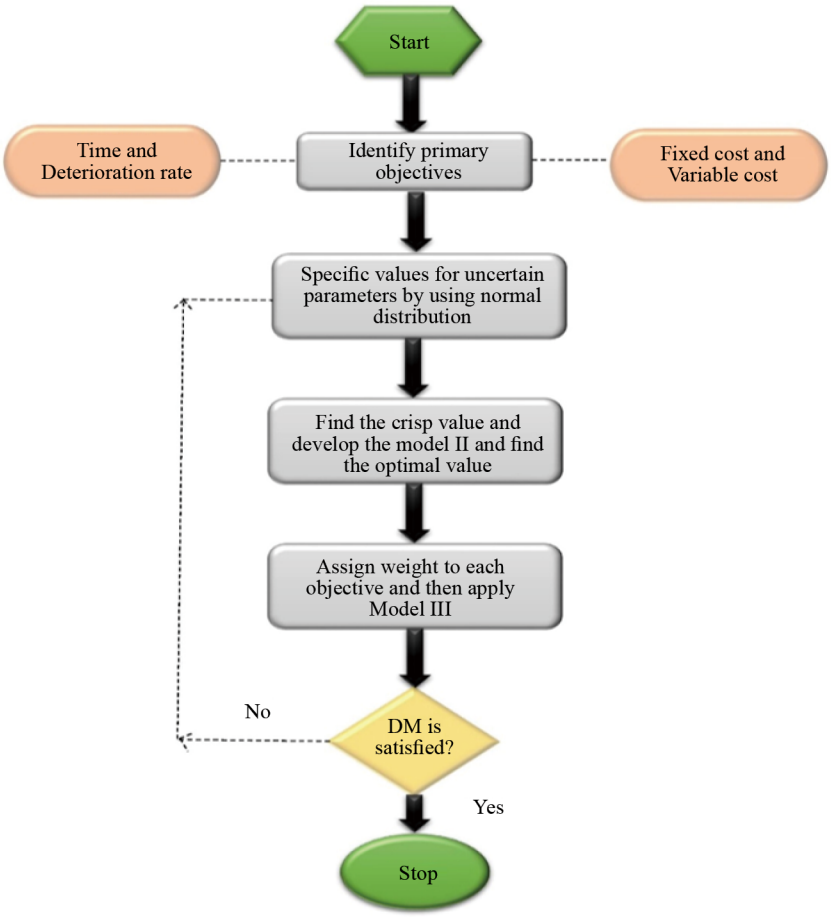


Figure 6. Flowchart: Proposed methodology

Step 8: Show Possible Solutions to DMs—Present potential solutions to the decision-makers. If they are satisfied, proceed to Step 9. If not, return to Steps 3-7 to refine the solutions until a compromise is reached. To avoid non-convexity artifacts and to guarantee nondominated outcomes, the authors generate the Pareto set using the ε-constraint method rather than relying on weight sweeping alone. The ε-constraint MIP skeleton (example: minimize objective $Z^{(1)}$ while bounding others) is:

For a chosen objective index k solve

$$\min_{x,\,\xi} Z^{(k)}(x,\,\xi)$$

$$\text{s.t. } Z^{(\ell)}(x,\,\xi) \le \varepsilon_\ell,\ \forall \ell \in \{1,\,\ldots,\,K\} \setminus \{k\}$$

flow conservation and capacity constraints

$$x_{ij} \le M_{ij}\xi_{ij},\quad \xi_{ij} \in \{0,\,1\},\quad x_{ij} \ge 0.$$

Here $Z^{(k)}$ denotes objective k (e.g. cost, damage, time, deterioration), $K = 4$ in this paper, and the ε_ℓ are upper bounds that we sweep over plausible ranges to sample the frontier.

Step 9: Evaluate Solutions Using the TOPSIS Method—Use the Technique for Order Preference by Similarity to Ideal Solutions TOPSIS method to generate and rank a list of the best alternatives for the TP. TOPSIS is applied solely to the nondominated pool returned by the ε-constraint sampling. No preference weights are used during Pareto generation, so preferences are not double-counted.

Step 10: Stop.

4. Illustrative scenario

A transportation company Trans Logistics Inc., is tasked with transporting goods from three manufacturing plants (S_1, S_2, S_3) to four regional distributions (D_1, D_2, D_3, D_4) centres. The company aims to optimize transportation planning based on multiple objectives to increase efficiency and reduce costs. The plants (source) are located in New York, Chicago and Los Angeles, while the distribution centres (destination) are in Houston, Miami, Atlanta and Denver. Trans Logistics Inc. has identified four key objectives for this transportation operation.

