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Research Article

Optimizing Multi-Commodity Fixed-Charge Solid Transportation Under Spherical Fuzzy Environment: A Multi-Objective Perspective

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Abstract: In today's dynamic and complex logistics networks, decision makers must simultaneously balance transporta-tion cost, delivery time, and service quality in multi-commodity systems involving fixed charges and uncertainty. This study proposes a novel multi-objective fixed-cost solid transportation problem formulated under the framework of spherical fuzzy sets, which effectively capture uncertainty in transportation costs, supplies, demands, and conveyance capacities. Although fixed costs are typically assumed to be constant in classical models, real-world factors such as fluctuating contractual conditions and administrative constraints often introduce indirect uncertainty, which is explicitly addressed in the proposed formulation. The model optimizes multiple conflicting objectives, including the minimization of transportation and fixed costs and delivery time, while simultaneously maximizing service reliability and customer satisfaction. An efficient solution methodology is developed to manage the resulting complexity. Numerical experiments and real-world case studies demonstrate that the proposed approach achieves an average reduction of approximately 15% in total transportation cost and an improvement of about 25% in computational efficiency compared with existing methods. These results highlight the robustness and effectiveness of the proposed framework in supporting reliable and informed decision-making. The methodology provides valuable insights for logistics, urban distribution, and supply chain systems seeking cost-effective and sustainable transportation solutions under uncertainty.

***Keywords*:** Multi-Objective Transportation Problem (MOTP), Spherical Fuzzy Numbers (SFNs), Solid Transportation Problem (STP), Fixed Charge Transportation Problem (FCTP), multi-commodity optimization, uncertainty environment

MSC: 90B06, 90C29, 90C70

Abbreviation

SFS	Spherical Fuzzy Set

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SFNs	Spherical Fuzzy Numbers
STP	Solid Transportation Problem
FCTP	Fixed Charge Transportation Problem
TP	Transportation Problem
MOTP	Multi-Objective Transportation Problem
TOPSIS	Technique for Order Preference by Similarity to Ideal Solution
MCDM	Multi-Criteria Decision-Making
FCSTP	Fixed Charge Solid Transportation Problem
DMs	Decision Makers
MOFCSTP	Multi-Objective Fixed Charge Solid Transportation Problem

1. Introduction

In operations research, the Transportation Problem (TP) is an optimization problem that focuses on the effective distribution and allocation of resources or goods from multiple supply locations to multiple demand sites to minimize transportation costs ($\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}$). This will show that m and n represent the number of sources and destinations, respectively. c_{ij} represents the cost of transportation from the source i to destination j. This sum covers all possible transportation routes. The supply points ($\sum_{i=1}^{m} a_i$), or locations where goods are available, the demand points ($\sum_{j=1}^{n} b_j$), or locations where goods are needed, the transportation cost, or price of sending a unit from the supply to the demand point, and decision factors (quantity of goods to be sent) are some of its essential elements. The total supply at each source (S_i) and the required demand (D_j) at each destination, constrain the problem, and non-negativity requirements ensure that deliveries cannot be negative. While in a balanced transportation problem, aggregate supply and aggregate demand are equal ($\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$) here, a_i denotes the supply at the source i, and b_j denotes the demand at the destination j, in an unbalanced problem ($\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$) modifications, such as adding dummy supply or demand points, may be necessary.

A short numerical example has now been added to illustrate this concept clearly. The ambiguity regarding "sum of supply points" and "sum of demand points" has been removed, and the mathematical formulation is now explicitly stated. Variants such as the Fixed-Charge Transportation Problem (FCTP) impose fixed costs for particular routes. In contrast, Multi-Objective Transportation Problems (MOTP) may have multiple competing objectives, such as minimizing environmental impact, time, and cost, along with the Solid Transportation Problem (STP). Defined as a transportation problem with three-Dimensional (3D) decision variables accounting for source, destination, and conveyance (or mode/layer), resulting in a solid (3D) structure. The transportation problem has many applications in logistics, supply chain management, urban planning, healthcare logistics, and humanitarian aid, and it provides an important basis for making resource distribution decisions. Hitchcock [1] introduced the TP, focusing on the optimal distribution of products from multiple sources to multiple destinations, laying the foundation for modern transportation and logistics optimization models. Koopmans [2] introduced the concept of optimal TP utilization, which focused on maximizing efficiency in the distribution of goods while minimizing transportation costs, which was a fundamental task in transportation economics and operations research. The traditional TP is extended by the MOTP, in which the distribution and allocation of resources from supply locations ($\sum_{i=1}^{m} a_i$) to demand points ($\sum_{j=1}^{n} b_j$) takes into account multiple competing objectives ($\sum_{k=1}^{K} Z_k$) at the same time. MOTP includes additional goals ($\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}$) such as reducing delivery time, increasing customer satisfaction, reducing environmental impact, or balancing resource use, in contrast to the single-objective TP, which only considers minimizing transportation costs. To find a solution that meets all these criteria at an acceptable level, compromises must be made because these goals often conflict. Multiple objectives and the need to balance them make the decision-making process in MOTP more complex. The general approach to solve this problem is to find a set of pareto-optimal and efficient solutions, where achieving one goal can only be at the expense of another. To solve MOTP, methods including fuzzy optimization, goal programming, weighted sum techniques, and evolutionary algorithms are often employed. Using these techniques, decision-makers can assess trade-offs and choose the best option based on their preferences or priorities. Applications for MOTP are numerous in real-world situations where multiple factors have to be taken into account simultaneously. Examples

include creating environmentally friendly supply chains, allocating resources to emergency relief efforts to maximize coverage and minimize delays, and optimizing logistics networks for cost and time efficiency. MOTP is an essential component of contemporary optimization and resource allocation due to its ability to capture the complexities of multi-criteria decision making. Recent advances in transportation problem modelling emphasize the use of fuzzy, intuitionistic, and neutrosophic techniques to manage ambiguity and multiple conflicting objectives. Abdel-Basset et al. [3] proposed a neutrosophic Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)-based framework for complex decision making. Bit et al. [4] developed a fuzzy programming approach to multi-objective STP, incorporating uncertainty into the objective functions and constraints, enabling more flexible and realistic decision making in transportation planning. Das et al. [5] addressed MOTP with interval cost, source, and destination parameters, incorporating uncertainty into cost structures and providing a more flexible approach to transportation optimization. Joshi et al. [6] proposed a fuzzy transportation planning approach using goal programming to address uncertainty and multi-objective decision making, which provides a structured method to optimize TP under uncertain conditions. Joshi et al. [7] explored multimodal routes for smart cities in their study, proposing an innovative TP that integrates multiple modes of transportation to improve urban mobility, sustainability, and efficiency in future urban environments. Kundu et al. [8] addressed the multi-objective multi-item solid transportation problem in uncertain environments, incorporating fuzzy logic to model uncertainty in cost and demand, offering a more flexible approach to optimizing transportation under uncertain conditions. A variation of the classical TP, the STP introduces more constraints and decision variables to account for the complexity of the real world. STP takes into account constraints $(\sum_{s=1}^{S} p_s)$ on transportation mode, storage capacity, and routing capacity, and involves multiple commodities, unlike the general transportation problem, which deals with delivering a single commodity from multiple supply locations to multiple demand points. This makes STP more important in situations where the flow of goods is further restricted by infrastructure, resources or operational conditions. STP typically aims to minimize the overall cost of transportation while meeting constraints such as supply availability at sources, destination demand, and capacity constraints on routes. When multiple commodities share a common supply, demand, or transportation network, the problem becomes more complex and requires solutions that balance the distribution of all commodities simultaneously. This complexity intensifies when adding real-world components such as time windows, fixed costs, or multi-objective criteria. In industries where it is necessary to effectively manage a variety of resources and goods, such as logistics, supply chain management, and industrial operations, STP is widely used. Examples include planning resource allocation in disaster relief efforts, managing supply chains for multiple products, and transporting various raw materials to manufacturing facilities. To address the difficulties presented by STP, a variety of solution approaches are used, including heuristic or metaheuristic algorithms, linear programming, and decomposition techniques. It is an important tool for contemporary optimization and resource management due to its ability to model complex, multi-commodity transportation systems. Dalman [9] proposed an uncertain programming model for the multi-item solid transportation problem, which addresses uncertainty in transportation costs and demand, offering a more adaptable framework for real-world logistics optimization. Haley [10] introduced new methods in mathematical programming to solve the STP, introducing techniques to optimize transportation costs while considering constraints related to product availability and demand. Jimenez and Verdegay [11] developed uncertain STP models that incorporate uncertainty in transportation costs and supply-demand parameters, improving the flexibility and accuracy of optimization in uncertain environments. Joshi et al. [12] applied neutrosophic decision making to eco-friendly logistics, focusing on fractional STP. Their approach addresses uncertainty in logistics decisions while promoting sustainable and efficient transportation solutions. Joshi et al. [13] focused on efficient healthcare delivery by addressing uncertainties through mixed-barriers partial transportation. Their approach optimizes resource allocation and transportation while managing uncertainties in healthcare logistics. Kundu et al. [14] extended the multi-item STP by incorporating type-2 fuzzy parameters, enabling the modelling of more complex uncertainties in transportation costs and demand, leading to more robust and flexible optimization solutions. The FCTP is a specialized form of the traditional TP that involves both variable charges proportional to the number of products transported and fixed costs associated with the use of transportation routes. This makes FCTP more realistic for simulate real-world conditions because many logistics and transportation systems have fixed startup expenses, including starting a shipment, renting a vehicle, or operating a route, regardless of the volume of transport. To meet supply and demand constraints, FCTP aims to minimize total costs, which include both fixed charges and variable transportation expenses. FCTP employs a binary decision variable