• The first objective Z_1 is to minimize overall transportation costs, which include both variable costs (such as fuel and labour) and fixed charges (such as tolls and handling fees at each destination).

• The second objective Z_2 is to reduce damage costs, which also include variable costs (such as damage during transit) and fixed charges (such as insurance premiums).

• The third objective Z_3 is to reduce transportation time to ensure timely delivery and increase customer satisfaction.

• The fourth objective Z_4 is to reduce the deterioration rate of perishable goods, which is important to maintain product quality during transit.

To achieve these objectives, Trans Logistics Inc. assign weights each objective based on their importance. Using a combination of robust optimization techniques and the TOPSIS methodology, Trans Logistics Inc. develops a MOTPs model. They incorporate uncertainty in demand and supply using general distribution parameters and develop a transportation plan that balances different objectives. After several iterations and adjustments based on feedback from the decision makers, they finalize a transportation plan that minimizes costs, minimizes losses, ensures timely delivery and maintains product quality. By implementing this optimized transportation plan, Trans Logistics Inc. not only increases its operational efficiency but also improves its service quality and customer satisfaction, leading to better overall performance and competitiveness in the logistics industry (show in Tables 5-10).

Table 5. Data for indeterminate (variable and fixed cost) transportation cost

	D_1	D_2	D_3	D_4
S_1	(2, 5), (36, 37)	(2, 7), (41, 43)	(4, 7), (37, 39)	(4, 9), (38, 39)
S_2	(6, 9), (41, 42)	(6, 11), (41, 43)	(7, 12), (42, 45)	(8, 13), (30, 32)
S_3	(10, 13), (31, 32)	(10, 15), (30, 33)	(11, 16), (35, 36)	(13, 16), (42, 43)

Table 6. Data for indeterminate (variable and fixed cost) damage cost

	D_1	D_2	D_3	D_4
S_1	(1, 6), (35, 38)	(3, 6), (40, 44)	(3, 8), (36, 40)	(5, 8), (36, 40)
S_2	(5, 10), (40, 46)	(7, 10), (40, 46)	(8, 11), (40, 46)	(9, 12), (29, 33)
S_3	(9, 14), (29, 33)	(11, 14), (29, 34)	(12, 15), (40, 46)	(12, 17), (41, 44)

Table 7. Data for indeterminate transportation time

	D_1	D_2	D_3	D_4
S_1	(2, 10)	(3, 10)	(1, 10)	(4, 10)
S_2	(3, 30)	(2, 30)	(4, 30)	(5, 30)
S_3	(4, 20)	(1, 20)	(3, 20)	(2, 20)

Table 8. Data for indeterminate deterioration rate

	D_1	D_2	D_3	D_4
S_1	(8, 10)	(19, 11)	(10, 12)	(10, 14)
S_2	(12, 14)	(13, 15)	(13, 17)	(14, 18)
S_3	(4, 7)	(4, 5)	(8, 10)	(8, 19)

Table 9. Data for indeterminate demand

b_1	b_2	b_3	b_4
(4, 6)	(7, 10)	(10, 15)	(6, 9)

Table 10. Data for indeterminate supply

a_1	a_2	a_3
(7, 8)	(9, 11)	(16, 19)

Now we considered 0.55 confidence interval and we get a crisp value of these objective function show in Tables 11-16.

Table 11. Data crisp value for (variable and fixed cost) transportation cost

	D_1	D_2	D_3	D_4
S_1	2.553178, 40.09352	2.774449, 45.75733	4.774449, 41.31479	4.99572, 42.31479
S_2	6.99572, 45.64669	7.216991, 45.75733	8.327627, 46.9786	9.438262, 33.54034
S_3	11.43826, 34.54034	11.65953, 33.65097	12.77017, 38.98288	14.77017, 46.75733

Table 12. Data crisp value for (variable and fixed cost) damage cost

	D_1	D_2	D_3	D_4
S_1	1.663813, 39.20415	3.663813, 44.86796	3.885084, 44.86796	5.885084, 45.08924
S_2	6.106356, 45.08924	8.106356, 44.86796	9.216991, 45.08924	10.32763, 32.65097
S_3	10.5489, 32.65097	12.5489, 44.86796	13.65953, 45.08924	13.8808, 45.86796

Table 13. Data crisp value for transportation time

	D_1	D_2	D_3	D_4
S_1	3.106356	4.106356	2.106356	5.106356
S_2	6.319067	5.319067	7.319067	8.319067
S_3	6.212711	3.212711	5.212711	4.212711

Table 14. Data crisp value for deterioration rate

	D_1	D_2	D_3	D_4
S_1	9.106356	10.21699	11.32763	11.5489
S_2	13.5489	14.65953	14.8808	15.99144
S_3	4.774449	4.553178	9.106356	10.10208

Table 15. Data crisp value for demand

b_1	b_2	b_3	b_4
4.663813	8.106356	11.65953	6.99572

Table 16. Data crisp value for supply

a_1	a_2	a_3
7.885084	10.21699	18.10208

The model for the proposed problem is:

Maximize $\dfrac{N}{\sqrt{\wp(1-w_k)}}$.

Subject to:

$$604.08\mathfrak{N} \leq 1{,}847.6 - (11.15x_{11} + 8.57x_{12} + 10.01x_{13} + 11.04x_{14} + 16.78x_{21} + 12.85x_{22}$$

$$+12.92x_{23} + 14.22x_{24} + 18.84x_{31} + 15.80x_{32} + 16.11x_{33} + 21.45x_{34})$$

$$120.72\mathfrak{N} \leq 523.27 - (10.07x_{11} + 9.35x_{12} + 9x_{13} + 11.66x_{14} + 15.77x_{21} + 13.66x_{22}$$

$$+13.62x_{23} + 14.99x_{24} + 18.19x_{31} + 16.58x_{32} + 16.92x_{33} + 20.44x_{34})$$

$$46.24\mathfrak{N} \leq 168.85 - (3.1x_{11} + 4.1x_{12} + 2.1x_{13} + 5.1x_{14} + 6.31x_{21} + 5.31x_{22}$$

$$+7.31x_{23} + 8.31x_{24} + 6.21x_{31} + 3.21x_{32} + 5.21x_{33} + 4.21x_{34})$$

$$54.82\mathfrak{N} \leq 333.88 - (9.1x_{11} + 10.21x_{12} + 11.32x_{13} + 11.54x_{14} + 13.54x_{21} + 14.65x_{22}$$

$$+14.88x_{23} + 15.99x_{24} + 4.37x_{31} + 4.55x_{32} + 9.1x_{33} + 10.1x_{34})$$

$$11.15x_{11} + 8.57x_{12} + 10.01x_{13} + 11.04x_{14} + 16.78x_{21} + 12.85x_{22} + 12.92x_{23} + 14.22x_{24}$$

$$+ 18.84x_{31} + 15.80x_{32} + 16.11x_{33} + 21.45x_{34} \leq 1847.6 + \sqrt{\wp(1-w_k)}$$

$$10.07x_{11} + 9.35x_{12} + 9.00x_{13} + 11.66x_{14} + 15.77x_{21} + 13.66x_{22} + 13.62x_{23} + 14.99x_{24}$$

$$+ 18.19x_{31} + 16.58x_{32} + 16.92x_{33} + 20.44x_{34} \leq 523.27 + \sqrt{\wp(1-w_k)}$$

$$3.10x_{11} + 4.10x_{12} + 2.10x_{13} + 5.10x_{14} + 6.31x_{21} + 5.31x_{22} + 7.31x_{23} + 8.31x_{24}$$

$$+ 6.21x_{31} + 3.21x_{32} + 5.21x_{33} + 4.21x_{34} \leq 122.61 + \sqrt{\wp(1-w_k)}$$

$$9.10x_{11} + 10.21x_{12} + 11.32x_{13} + 11.54x_{14} + 13.54x_{21} + 14.65x_{22} + 14.88x_{23} + 15.99x_{24}$$

$$+ 4.37x_{31} + 4.55x_{32} + 9.10x_{33} + 10.10x_{34} \leq 279.06 + \sqrt{\wp(1-w_k)}$$

$$x_{11} + x_{12} + x_{13} + x_{14} = 7.88$$

$$x_{21} + x_{22} + x_{23} + x_{24} = 10.21$$

$$x_{31} + x_{32} + x_{33} + x_{34} = 18.10$$

$$x_{11} + x_{21} + x_{31} = 4.66$$

$$x_{12} + x_{22} + x_{32} = 8.10$$

$$x_{13} + x_{23} + x_{33} = 11.65$$

$$x_{14} + x_{24} + x_{34} = 6.99$$

$$0 \leq \mathfrak{N} \leq 1, \ \wp > 0$$

$$x_{ij} \geq 0 \ \ \forall \, i = 1, 2 \ldots m, \ \ j = 1, 2 \ldots n.$$

$$\sum_{k=1}^{K} w_k = 1, \quad 0 \leq w_k \leq 1 \ \ \forall\, k = 1, 2 \ldots K.$$

5. Result and decision

The proposed decision-maker-centric model was rigorously tested and compared with previous algorithms and several established methods, including Uddin's method, which uses LINGO 18 software (see Table 17). The comparison was visualized through a solution Table 17 using different weights given by $w_1 = (0.3, 0.2, 0.3, 0.2)$, $w_2 = (0.1, 0.4, 0.2, 0.3)$, $w_3 = (0.4, 0.1, 0.4, 0.1)$, $w_4 = (0.5, 0.0, 0.1, 0.4)$, and $w_5 = (0.1, 0.7, 0.2, 0.0)$.