to determine whether the fixed cost of a transport route has been paid. Once a route is activated, the volume shipped on that route determines the variable cost. The problem becomes more computationally complex than the traditional transportation problem due to the introduction of non-linearity resulting from fixed charges. Therefore, solving FCTPs often requires complex optimization techniques such as branch-and-bound methods, decomposition approaches, mixed-integer programming, or metaheuristic algorithms such as simulated annealing or genetic algorithms. FCTP has many applications in supply chain management, network design, and logistics, all of which involve high fixed setup costs. For example, it can be used to model how products are distributed from warehouses to retailers, how transportation networks are planned, or how resources are allocated in large-scale industrial systems. In complex logistics and operational systems, FCTP provides a more accurate and useful framework for decision making, accounting for the fixed costs associated with transit routes. Balinski [15] introduced the fixed cost transportation problem, focusing on optimizing transportation networks with fixed costs. His work laid the foundation for later advances in mixed-integer programming and transportation optimization models. Gray [16] presented an exact solution method for the fixed-charge transportation problem, introducing optimization techniques that efficiently handle both variable and fixed costs in transportation network models. Ghosh and Roy [17] developed a multi-objective fully intuitionistic fuzzy fixed-charge solid transportation problem, which integrates intuitionistic fuzzy sets to model uncertainty in both cost and transportation parameters for better decision making. Hirsch and Dantzig [18] introduced the FCTP focusing on optimizing transportation and network design taking into account both fixed and variable costs, which had a significant impact on operations research and network optimization strategies. Roy et al. [19] tackled the multi-objective multi-item Fixed Charge Solid Transportation Problem (FCSTP) under twofold uncertainty, addressing both cost and demand uncertainties using advanced modelling techniques to optimize transportation and logistics decisions under uncertain conditions. Roy and Midya [20] explored the Multi-Objective Fixed Charge Solid Transportation Problem (MOFCSTP) with product blending under intuitionistic fuzzy environments, integrating uncertainty in both transportation costs and blending parameters to optimize complex logistics scenarios. Yang and Liu [21] developed a solution to the fuzzy FCTP, introducing an algorithm that incorporates fuzzy logic to model uncertainties in fixed and variable transportation costs, allowing decision making in uncertain environments. Zhang et al. [22] addressed the FCSTP problem in uncertain environments, proposing an algorithm that incorporates uncertainty in cost and demand, providing an efficient solution to optimize transportation in unpredictable scenarios. An extension of fuzzy set theory, Spherical Fuzzy Numbers (SFNs), aims to better manage ambiguity and uncertainty in complex decision-making situations. Truth, indeterminacy, and falsity are three membership components that are interrelated and bounded on the surface of a unit sphere, giving SFNs more flexibility than intuitionistic fuzzy sets and classical fuzzy sets (as shown in Table 1 and Figure 1). The introduction of this idea addresses the constraints of intuitionistic fuzzy numbers and traditional fuzzy numbers in representing fuzzy information. SFNs introduce an additional dimension to model uncertainty, making them ideal for applications requiring accurate modelling of complex uncertainties. Traditional fuzzy numbers employ a single membership degree, while intuitionistic fuzzy numbers incorporate non-membership degrees. The degree of veracity or closeness is represented by the membership (θ) value. Indeterminacy (δ) shows how uncertain or hesitant an individual is. The degree of falsity or non-relation is denoted by non-membership (φ). The constraint $\theta^2+\varphi^2+\delta^2 \leq 1$ ensures that their values lie within a unit region. Decision makers can more easily model hesitation or ambiguity by explicitly including uncertainty in SFNs, which is often ignored in simpler ambiguous systems. SFNs are particularly helpful when dealing with optimization problems, Multi-Criteria Decision Making (MCDM), and situations where interactions among truth, falsity, and uncertainty are important. Upon reaching $\varphi=0$, the SFN becomes an intuitionistic fuzzy number. It becomes a conventional fuzzy number when $\delta=1-\theta$ and $\varphi=0$. In domains such as logistics and transportation, rounded fuzzy numbers have become popular for modelling uncertainty in transportation capacities and costs. They are applied in supply chain management to evaluate supplier performance under unexpected circumstances, as well as in healthcare decision-making to assess treatment plans amid unclear or insufficient data. They are also utilized in engineering and optimization for challenges involving risk assessment and resource allocation. With varying degrees of hesitation and ambiguity, group decision-making is used to compile expert viewpoints. An important development in fuzzy set theory is represented by spherical fuzzy numbers, which provide a strong mathematical foundation for solving real-world problems with inherent uncertainties. Because they can represent truth, uncertainty, and falsity on a spherical surface, they are a flexible tool for a range of optimization and decision-making applications. Ahmed and Adhami [23] introduced spherical fuzzy multi-objective linear programming,

which enhances uncertainty modelling with membership, non-membership, and hesitation values, offering a flexible approach to decision making in complex optimization problems. Chakraborty [24] conducted a comparative analysis of TOPSIS and modified TOPSIS, highlighting their strengths and weaknesses in solving multi-criteria decision-making problems while providing insights into their applicability and performance. Gündoğdu and Kahraman [25] introduced a novel fuzzy TOPSIS method using interval-valued Spherical Fuzzy Set (SFS) to enhance decision making by incorporating uncertainty and hesitation in multi-criteria decision problems for improved accuracy. Kahraman and Gündogdu [26] explored decision making with SFS, presented an advanced method for addressing uncertainty and hesitation in MCDM problems to yield more accurate and reliable decision results. Gündoğdu and Kahraman [27, 28] focused on SFSs and their applications in decision making, providing a robust framework for dealing with uncertainty and imprecision in complex multi-criteria decision problems. In another study, Gündoğdu and Kahraman [27] examined the properties and arithmetic operations of SFSs, providing a comprehensive analysis of how these sets can be manipulated to improve decision making under uncertainty across various domains. Furthermore, Gündoğdu and Kahraman [29] applied the interval-valued SF Analytic Hierarchy Process (AHP) to hospital performance assessment, integrating fuzzy sets to evaluate and rank hospitals based on multiple criteria under uncertainty, thereby improving healthcare management decision-making. Gündoğdu and Kahraman [30] introduced a novel VIKOR method using SFSs for decision making, applying it to warehouse site selection to reduce uncertainty and improve the ranking of potential sites based on multiple conflicting criteria (shown in Table 2). These contributions demonstrate that supply chain optimization and modern transportation are moving toward more adaptable, uncertain, and sustainable methods.

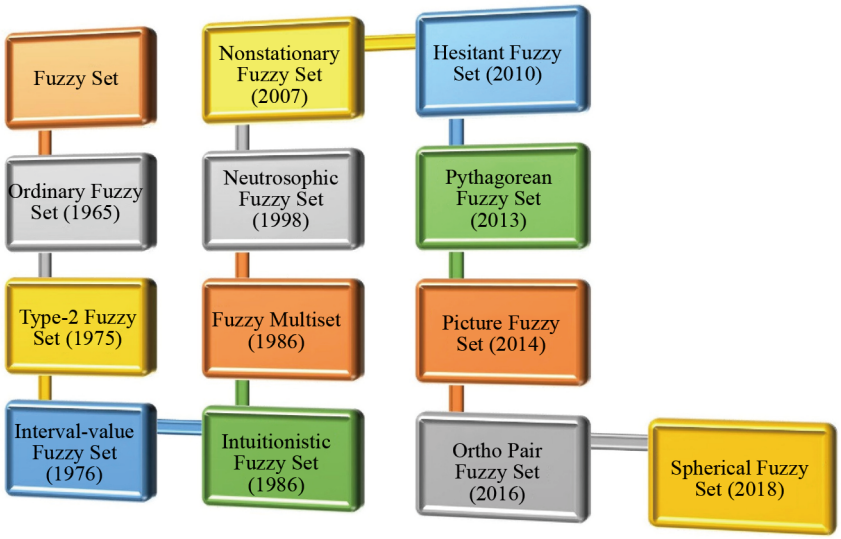


Figure 1. Development of fuzzy sets: timeline and characteristics

Table 1. Introduction to different types of fuzzy sets and their essential characteristics [26]

Fuzzy set concept	Introduced by	Year	Key-idea
Ordinary Fuzzy Set	L. A. Zadeh	1975	Extends classical set theory by modelling uncertainty with membership degrees ranging from 0 to 1.
Type-2 Fuzzy Set	L. A. Zadeh	1975	The ambiguity of membership values themselves embodies more complex uncertainty in real-world situations.
Interval-value Fuzzy Set	L. A. Zadeh, Grattan-Guiness	1976	Membership values are not exact prices; Rather, they are intervals that represent the range of uncertainty.