Table 17. Detailed showcase of objective values across a range of confidence levels for desired goals

Φ	Weight	Proposed method				Joshi's model				Uddin's method			
		Z_1	Z_2	Z_3	Z_4	Z_1	Z_2	Z_3	Z_4	Z_1	Z_2	Z_3	Z_4
0.5	w_1	357.25	366.28	70.88	231.47	355.41	369.96	71.34	237.77				
	w_2	361.77	359.96	69.75	225.82	360.41	257.24	70.09	227.52				
	w_3	357.25	374.12	69.76	236.37	353.33	356.47	71.86	240.07	333	335	72	237
	w_4	363.05	354.68	67.96	219.88	362.28	347.94	69.43	224.22				
	w_5	357.6	368.68	71.18	232.97	356.05	374.15	74.3	245.15				
	Average	359.38	364.74	69.906	229.3	357.5	341.15	71.404	234.95	333	335	72	237
	Ideal	414	375	47	206	414	375	47	206	414	375	47	206
	Anti-Ideal	312	261	84	254	312	261	84	254	312	261	84	254
0.55	w_1	1,728.5	516.64	130.37	287.89	1,245.6	400.09	144.41	302.2				
	w_2	1,747.2	520.41	131.73	286.91	1,259.5	443.6	143.96	298.82				
	w_3	1,700.2	510.61	129.13	288.78	1,232.9	416.89	144.33	305.36	1,218.2	406.19	144.51	304.94
	w_4	1,788.7	530.14	133.09	285.93	1,273.4	427.11	146.22	299.85				
	w_5	1,607.5	491.17	129.97	288.17	1,241.5	399.06	144.76	303.66				
	Average	1,714.4	513.79	130.86	287.54	1,250.5	417.35	144.74	301.98	1,218.2	406.19	144.51	304.94
	Ideal	1,847.6	523.27	122.61	279.06	1,847.6	523.27	122.61	279.06	1,847.6	523.27	122.61	279.06
	Anti-Ideal	1,243.5	402.55	168.85	333.88	1,243.5	402.55	168.85	333.88	1,243.5	402.55	168.85	333.88
0.6	w_1	610.45	617.73	236.53	381.6	503.9	578.75	245.98	388.44				
	w_2	611.6	622.09	237.57	380.62	506.73	584.95	246.54	387.36				
	w_3	610.45	617.73	235.45	382.62	500.95	572.33	245.98	388.44	476.45	495.6	254.86	398.97
	w_4	605.95	636.07	238.69	376.56	509.78	591.61	253.77	386.98				
	w_5	621.89	631.08	236.17	381.94	502.91	576.61	252.79	396.8				
	Average	612.07	624.94	236.88	380.67	504.85	580.85	249.01	389.6	476.45	495.6	254.86	398.97
	Ideal	635.86	631.2	221.67	365.8	635.86	631.2	221.67	365.8	635.86	631.2	221.67	365.8
	Anti-Ideal	433.42	467.51	276.81	427.11	433.42	467.51	276.81	427.11	433.42	467.51	276.81	427.11

Table 17. (cont.)