Table 1. (cont.)

Fuzzy set concept	Introduced by	Year	Key-idea
Intuition Fuzzy Set	Krzysztof Atanassov	1983	Introduces degrees of membership and non-membership as symbols of unclear, inadequate, or conflicting information.
Fuzzy Multiset	Yager	1986	Fuzzy sets are extended, allowing an element with a fuzzy membership value to appear more than once.
Neutrosophic Fuzzy Set	Smarandache	1995	Provides a comprehensive perspective on uncertainty by modelling truth, uncertainty, and falsehood.
Nonstationary Fuzzy Set	Garibaldi	1997	Membership functions are dynamic, time-varying uncertainties that vary with space or time.
Hesitant Fuzzy Set	Torra	2010	Models' uncertainty in decision making by allowing hesitation between multiple membership values.
Pythagorean Fuzzy Set	Yager	2013	Intuitionistic fuzzy sets are enhanced by guaranteeing that the sum of class membership and non-membership is less than 1.
Picture Fuzzy Set	Cuong	2015	Shows positive, neutral, and negative membership levels to model multidimensional or conflicting phenomena.
Ortho Pair Fuzzy Set	Yager	2016	Represents uncertainty in uncertain contexts by modelling both positive and negative membership values.
Spherical Fuzzy Set	Gundogdu and Kahraman	2018	To deal with increased ambiguity, truth, falsity, and uncertainty are modelled geometrically using spherical relationships.

Table 2. Comparative analysis of existing literature with the proposed study

Reference	Year	Objective	Environment	STP	FCTP	FCSTP	Multi-item	Weight	Solution technique
Ahmad and Adhami [23]	2021	2	Spherical Fuzzy	✓				✓	Linear Programming
Bit et al. [4]	1993	2	Fuzzy	✓				✓	Fuzzy Programming
Das et al. [5]	1999	3	Interval Based	✓				✓	Multi-Objective Optimization
Dalman [9]	2018	3	Uncertainty	✓			✓	✓	Uncertain programming
Gray [16]	1971	1	Deterministic		✓				Exact Solution
Haley [10]	1962	1	Deterministic	✓			✓		Exact Method
Hirsch and Dantzig [18]	1968	1	Deterministic		✓				Optimization Model
Jiménez and Verdegay [11]	1998	1	Fuzzy	✓			✓		Fuzzy Programming
Joshi et al. [6]	2023	3	Uncertainty					✓	Fuzzy Goal Programming
Joshi et al. [7]	2024	3	Uncertainty					✓	Multi-Model Optimization
Joshi et al. [12]	2023	3	Neutrosophic	✓	✓	✓		✓	Neutrosophic Decision Making
Kahraman and Gündogdu [26]	2021	1	Spherical Fuzzy						Decision-Making Model
Kutlu et al. [27]	2023	1	Spherical Fuzzy						AHP
Kundu et al. [8]	2013	2	Fuzzy	✓			✓		Fuzzy Optimization
Roy et al. [19]	2019	3	Uncertainty	✓	✓	✓	✓		Multi-Objective Optimization
Roy and Midya [20]	2019	2	intuitionistic fuzzy	✓	✓	✓			Intuitionistic Fuzzy Programming
Yang and Liu [21]	2007	2	Fuzzy	✓	✓	✓			Fuzzy Optimization
This paper		3	Spherical Fuzzy	✓	✓	✓	✓	✓	Spherical Fuzzy Programming

Our proposed approach focuses on using a sophisticated framework to solve the MOTP while considering multiple products or commodities. This framework combines elements of the STP and the FCTP. With this method, it is aimed to tackle real-world complexity while addressing a variety of goals, such as reducing overall costs, minimizing environmental impact, or increasing delivery efficiency, in addition to practical limitations such as fixed cost, multi-item distribution, and routing capabilities. To deal with the fundamental complexities of real transportation systems, the authors have used a circular fuzzy optimization framework with a ranking approach to guide decision makers toward ideal solutions that balance multiple, often incompatible goals. The use of SFS constitutes an important advancement in our methodology to address ambiguity and uncertainty in decision making. By extending regular fuzzy sets, SFS enable decision-makers to incorporate uncertainty and ambiguity into their assessments through a more adaptable representation of uncertainty. This is particularly useful when transportation characteristics, such as costs, demand, or capacities, are unpredictable or imprecise. A more thorough investigation of ambiguous data is made possible by this sophisticated fuzzy modelling, which reflects the discontinuity of membership and non-membership degrees separately. A weighted aggregation technique is used to assign relative importance to multiple objectives, according to the preferences of decision makers, such as maximizing resource use, minimizing environmental impact, improving service quality, or minimizing transportation costs. This ranking system provides clear direction to decision makers, helping them choose solutions that best fit strategic priorities and successfully handle trade-offs between conflicting goals. Our proposed approach provides a realistic, adaptable, and structured model that accounts for the intrinsic complexity of contemporary transportation systems, thus improving decision-making. This approach combines spherical fuzzy logic, fixed cost, concrete transportation theory, and MOTP to help decision makers make more informed and efficient choices in complex and uncertain situations. This methodology provides a systematic tool for them to identify solutions that closely match their strategic goals and priorities, while also capturing the complexity and unpredictability of real-world transportation networks. Our proposed methodology combines MOTP, FCTP, SFS, STP, and a ranking strategy to provide a powerful tool for decision makers in various industries. This method helps in strategic planning in unpredictable and changing circumstances, while also enhancing operational efficiency and cost-effectiveness. It also has many real-world applications across various fields. It can be used in supply chain optimization to balance delivery time, cost, and environmental factors to create effective multi-item distribution networks. This method assists in the allocation of resources such as food, gasoline, or medical supplies in urban transportation planning, considering route capacity and fixed costs. This approach is particularly helpful in humanitarian logistics, where it can maximize coverage and reduce delivery times by optimizing the delivery of critical goods such as food, water, and medical equipment to disaster-affected areas. Additionally, this approach is well-suited for retail and e-commerce, allowing cost-effective distribution of multiple items while increasing customer satisfaction and ensuring on-time delivery. Furthermore, in energy distribution networks, it can optimize the movement of different energy supplies, including gas, coal, and renewable energy, between regions while considering environmental impact and fixed setup costs. The proposed strategies help decision makers find practical, economical, and sustainable solutions for transportation systems by addressing the complexities and unpredictability of these situations. Multi-Objective Solid Transportation Problems (MOSTPs) are essential in logistics systems involving multiple commodities, fixed charges, and capacity constraints. In practice, MOSTPs are difficult to solve due to conflicting objectives and pervasive uncertainty in transportation costs, supplies, demands, and operational conditions. Classical deterministic models are often inadequate for capturing such uncertainty. Spherical Fuzzy Sets (SFSs), which incorporate membership, non-membership, and hesitancy degrees, provide a flexible modeling framework but introduce additional computational and methodological challenges, particularly when combined with fixed-cost and solid transportation structures. To address these issues, this study proposes a novel multi-objective fixed-cost solid transportation model under a spherical fuzzy environment. The main contributions include realistic uncertainty modeling, an efficient multi-objective solution framework, and numerical validation demonstrating improved cost efficiency and computational performance.

2. Preliminaries

Definition 2.1 [1] (show in Figure 2) A compromise optimal solution to a multi-objective optimization problem is a feasible solution $x^c \in X$. That balances conflicting objectives $g_1(x)$, $g_2(x)$, …, $g_k(x)$. Minimize the deviation from the ideal solution $g^* = (g_1^*, g_2^*, \ldots, g_k^*)$. The ideal solution represents the theoretical best value for each objective, often unattainable due to trade-offs or constraints. In contrast, an optimal solution minimizes or maximizes a single objective function. $g(x)$ in a single objective optimization problem.