Φ	Weight	Proposed method				Joshi's model				Uddin's method			
		Z_1	Z_2	Z_3	Z_4	Z_1	Z_2	Z_3	Z_4	Z_1	Z_2	Z_3	Z_4
0.65	w_1	745.35	793.44	365.64	487.56	607.6	667.51	373.96	490.41				
	w_2	692.45	775.25	366.84	486.25	730.76	664.89	400.29	524.98				
	w_3	771.46	763.97	364.44	488.98	608.84	672.62	373.96	490.41	640.26	677.8	389.79	508.62
	w_4	696.55	676.42	367.96	484.82	605.11	668.82	381.38	490.61				
	w_5	745.35	775.25	365.24	488.03	735.24	766.71	372.72	491.15				
	Average	730.23	756.87	366.02	487.13	657.51	688.11	380.46	497.51	640.26	677.8	389.79	508.62
	Ideal	773.09	779.49	347.54	468.23	773.09	779.49	347.54	468.23	773.09	779.49	347.54	468.23
	Anti-Ideal	501.1	555.71	411.62	536.53	501.1	555.71	411.62	536.53	501.1	555.71	411.62	536.53
0.7	w_1	921.12	967.46	528.91	611.83	885.24	926.23	537.43	618.04				
	w_2	735.24	933.33	531.29	613.17	894.46	893.9	538.43	617.22				
	w_3	885.24	914.28	528.81	616.73	730.87	916.23	537.43	618.04	774.87	917.74	558.41	651.59
	w_4	739.46	947.71	532.49	611.44	726.51	959.45	544.9	615.99				
	w_5	916.92	927.34	529.65	615.53	729.47	899.92	547.5	621.32				
	Average	839.6	938.02	530.23	613.74	793.31	919.15	541.14	618.12	774.87	917.74	558.41	651.59
	Ideal	934	941	508.75	592.16	934	941	508.75	592.16	934	941	508.75	592.16
	Anti-Ideal	612.75	648.77	581.49	688.3	612.75	648.77	581.49	688.3	612.75	648.77	581.49	688.3
0.75	w_1	1,080.6	1,177.8	741.66	771.33	1,071.5	1,121	749.71	773.37				
	w_2	1,085.6	1,083	742.94	769.14	1,069	1,020.8	753.51	776.94				
	w_3	1,075.3	972.42	740.34	773.58	1,053.6	1,006.3	747.99	775.19	1,063.5	1,039	784.1	824.67
	w_4	1,188.8	1,176	737.47	763.02	1,122.2	1,059.5	767.97	783.44				
	w_5	1,188.8	976	741.22	772.08	1,118.5	1,043.3	760.47	781.39				
	Average	1,123.8	1,077	740.73	769.83	1,087	1,050.2	755.93	778.07	1,063.5	1,039	784.1	824.67
	Ideal	1,131.5	1,137.1	716.97	744.89	1,131.5	1,137.1	716.97	744.89	1,131.5	1,137.1	716.97	744.89
	Anti-Ideal	659.81	814.23	802.52	851.76	659.81	814.23	802.52	851.76	659.81	814.23	802.52	851.76
0.8	w_1	1,268.9	1,263.5	1,023.4	971.93	1,261.6	1,070.5	1,028.6	977.08				
	w_2	1,328.7	1,315.3	1,021	905.93	1,213.1	1,131.2	1,036.8	976.22				
	w_3	1,216.8	1,263.5	1,015.5	966.41	1,261.6	1,119.2	1,028.6	977.08				
	w_4	1,380.4	1,351.9	1,025.8	966.75	1,349.7	1,082.1	1,036.9	969.33	1,243.3	1,045.5	1,064.3	1,025.8
	w_5	1,366.9	1,362.3	1,013.1	972.98	1,260.1	1,068.5	1,043.8	986.15				
	Average	1,312.3	1,311.3	1,019.8	956.8	1,269.2	1,094.3	1,035	977.17	1,243.3	1,045.5	1,064.3	1,025.8
	Ideal	1,381.8	1,384.4	995.17	941.67	1,381.8	1,384.4	995.17	941.67	1,381.8	1,384.4	995.17	941.67
	Anti-Ideal	854.72	853.96	1,086.3	1,069.9	854.72	853.96	1,086.3	1,069.9	854.72	853.96	1,086.3	1,069.9

Table 17. (cont.)

Φ	Weight	Proposed method				Joshi's model				Uddin's method			
		Z_1	Z_2	Z_3	Z_4	Z_1	Z_2	Z_3	Z_4	Z_1	Z_2	Z_3	Z_4
0.85	w_1	1,385.3	1,399.6	1,357.5	1,334.6	1,275.3	1,279.6	1,457.5	1,340.6				
	w_2	1,385.3	1,399.6	1,357.5	1,334.6	1,275.3	1,279.6	1,457.5	1,340.6				
	w_3	1,385.3	1,399.6	1,357.5	1,334.6	1,275.3	1,279.6	1,457.5	1,340.6	1,169.1	1,173.5	1,474.9	1,346.5
	w_4	1,385.3	1,399.6	1,357.5	1,334.6	1,275.3	1,279.6	1,457.5	1,340.6				
	w_5	1,385.3	1,399.6	1,357.5	1,334.6	1,275.3	1,279.6	1,457.5	1,340.6				
	Average	1,385.3	1,399.6	1,357.5	1,334.6	1,275.3	1,279.6	1,457.5	1,340.6	1,169.1	1,173.5	1,474.9	1,346.5
	Ideal	1,719.6	1,724	1,389.5	1,324.6	1,719.6	1,724	1,389.5	1,324.6	1,719.6	1,724	1,389.5	1,324.6
	Anti-Ideal	909.54	913	1,491	1,343.5	909.54	913	1,491	1,343.5	909.54	913	1,491	1,343.5