Definition 2.2 [1] (show in Figure 2) An ideal solution refers to the best achievable result in the objective space, where all objectives $g_1(x)$, $g_2(x)$, …, $g_k(x)$. They are optimized together without any compromise. It represents the point $g_{ideal} = (\min_{x\in X} g_1(x), \min_{x\in X} g_2(x), \ldots, \min_{x\in X} g_k(x))$, assuming X. Is the feasible set. This solution achieves the minimum or maximum, depending on the problem value for each objective. However, in practice, the ideal solution is usually unattainable due to conflicting objectives or resource limitation, as improvement in one parameter often outcomes in a trade-off with others.

Definition 2.3 [1] (show in Figure 2) An anti-ideal solution refers to the worst possible outcome in the objective space, where all objectives $g_1(x)$, $g_2(x)$, …, $g_k(x)$ are simultaneously maximized or minimized depending on the problem to their worst possible values. This point represents $g_{anti-ideal} = (\max_{x\in X} g_1(x), \max_{x\in X}\ g_2(x), \ldots, \max_{x\in X} g_k(x))$, assuming that X. It is a feasible set. In contrast to the ideal solution, the anti-ideal solution serves as a reference to evaluate and compare the performance of other feasible solutions, highlighting the range of possible outcomes within the objective space.

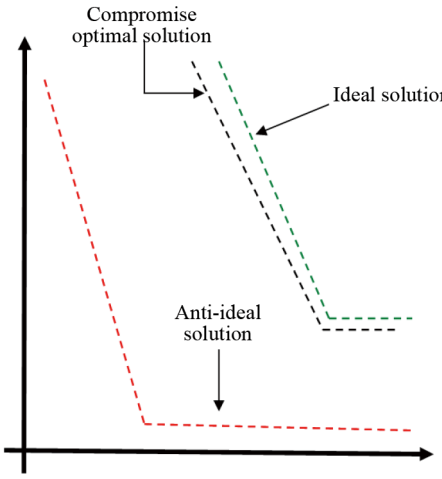


Figure 2. Representation of compromise optimal solution, ideal and anti-ideal solution

Definition 2.4 [1] (show in Figure 3) Pareto dominance describes the relationship between two solutions $x_1 \in X$ and $x_2 \in X$ in a MOTP. A solution x_1 is said to dominate x_2 (denoted $x_1 < x_2$) if only if $g_i(x_1) \leq g_i(x_2)$, $\forall i \in \{1, 2, \ldots, n\}$ and $\exists j \in \{1, 2, \ldots, n\}$, $g_j(x_1) < g_j(x_2)$. This means that x_1 is no worse than x_2 in all objectives and is strictly better in at least one objective, making it better than x_2.

Definition 2.5 [1] (show in Figure 3) The "Pareto-optimal solution" describes a scenario in which further improvements in one criterion would have a detrimental effect on at least one other criterion. This means that every change made to improve one part of the solution will make another aspect worse. If there is no other solution, a solution $x^* \in X$ is Pareto optimal. If $\nexists\, x \in X$ such that $x \leq x^*$, then the $x \in X$ that dominates x^* is the Pareto optimal choice. Otherwise stated, it is a compromise between conflicting objectives, in which enhancing one characteristic cannot be achieved without compromising the other.

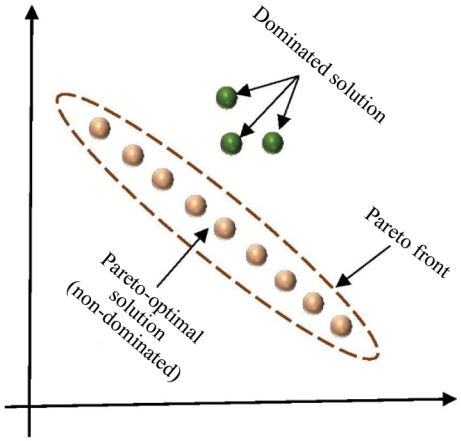


Figure 3. Reorientation of pareto front, optimal and dominated solution

Definition 2.6 [1] (show in Figure 3) The pareto front is the set of objective function values corresponding to the Pareto-optimal solution in a MOTP. It represents the limit in objective space where no improvement in one objective can be made without worsening the other. Mathematically, the Pareto front is defined as $F_{Pareto} = \{f(x)|x \in X_{Pareto}\}$. Where: $X_{Pareto} = \{x^* \in X|\nexists\, x \in X \text{ such that } x < x^*\}$. These solutions reflect trade-offs between conflicting objectives, offering a set of non-dominated options to decision makers.

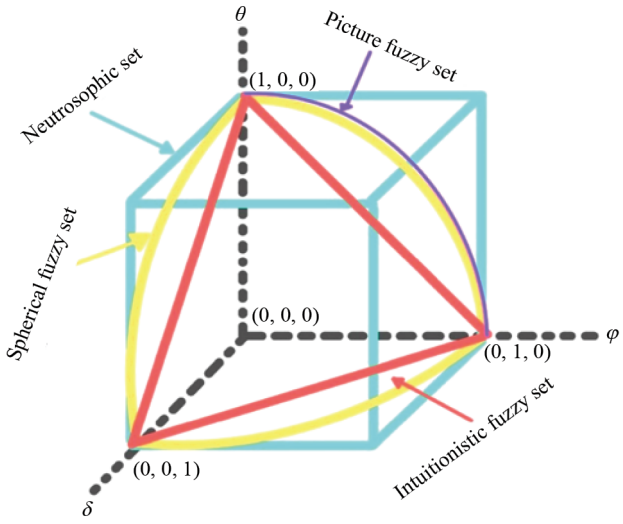


Figure 4. Representation of SFS

Definition 2.7 [26] (show in Figure 4) A SFS $\mathscr{H}^{sf}$ is defined as follows, let $\varkappa$ be a universe of discourse. The universe of discourse is one-dimensional, defined over real numbers $\mathbb{R}$, where each element is associated with a spherical fuzzy membership triplet (μ, ν, π). A SFS $\mathscr{H}^{sf}$ in $\varkappa$ is expressed as: $\mathscr{H}^{sf} = \{\langle x, \theta_{\mathscr{H}^{sf}}(x), \varphi_{\mathscr{H}^{sf}}(x), \delta_{\mathscr{H}^{sf}}(x)\rangle\,|x \in \varkappa\}$.

Where:

- $\theta_{\mathscr{H}^{sf}}(x) : \varkappa \to [0, 1]$ is the membership degree of $x \in \mathscr{H}^{sf}$.
- $\varphi_{\mathscr{H}^{sf}}(x) : \varkappa \to [0, 1]$ is the non-membership degree of $x \in \mathscr{H}^{sf}$.
- $\delta_{\mathscr{H}^{sf}}(x) : \varkappa \to [0, 1]$ is the indeterminacy (hesitancy) degree of $x \in \mathscr{H}^{sf}$.

This value must satisfy the SF constraint:

$$\theta_{\mathscr{H}^{sf}}(x)^2 + \varphi_{\mathscr{H}^{sf}}(x)^2 + \delta_{\mathscr{H}^{sf}}(x)^2 \leq 1 \; \forall \; x \in \varkappa$$

This condition ensures that the three degrees represent valid ratios within the range [0, 1] and collectively describe the uncertainty or ambiguity associated with the element x.

• $\theta_{\mathscr{H}^{sf}}(x)^2 + \varphi_{\mathscr{H}^{sf}}(x)^2 + \delta_{\mathscr{H}^{sf}}(x)^2 \leq 1 \; \forall \; x \in \varkappa$ is defines a spherical boundary in the 3D space of membership (θ), non-membership (φ) and indeterminacy (δ) values.

• $\forall x \in \varkappa$, the sum $\theta_{\mathscr{H}^{sf}}(x) + \varphi_{\mathscr{H}^{sf}}(x) + \delta_{\mathscr{H}^{sf}}(x)$ can be less than or equal to 1 (but not necessarily equal to 1).

When

• $\delta_{\mathscr{H}^{sf}}(x) = 1 - (\theta_{\mathscr{H}^{sf}}(x) + \varphi_{\mathscr{H}^{sf}}(x))$ SFS reduce to intuitionistic FS.

• $\delta_{\mathscr{H}^{sf}}(x) = 0$ SFS reduce to Pythagorean FS with the $\theta_{\mathscr{H}^{sf}}(x)^2 + \varphi_{\mathscr{H}^{sf}}(x)^2 \leq 1 \; \forall x \in \varkappa$.

Definition 2.8 [26] (show in Figure 5) The spherical distance metric is used to measure the dissimilarity to between two SFS denoted as $\mathscr{H}^{sf}$ and $\mathfrak{R}^{sf}$. The SFS is characterized by three components for each element in the discourse universe $\varkappa$: membership (θ), non-membership (φ), and indeterminacy (δ).

The distance is defined based on the difference in membership (θ), non-membership (φ), and indeterminacy (δ) values between two fuzzy sets. The distance formula is given as:

$$dis\ \left(\mathscr{H}^{sf}, \mathfrak{R}^{sf}\right) = \arccos\left\{1 - \frac{1}{2}((\theta_{\mathscr{H}^{sf}} - \theta_{\mathfrak{R}^{sf}})^2 + (\varphi_{\mathscr{H}^{sf}} - \varphi_{\mathfrak{R}^{sf}})^2 + (\delta_{\mathscr{H}^{sf}} - \delta_{\mathfrak{R}^{sf}})^2\right\}$$

Where:

• $\theta_{\mathscr{H}^{sf}}$, $\varphi_{\mathscr{H}^{sf}}$, $\delta_{\mathscr{H}^{sf}}$ are the membership (θ), non-membership (φ), and indeterminacy (δ) function for the element x in $\mathscr{H}^{sf}$.

• $\theta_{\mathfrak{R}^{sf}}$, $\varphi_{\mathfrak{R}^{sf}}$, $\delta_{\mathfrak{R}^{sf}}$ are the membership (θ), non-membership (φ), and indeterminacy (δ) function for the element x in $\mathfrak{R}^{sf}$.

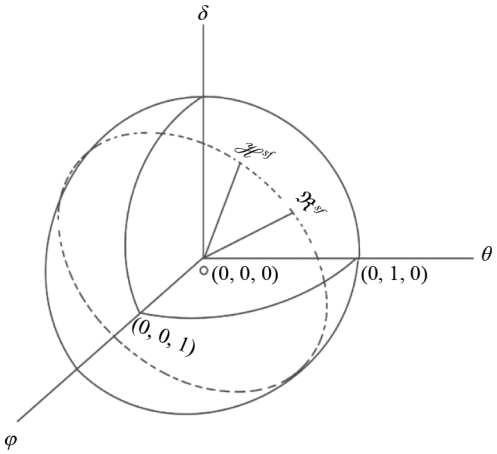


Figure 5. Graphical representation of SFS

The difference in membership (θ), non-membership (φ), and indeterminacy (δ) values for each element in the discourse universe $\varkappa$ is used in this equation to calculate the spherical distance between two SFS $\mathscr{H}^{sf}$ and $\mathfrak{R}^{sf}$. If the fuzzy sets cover a set of n elements in the universe $\varkappa$, the distance is calculated by summing the individual spherical

distances over all elements, scaled by $\frac{2}{\pi}$. The following formula is used to determine the spherical distance of each element in the universe $\varkappa$:

$$dis\ \left(\mathscr{H}^{sf}, \Re^{sf}\right)$$

$$= \frac{2}{\pi}\sum_{i=1}^{n} \arccos\left\{1 - \frac{1}{2}((\theta_{\mathscr{H}^{sf}}(x_i) - \theta_{\Re^{sf}}(x_i))^2 + (\varphi_{\mathscr{H}^{sf}}(x_i) - \varphi_{\Re^{sf}}(x_i))^2 + (\delta_{\mathscr{H}^{sf}}(x_i) - \delta_{\Re^{sf}}(x_i))^2\right\}$$

The resulting distance scale is set between $[0, 1]$ and $\left[0, \frac{\pi}{2}\right]$. Since $\theta_{\mathscr{H}^{sf}}(x)^2 + \varphi_{\mathscr{H}^{sf}}(x)^2 + \delta_{\mathscr{H}^{sf}}(x)^2 \leq 1\ \forall x \in \varkappa$, and given as:

$$dis\ \left(\mathscr{H}^{sf}, \Re^{sf}\right) = \frac{2}{\pi}\sum_{i=1}^{n} \arccos\begin{pmatrix} \theta_{\mathscr{H}^{sf}}(x_i) * \theta_{\Re^{sf}}(x_i) \\ \varphi_{\mathscr{H}^{sf}}(x_i) * \varphi_{\Re^{sf}}(x_i) \\ \delta_{\mathscr{H}^{sf}}(x_i) * \delta_{\Re^{sf}}(x_i) \end{pmatrix}$$

Normalization is applied to ensure that distance values are within a limited range. The normalized spherical distance is given by:

$$dis\ \left(\mathscr{H}^{sf}, \Re^{sf}\right) = \frac{2}{n\pi}\sum_{i=1}^{n} \arccos\begin{pmatrix} \theta_{\mathscr{H}^{sf}}(x_i) * \theta_{\Re^{sf}}(x_i) \\ \varphi_{\mathscr{H}^{sf}}(x_i) * \varphi_{\Re^{sf}}(x_i) \\ \delta_{\mathscr{H}^{sf}}(x_i) * \delta_{\Re^{sf}}(x_i) \end{pmatrix}$$

Normalization ensures that the distance is within the range $[0, 1]$ where 0 means that the sets are equal, and 1 indicates maximum dissimilarity. In this case, when the factor $\frac{2}{n\pi}$ is used, the distance values are undeniably in the range $[0, 1]$ rather than $[0, \pi]$.

• The denormalized distance is $0 \leq dis\ \left(\mathscr{H}^{sf}, \Re^{sf}\right) \leq n$.

• For normalized distance $0 \leq dis\ \left(\mathscr{H}^{sf}, \Re^{sf}\right) \leq 1$.

These distances can be used to measure indeterminacy degree (δ), non-membership degree (φ), and the membership degree (θ) of two spherical fuzzy sets covering the entire discourse universe. Ambiguity exists in many domains, including risk assessment, fuzzy optimization, and decision making. These distances provide a practical means of evaluating and comparing variations in spherical fuzzy sets.

Definition 2.9 [26] Given two SFS $\mathscr{H}^{sf}$ and $\Re^{sf}$. On a $\varkappa$, where $\forall x \in \varkappa$ is associated with triplets $(\theta_{\mathscr{H}^{sf}}(x), \varphi_{\mathscr{H}^{sf}}(x)$, and $\delta_{\mathscr{H}^{sf}}(x))$ for $\mathscr{H}^{sf}$ and $(\theta_{\Re^{sf}}(x), \varphi_{\Re^{sf}}(x)$, and $\delta_{\Re^{sf}}(x))$ for $\Re^{sf}$ the following operations are defined:

1. Union:

$$\mathscr{H}^{sf} \cup \mathfrak{R}^{sf} = \left\{ \begin{array}{l} \max\{\theta_{\mathscr{H}^{sf}}, \theta_{\mathfrak{R}^{sf}}\}, \ \min\{\varphi_{\mathscr{H}^{sf}}, \varphi_{\mathfrak{R}^{sf}}\}, \\ \min\left\{\sqrt{\left(1-((max\ \{\theta_{\mathscr{H}^{sf}}, \theta_{\mathfrak{R}^{sf}}\})^2+(\min\{\varphi_{\mathscr{H}^{sf}}, \varphi_{\mathfrak{R}^{sf}}\})^2\right)}, \ \max\{\delta_{\mathscr{H}^{sf}}, \delta_{\mathfrak{R}^{sf}}\}\right\} \end{array} \right\}$$

2. Intersection:

$$\mathscr{H}^{sf} \cap \mathfrak{R}^{sf} = \left\{ \begin{array}{l} \min\{\theta_{\mathscr{H}^{sf}}, \theta_{\mathfrak{R}^{sf}}\}, \ \max\{\varphi_{\mathscr{H}^{sf}}, \varphi_{\mathfrak{R}^{sf}}\}, \\ \max\left\{\sqrt{\left(1-((min\ \{\theta_{\mathscr{H}^{sf}}, \theta_{\mathfrak{R}^{sf}}\})^2+(\max\{\varphi_{\mathscr{H}^{sf}}, \varphi_{\mathfrak{R}^{sf}}\})^2\right)}, \ \min\{\delta_{\mathscr{H}^{sf}}, \delta_{\mathfrak{R}^{sf}}\}\right\} \end{array} \right\}$$