In solution Table 17, the proposed model consistently outperformed others, producing results that were close to the optimal values across a variety of scenarios. This demonstrates the proposed method's efficacy and durability in dealing with uncertainty and achieving transportation goals. Detailed graphs (Figures 7-10) show how closely each method's findings match the ideal answer. The comparison focuses on four major objectives: transportation costs (both variable and constant), damage costs (both variable and fixed), transit time, and degradation rate. For each goal, we created a graph comparing the performance of the proposed methodology, past methods, Uddin's technique, ideal solutions, and suboptimal solutions.

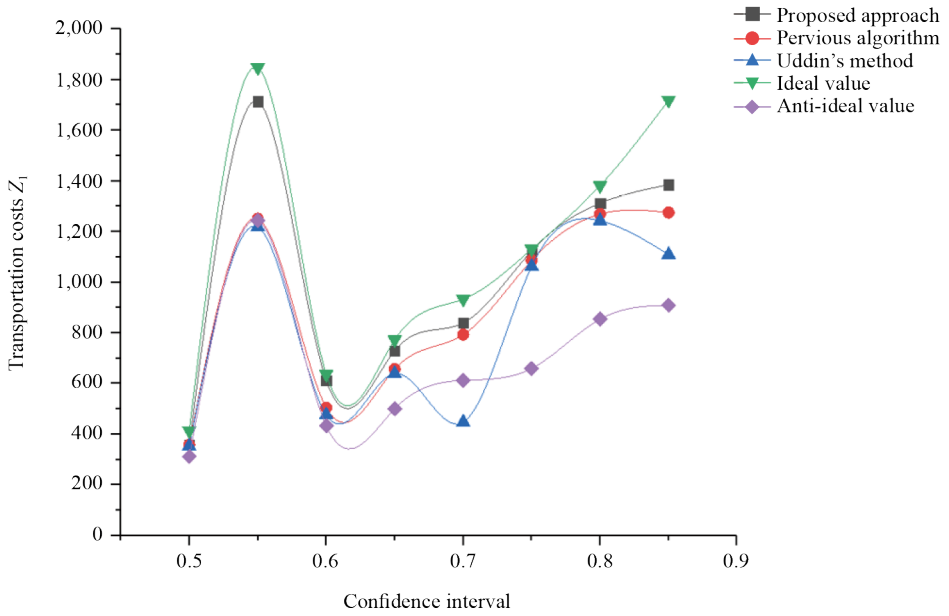


Figure 7. Comparison between transportation costs Z_1 and confidence level

This graph (Figure 7) shows the total transportation expenditures, including both the variable and fixed costs. Prior algorithms including Uddin's technique performed poorly compared to the results consistently produced by the suggested

model, which was the optimal option. From this, it is evident that this system successfully reduces transportation expenses overall.

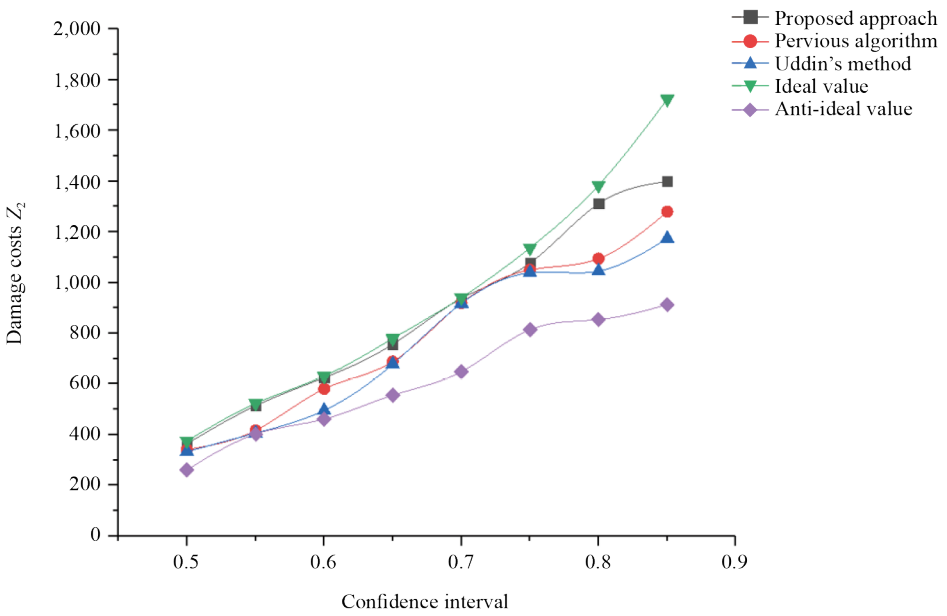


Figure 8. Comparison between damage costs Z_2 and confidence level

Figure 8 is a graph that highlights the damage costs, which account for both stationary and dynamic components. The proposed approach outperformed the competition and was very near to the best solution in another case. Consequently, this proves that the model is trustworthy when it comes to reducing costs related to shipping damage.