3. Addition:

$$\mathscr{H}^{sf} \oplus \mathfrak{R}^{sf} = \left\{ \begin{array}{l} \sqrt{\theta_{\mathscr{H}^{sf}}{}^2+\theta_{\mathfrak{R}^{sf}}{}^2-\theta_{\mathscr{H}^{sf}}{}^2\theta_{\mathfrak{R}^{sf}}{}^2}, \ \varphi_{\mathscr{H}^{sf}}\varphi_{\mathfrak{R}^{sf}}, \\ \sqrt{\left(1-\theta_{\mathfrak{R}^{sf}}{}^2\right)\delta_{\mathscr{H}^{sf}}{}^2+\left(1-\theta_{\mathfrak{R}^{sf}}{}^2\right)\delta_{\mathfrak{R}^{sf}}{}^2-\delta_{\mathscr{H}^{sf}}{}^2\delta_{\mathfrak{R}^{sf}}{}^2} \end{array} \right\}$$

4. Multiplication:

$$\mathscr{H}^{sf} \otimes \mathfrak{R}^{sf} = \left\{ \begin{array}{l} \theta_{\mathscr{H}^{sf}}\theta_{\mathfrak{R}^{sf}}, \ \sqrt{\varphi_{\mathscr{H}^{sf}}{}^2+\varphi_{\mathfrak{R}^{sf}}{}^2-\varphi_{\mathscr{H}^{sf}}{}^2\varphi_{\mathfrak{R}^{sf}}{}^2}, \\ \sqrt{(1-\varphi_{\mathfrak{R}^{sf}}{}^2)\delta_{\mathscr{H}^{sf}}{}^2+(1-\varphi_{\mathscr{H}^{sf}}{}^2)\delta_{\mathfrak{R}^{sf}}{}^2-\delta_{\mathscr{H}^{sf}}{}^2\delta_{\mathfrak{R}^{sf}}{}^2} \end{array} \right\}$$

5. Multiplication $\mu > 0$:

$$\mu\mathscr{H}^{sf} = \left\{ \begin{array}{l} \sqrt{1-\left(1-\theta_{\mathscr{H}^{sf}}{}^2\right)^{\mu}}, \ \varphi_{\mathscr{H}^{sf}}{}^{\mu}, \\ \sqrt{(1-\theta_{\mathscr{H}^{sf}}^2)^{\mu}-(1-\theta_{\mathscr{H}^{sf}}{}^2-\delta_{\mathscr{H}^{sf}}{}^2)^{\mu}} \end{array} \right\}$$

6. μ power of SFS $\mu > 0$:

$$(\mathscr{H}^{sf})^{\mu} = \left\{ \begin{array}{l} \theta_{\mathscr{H}^{sf}}^{\mu}, \sqrt{1-(1-\varphi_{\mathscr{H}^{sf}}{}^2)^{\mu}}, \\ \sqrt{(1-\varphi_{\mathscr{H}^{sf}}{}^2)^{\mu}-(1-\varphi_{\mathscr{H}^{sf}}{}^2-\delta_{\mathscr{H}^{sf}}^2)^{\mu}} \end{array} \right\}$$

Definition 2.10 [26] For two SFS $\mathscr{H}^{sf} = (\theta_{\mathscr{H}^{sf}}(x), \varphi_{\mathscr{H}^{sf}}(x)$ and $\delta_{\mathscr{H}^{sf}}(x))$ and $\mathfrak{R}^{sf} = (\theta_{\mathfrak{R}^{sf}}(x), \varphi_{\mathfrak{R}^{sf}}(x)$, and $\delta_{\mathfrak{R}^{sf}}(x))$, the following properties hold under the condition $\mu, \mu_1, \mu_2 \geq 0$.

(1) $\mathscr{H}^{sf} \oplus \mathfrak{R}^{sf} = \mathfrak{R}^{sf} \oplus \mathscr{H}^{sf}$.

(2) $\mathscr{H}^{sf} \otimes \mathfrak{R}^{sf} = \mathfrak{R}^{sf} \otimes \mathscr{H}^{sf}$.
(3) $\mu(\mathscr{H}^{sf} \oplus \mathfrak{R}^{sf}) = \mu\mathscr{H}^{sf} \oplus \mu\mathfrak{R}^{sf}$.
(4) $\mu_1\mathscr{H}^{sf} \oplus \mu_2\mathscr{S}^{sf} = (\mu_1 + \mu_2)\mathscr{S}^{sf}$.
(5) $(\mathscr{H}^{sf} \bigotimes \mathfrak{R}^{sf})^{\mu} = \mathscr{H}^{sf\mu} \bigotimes \mathfrak{R}^{sf\mu}$.
(6) $\mathscr{H}^{sf\mu_1} \bigotimes \mathscr{H}^{sf\mu_2} = \mathscr{H}^{sf\mu_1+\mu_2}$.
1. Commutativity of addition:

$$\mathscr{H}^{sf} \oplus \mathfrak{R}^{sf} = \mathfrak{R}^{sf} \oplus \mathscr{H}^{sf}$$

Proof.

$$\mathscr{H}^{sf} \oplus \mathfrak{R}^{sf}$$

$$= \left\{ \sqrt{\theta_{\mathscr{H}^{sf}}{}^2 + \theta_{\mathfrak{R}^{sf}}{}^2 - \theta_{\mathscr{H}^{sf}}{}^2\theta_{\mathfrak{R}^{sf}}{}^2} \, , \varphi_{\mathscr{H}^{sf}}\varphi_{\mathfrak{R}^{sf}} \, , \sqrt{\left(1-\theta_{\mathfrak{R}^{sf}}{}^2\right)\delta_{\mathscr{H}^{sf}}{}^2 + \left(1-\theta_{\mathscr{H}^{sf}}{}^2\right)\delta_{\mathfrak{R}^{sf}}{}^2 - \delta_{\mathscr{H}^{sf}}{}^2\delta_{\mathfrak{R}^{sf}}{}^2} \right\}$$

$$\mathfrak{R}^{sf} \oplus \mathscr{H}^{sf}$$

$$= \left\{ \sqrt{\theta_{\mathfrak{R}^{sf}}{}^2 + \theta_{\mathscr{H}^{sf}}{}^2 - \theta_{\mathfrak{R}^{sf}}{}^2\theta_{\mathscr{H}^{sf}}{}^2} \, , \varphi_{\mathfrak{R}^{sf}}\varphi_{\mathscr{H}^{sf}} \, , \sqrt{\left(1-\theta_{\mathscr{H}^{sf}}{}^2\right)\tau^2_{\mathscr{P}^{sf}} + \left(1-\theta_{\mathfrak{R}^{sf}}{}^2\right)\delta_{\mathscr{H}^{sf}}{}^2 - \delta_{\mathfrak{R}^{sf}}{}^2\delta_{\mathscr{H}^{sf}}{}^2} \right\}$$

And so, we can say that $\mathscr{H}^{sf} \oplus \mathfrak{R}^{sf} = \mathfrak{R}^{sf} \oplus \mathscr{H}^{sf}$.
2. Commutativity of multiplication:

$$\mathscr{H}^{sf} \otimes \mathfrak{R}^{sf} = \mathfrak{R}^{sf} \otimes \mathscr{H}^{sf}$$

Proof.

$$\mathscr{H}^{sf} \otimes \mathfrak{R}^{sf}$$

$$= \left\{ \theta_{\mathscr{H}^{sf}}\theta_{\mathfrak{R}^{sf}}, \ \sqrt{\varphi_{\mathscr{H}^{sf}}{}^2 + \varphi_{\mathfrak{R}^{sf}}{}^2 - \varphi_{\mathscr{H}^{sf}}{}^2\varphi_{\mathfrak{R}^{sf}}{}^2}, \ \sqrt{\left(1-\varphi_{\mathfrak{R}^{sf}}{}^2\right)\delta_{\mathscr{H}^{sf}}{}^2 + \left(1-\varphi_{\mathscr{H}^{sf}}{}^2\right)\delta_{\mathfrak{R}^{sf}}{}^2 - \delta_{\mathscr{H}^{sf}}{}^2\delta_{\mathfrak{R}^{sf}}{}^2} \right\}$$

$$\mathfrak{R}^{sf} \otimes \mathscr{H}^{sf}$$

$$= \left\{ \theta_{\mathfrak{R}^{sf}}\theta_{\mathscr{H}^{sf}}, \ \sqrt{\varphi_{\mathscr{H}^{sf}}{}^2 + \varphi_{\mathscr{H}^{sf}}{}^2 - \varphi_{\mathfrak{R}^{sf}}{}^2\varphi_{\mathscr{H}^{sf}}{}^2}, \ \sqrt{\left(1-\varphi_{\mathscr{H}^{sf}}{}^2\right)\varphi_{\mathfrak{R}^{sf}}{}^2 + \left(1-\varphi_{\mathfrak{R}^{sf}}{}^2\right)\delta_{\mathscr{H}^{sf}}{}^2 - \delta_{\mathfrak{R}^{sf}}{}^2\delta_{\mathscr{H}^{sf}}{}^2} \right\}$$