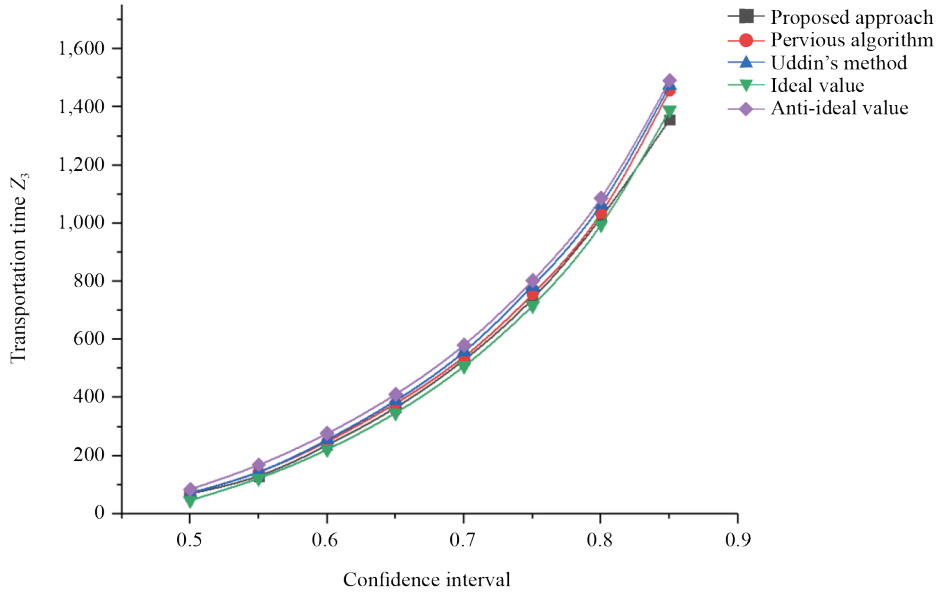


Figure 9. Comparison between transportation times Z_3 and confidence level

Figure 9 shows the curve representing the amount of time spent traveling. In contrast to other methods, the proposed model not only offered a more balanced approach, but it also approached the best solution quite closely. Ensuring timely delivery without compromising other obligations is made possible by maintaining this equilibrium.

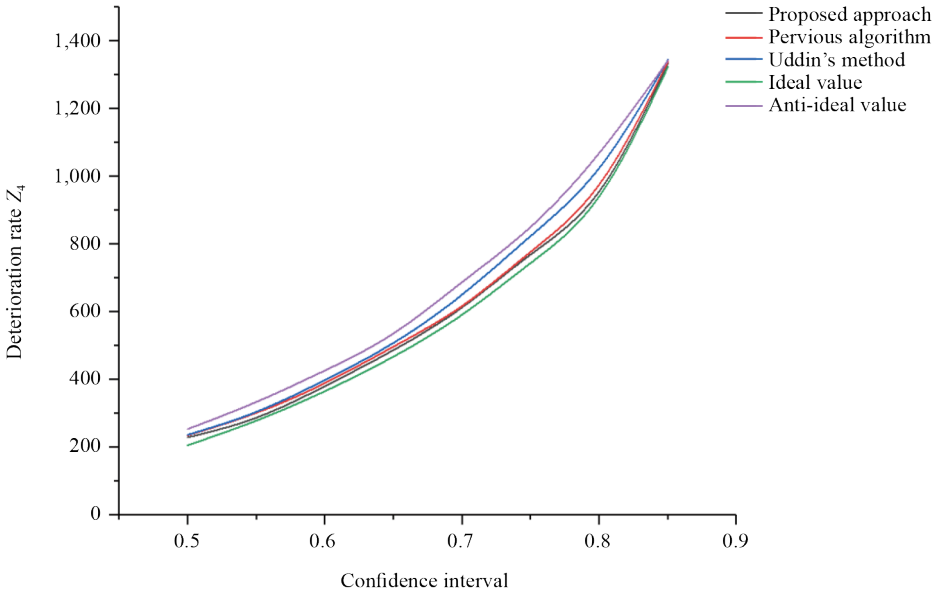


Figure 10. Comparison between deterioration rates Z_4 and confidence level

Figure 10 shows a graph showing the degradation rate. The fact that the suggested strategy achieved results that were close to the optimal solution indicated its efficiency in controlling the quality and condition of products throughout time. Visually comparing these four goals in great detail highlights the benefit of the proposed approach. Because of its closeness to the optimum answers in every graph, its effectiveness and reliability are highlighted. It is clearly shown in Figures 7-10 that the model is the one that is closest to the optimum value. Because of this, decision-makers are able to make quick and educated choices. The selections are informed by the TOPSIS technique, which provides a sophisticated rating system. Decision making is enhanced by this process. This technique ensures the greatest possible choice of alternatives by evaluating how closely each one is to the ideal response. Figure 11 shows that the outcome is a reliable and efficient decision-making process.

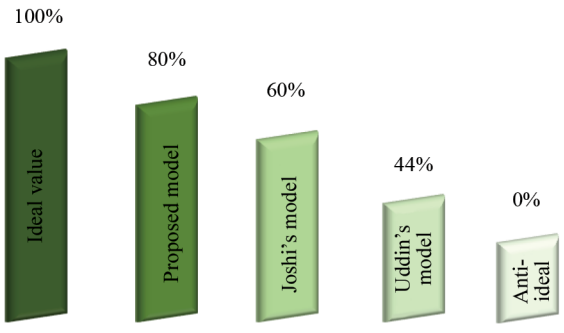


Figure 11. Comparison between the proposed method and existing methods

It is clear that the proposed method is closest to the optimal value, paving the way for an exceptionally promising alternative (shown in Figures 11-12).

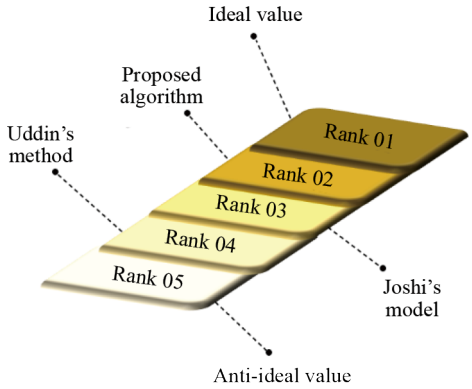


Figure 12. Ranking chart

Because it incorporates a variety of confidence intervals and weight assignments, the model also offers flexibility with various choices. The suggested model is a useful tool for DM in the TP, as this thorough comparison shows its dependability and efficiency. The model equips DM with the confidence and options necessary to handle complicated TP scenarios effectively by delivering a variety of solutions close to ideal value. To summarize, this comparison serves to confirm the effectiveness of the suggested approach and to demonstrate its real-world practicality. With the use of a potent tool, DM may streamline difficult decision-making processes, transforming uncertainty into strategic chances for transportation logistics optimization and success.

6. Conclusion

Finally, the proposed FCMOTP model integrates interval goals and weight assignments to address uncertainty and maximize numerous conflicting objectives. The proposed model is resilient and efficient, and its results are near to ideal solutions, making it useful for decision makers. The proposed methodology is flexible and adaptable to real-world circumstances since interval goals accommodate uncertainty and weight assignment prioritizes objectives. The proposed methodology benefits DM in logistics, supply chain management, and urban transportation planning. A retail corporation with a complex distribution network can use this model to optimize transportation routes, reduce costs, and ensure on-time delivery despite demand and cost fluctuations, and municipal authorities can use it to improve public transportation systems, reduce congestion, and reduce environmental impacts. Interval goals and weight assignment allow decision makers to adapt to changing circumstances, prioritize goals, and make quick, educated judgments. The model's flexibility, adaptability, and strategic optimization enable quick, informed decisions and better transportation and damage cost, time, and degradation rate management. This integrated methodology simplifies FCMOTPs into strategic possibilities, enabling DM to manage risks and achieve optimal results. The proposed TOPSIS-based research reveals that the paradigm benefits decision makers. It routinely rates higher than other methods, making uncertainty management and optimization planning easier.

Author contribution

Vishwas Deep and Priya: conceptualization, investigation, methodology and writing—original draft. Lenka and Huda: resources, investigation, validation, and writing—review and editing. Ajay Kumar: conceptualization, methodology,

investigation, supervision, and writing—review and editing. B. Swarna: conceptualization, supervision and writing—review and editing, visualization.

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Data availability statement

The authors declare that the data supporting the findings of this study are available within the paper.

Conflict of interest

The authors declare no competing financial interest.

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