And so, we can say that $\mathscr{H}^{sf} \otimes \mathfrak{R}^{sf} = \mathfrak{R}^{sf} \otimes \mathscr{H}^{sf}$.
3. Scalar multiplication with addition:

$$\mu\left(\mathscr{H}^{sf}\oplus\mathfrak{R}^{sf}\right)=\mu\mathscr{H}^{sf}\oplus\mu\mathfrak{R}^{sf}$$

Proof.

$$\mu\left(\mathscr{H}^{sf}\oplus\mathfrak{R}^{sf}\right)=\mu\left\{\sqrt{\theta_{\mathscr{H}^{sf}}{}^{2}+\theta_{\mathfrak{R}^{sf}}{}^{2}-\theta_{\mathscr{H}^{sf}}{}^{2}\theta_{\mathfrak{R}^{sf}}{}^{2}},\ \varphi_{\mathscr{H}^{sf}}\varphi_{\mathfrak{R}^{sf}},\right.$$

$$\left.\sqrt{\left(1-\theta_{\mathfrak{R}^{sf}}{}^{2}\right)\delta_{\mathscr{H}^{sf}}{}^{2}+\left(1-\theta_{\mathscr{H}^{sf}}{}^{2}\right)\delta_{\mathfrak{R}^{sf}}{}^{2}-\delta_{\mathscr{H}^{sf}}{}^{2}\delta_{\mathfrak{R}^{sf}}{}^{2}}\right\}$$

$$=\left\{\sqrt{1-\left(1-\left(\theta_{\mathscr{H}^{sf}}{}^{2}+\theta_{\mathfrak{R}^{sf}}{}^{2}-\theta_{\mathscr{H}^{sf}}{}^{2}\theta_{\mathfrak{R}^{sf}}{}^{2}\right)\right)^{\mu}},\ \varphi_{\mathscr{H}^{sf}}\varphi_{\mathfrak{R}^{sf}},\right.$$

$$\sqrt{\left(1-\left(\theta_{\mathscr{H}^{sf}}{}^{2}+\theta_{\mathfrak{R}^{sf}}{}^{2}-\theta_{\mathscr{H}^{sf}}{}^{2}\theta_{\mathfrak{R}^{sf}}{}^{2}\right)\right)^{\mu}-\left(1-\left(\theta_{\mathscr{H}^{sf}}{}^{2}+\theta_{\mathfrak{R}^{sf}}{}^{2}-\theta_{\mathscr{H}^{sf}}{}^{2}\theta_{\mathfrak{R}^{sf}}{}^{2}\right)\right.}$$

$$\left.\left.-\left(1-\theta_{\mathfrak{R}^{sf}}{}^{2}\right)\delta_{\mathscr{H}^{sf}}{}^{2}-\left(1-\theta_{\mathscr{H}^{sf}}{}^{2}\right)\delta_{\mathfrak{R}^{sf}}{}^{2}-\delta_{\mathscr{H}^{sf}}{}^{2}\delta_{\mathfrak{R}^{sf}}{}^{2}\right)^{\mu}\right\}$$

$$\mu\mathscr{H}^{sf}\oplus\mu\mathfrak{R}^{sf}=\left\{\sqrt{1-\left(1-\theta_{\mathscr{H}^{sf}}{}^{2}\right)^{\mu}}\ ,\ \varphi_{\mathscr{H}^{sf}}{}^{\mu},\ \sqrt{\left(1-\theta_{\mathscr{H}^{sf}}{}^{2}\right)^{\mu}-\left(1-\theta_{\mathscr{H}^{sf}}{}^{2}-\delta_{\mathscr{H}^{sf}}{}^{2}\right)^{\mu}}\right\}$$

$$\oplus\left\{\sqrt{1-\left(1-\theta_{\mathfrak{R}^{sf}}{}^{2}\right)^{\mu}}\ ,\ \varphi_{\mathfrak{R}^{sf}}{}^{\mu},\ \sqrt{\left(1-\theta_{\mathfrak{R}^{sf}}{}^{2}\right)^{\mu}-\left(1-\theta_{\mathfrak{R}^{sf}}{}^{2}-\delta_{\mathfrak{R}^{sf}}{}^{2}\right)^{\mu}}\right\}$$

$$=\left\{\begin{array}{l}\sqrt{1-\left(1-\theta_{\mathscr{H}^{sf}}{}^{2}\right)^{\mu}+1-\left(1-\theta_{\mathfrak{R}^{sf}}{}^{2}\right)^{\mu}-\left(1-\left(1-\theta_{\mathscr{H}^{sf}}{}^{2}\right)^{\mu}\right)\left(1-\left(1-\theta_{\mathfrak{R}^{sf}}{}^{2}\right)^{\mu}\right)},\\ \varphi_{\mathscr{H}^{sf}}\varphi_{\mathfrak{R}^{sf}},\\ \sqrt{\begin{array}{l}1-\left(1-\left(1-\theta_{\mathfrak{R}^{sf}}{}^{2}\right)^{\mu}\right)\\ \left(\left(1-\theta_{\mathscr{H}^{sf}}{}^{2}\right)^{\eta}-\left(1-\theta_{\mathscr{H}^{sf}}{}^{2}-\delta_{\mathscr{H}^{sf}}{}^{2}\right)^{\mu}\right)+\\ 1-\left(1-\left(1-\theta_{\mathscr{H}^{sf}}{}^{2}\right)^{\mu}\right)\\ (\left(1-\theta_{\mathfrak{R}^{sf}}{}^{2}\right)^{\mu}-\\ (\left(1-\theta_{\mathscr{H}^{sf}}{}^{2}-\delta_{\mathfrak{R}^{sf}}{}^{2}\right)^{\mu}))-((\left(1-\theta_{\mathscr{H}^{sf}}{}^{2}\right)^{\mu}-\left(1-\theta_{\mathscr{H}^{sf}}{}^{2}-\delta_{\mathscr{H}^{sf}}{}^{2}\right)^{\mu})\\ ((\left(1-\theta_{\mathfrak{R}^{sf}}{}^{2}\right)^{\mu}-\left(1-\theta_{\mathfrak{R}^{sf}}{}^{2}-\delta_{\mathfrak{R}^{sf}}{}^{2}\right)^{\mu})\end{array}}\end{array}\right\}$$

$$= \left\{ \begin{array}{l} \sqrt{1-\left(1-\theta_{\mathscr{H}^{sf}}{}^{2}\right)^{\eta}\left(1-\theta_{\mathfrak{R}^{sf}}{}^{2}\right)^{\mu}},\ \varphi_{\mathscr{H}^{sf}}\varphi_{\mathfrak{R}^{sf}}, \\ \sqrt{\begin{array}{l} \left(1-\theta_{\mathscr{H}^{sf}}{}^{2}\right)^{\mu}\left(1-\theta_{\mathfrak{R}^{sf}}{}^{2}\right)^{\mu}-\left(\left(1-\theta_{\mathscr{H}^{sf}}{}^{2}\right)-\delta_{\mathscr{H}^{sf}}{}^{2}\right)^{\mu} \\ \left(\left(1-\theta_{\mathfrak{R}^{sf}}{}^{2}\right)-\delta_{\mathfrak{R}^{sf}}{}^{2}\right)^{\mu} \end{array}} \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \sqrt{1-\left(1-\left(\theta_{\mathscr{H}^{sf}}{}^{2}+\theta_{\mathfrak{R}^{sf}}{}^{2}-\theta_{\mathscr{H}^{sf}}{}^{2}\theta_{\mathfrak{R}^{sf}}{}^{2}\right)\right)^{\mu}},\ \varphi_{\mathscr{H}^{sf}}\varphi_{\mathfrak{R}^{sf}} \\ \sqrt{\begin{array}{l} \left(1-\left(\theta_{\mathscr{H}^{sf}}{}^{2}+\theta_{\mathfrak{R}^{sf}}{}^{2}-\theta_{\mathscr{H}^{sf}}{}^{2}\theta_{\mathfrak{R}^{sf}}{}^{2}\right)\right)^{\mu}-\left(\begin{array}{l} 1-\left(\theta_{\mathscr{H}^{sf}}{}^{2}+\theta_{\mathfrak{R}^{sf}}{}^{2}-\theta_{\mathscr{H}^{sf}}{}^{2}\theta_{\mathfrak{R}^{sf}}{}^{2}\right) \\ -\left(1-\theta_{\mathfrak{R}^{sf}}{}^{2}\right)\delta_{\mathscr{H}^{sf}}{}^{2} \\ -\left(1-\theta_{\mathscr{H}^{sf}}{}^{2}\right)\delta_{\mathfrak{R}^{sf}}{}^{2}+\delta_{\mathscr{H}^{sf}}{}^{2}\delta_{\mathfrak{R}^{sf}}{}^{2} \end{array}\right)^{\mu} \end{array}} \end{array} \right\}$$

And so, we can say that, $\mu\left(\mathscr{H}^{sf}\oplus\mathfrak{R}^{sf}\right)=\mu\mathscr{H}^{sf}\oplus\mu\mathfrak{R}^{sf}$.

4. Distributivity of scalar multiplication over addition:

$$\mu_1\mathscr{H}^{sf}\oplus\mu_2\mathscr{S}^{sf}=(\mu_1+\mu_2)\mathscr{S}^{sf}$$

5. Exponentiation of multiplication:

$$\left(\mathscr{H}^{sf}\bigotimes\mathfrak{R}^{sf}\right)^{\mu}=\mathscr{H}^{sf\mu}\bigotimes\mathfrak{R}^{sf\mu}$$

Proof.

$$\left(\mathscr{H}^{sf}\bigotimes\mathfrak{R}^{sf}\right)^{\mu}$$

$$= \left\{ \theta_{\mathscr{H}^{sf}}\theta_{\mathfrak{R}^{sf}},\ \sqrt{\left(\varphi_{\mathscr{H}^{sf}}{}^{2}+\varphi_{\mathfrak{R}^{sf}}{}^{2}-\varphi_{\mathscr{H}^{sf}}{}^{2}\varphi_{\mathfrak{R}^{sf}}{}^{2}\right)},\ \sqrt{\begin{array}{l} \left(1-\varphi_{\mathfrak{R}^{sf}}{}^{2}\right)\delta_{\mathscr{H}^{sf}}{}^{2} \\ -\left(1-\varphi_{\mathscr{H}^{sf}}{}^{2}\right)\delta_{\mathfrak{R}^{sf}}{}^{2}+\delta_{\mathscr{H}^{sf}}{}^{2}\delta_{\mathfrak{R}^{sf}}{}^{2} \end{array}}^{\eta} \right\}$$

$$=\left\{\begin{array}{l}\theta_{\mathscr{H}^{sf}}{}^{\eta}\theta_{\mathfrak{R}^{sf}}{}^{\eta},\sqrt{\left(1-\left(1-\left(\varphi_{\mathscr{H}^{sf}}{}^{2}+\varphi_{\mathfrak{R}^{sf}}{}^{2}-\varphi_{\mathscr{H}^{sf}}{}^{2}\varphi_{\mathfrak{R}^{sf}}{}^{2}\right)\right)\right)^{\eta}},\\ \sqrt{\begin{array}{l}\left(1-\left(\varphi_{\mathscr{H}^{sf}}{}^{2}+\varphi_{\mathfrak{R}^{sf}}{}^{2}-\varphi_{\mathscr{H}^{sf}}{}^{2}\varphi_{\mathfrak{R}^{sf}}{}^{2}\right)\right)^{\eta}\\ -\left(\left(1-\left(\varphi_{\mathscr{H}^{sf}}{}^{2}+\varphi_{\mathfrak{R}^{sf}}{}^{2}-\varphi_{\mathscr{H}^{sf}}{}^{2}\varphi_{\mathfrak{R}^{sf}}{}^{2}\right)\right)\right.\\ -\left(1-\left(\varphi_{\mathscr{H}^{sf}}{}^{2}+\varphi_{\mathfrak{R}^{sf}}{}^{2}-\varphi_{\mathscr{H}^{sf}}{}^{2}\varphi_{\mathfrak{R}^{sf}}{}^{2}\right)-\left(1-\varphi_{\mathfrak{R}^{sf}}{}^{2}\right)\delta_{\mathscr{H}^{sf}}{}^{2}\right.\\ \left.-\left(1-\varphi_{\mathscr{H}^{sf}}{}^{2}\right)\delta_{\mathfrak{R}^{sf}}{}^{2}+\delta_{\mathscr{H}^{sf}}{}^{2}\delta_{\mathfrak{R}^{sf}}{}^{2}\right))^{\eta}\end{array}}\end{array}\right\}$$

$$\mathscr{H}^{sf\mu}\bigotimes\mathfrak{R}^{sf\mu}$$

$$=\left\{\begin{array}{l}\theta_{\mathscr{H}^{sf}}{}^{\eta},\sqrt{1-\left(1-\varphi_{\mathscr{H}^{sf}}{}^{2}\right)^{\eta}},\\ \sqrt{\left(1-\varphi_{\mathscr{H}^{sf}}{}^{2}\right)^{\eta}-\left(1-\theta_{\mathscr{H}^{sf}}{}^{2}-\delta_{\mathscr{H}^{sf}}{}^{2}\right)^{\eta}}\end{array}\right\}\bigotimes\left\{\begin{array}{l}\theta_{\mathfrak{R}^{sf}}{}^{\eta},\sqrt{1-\left(1-\varphi_{\mathfrak{R}^{sf}}{}^{2}\right)^{\eta}},\\ \sqrt{\left(1-\varphi_{\mathfrak{R}^{sf}}{}^{2}\right)^{\eta}-\left(1-\theta_{\mathfrak{R}^{sf}}{}^{2}-\delta_{\mathfrak{R}^{sf}}{}^{2}\right)^{\eta}}\end{array}\right\}$$

$$=\left\{\begin{array}{l}\theta_{\mathscr{H}^{sf}}{}^{\eta}\theta_{\mathfrak{R}^{sf}}{}^{\eta},\sqrt{1-\left(1-\varphi_{\mathscr{H}^{sf}}{}^{2}\right)^{\eta}+1-\left(1-\varphi_{\mathfrak{R}^{sf}}{}^{2}\right)^{\eta}-\left(1-\left(1-\varphi_{\mathscr{H}^{sf}}{}^{2}\right)^{\eta}\right)\left(1-\left(1-\varphi_{\mathfrak{R}^{sf}}{}^{2}\right)^{\eta}\right)},\\ \sqrt{\begin{array}{l}1-\left(1-\left(1-\varphi_{\mathfrak{R}^{sf}}{}^{2}\right)^{\eta}\right)\left(\left(1-\varphi_{\mathscr{H}^{sf}}{}^{2}\right)^{\eta}-\left(1-\theta_{\mathfrak{R}^{sf}}{}^{2}-\delta_{\mathfrak{R}^{sf}}{}^{2}\right)^{\eta}\right)\\ -\left(\left(1-\varphi_{\mathscr{H}^{sf}}{}^{2}\right)^{\eta}-\left(1-\theta_{\mathscr{H}^{sf}}{}^{2}-\delta_{\mathscr{H}^{sf}}{}^{2}\right)^{\eta}\right)\left(\left(1-\varphi_{\mathfrak{R}^{sf}}{}^{2}\right)^{\eta}-\left(1-\theta_{\mathfrak{R}^{sf}}{}^{2}-\delta_{\mathfrak{R}^{sf}}{}^{2}\right)^{\eta}\right)\end{array}}\end{array}\right\}$$

$$=\left\{\begin{array}{l}\theta_{\mathscr{H}^{sf}}{}^{\eta}\theta_{\mathfrak{R}^{sf}}{}^{\eta},\sqrt{\left(1-\left(1-\varphi_{\mathscr{H}^{sf}}{}^{2}\right)^{\eta}\left(1-\varphi_{\mathfrak{R}^{sf}}{}^{2}\right)^{\eta}\right)}\\ \sqrt{\left(\left(1-\varphi_{\mathscr{H}^{sf}}{}^{2}\right)^{\eta}\left(1-\varphi_{\mathfrak{R}^{sf}}{}^{2}\right)^{\eta}-\left(\left(1-\varphi_{\mathscr{H}^{sf}}{}^{2}\right)\delta_{\mathscr{H}^{sf}}{}^{2}\right)\left(1-\varphi_{\mathfrak{R}^{sf}}{}^{2}\right)\delta_{\mathfrak{R}^{sf}}{}^{2}\right)}\end{array}\right\}$$

$$=\left\{\begin{array}{l}\theta_{\mathscr{H}^{sf}}{}^{\eta}\theta_{\mathfrak{R}^{sf}}{}^{\eta},\sqrt{1-\left(1-\left(\varphi_{\mathscr{H}^{sf}}{}^{2}+\varphi_{\mathfrak{R}^{sf}}{}^{2}-\varphi_{\mathscr{H}^{sf}}{}^{2}\varphi_{\mathfrak{R}^{sf}}{}^{2}\right)\right)^{\eta}},\\ \sqrt{\begin{array}{l}\left(1-\left(\varphi_{\mathscr{H}^{sf}}{}^{2}+\varphi_{\mathfrak{R}^{sf}}{}^{2}-\varphi_{\mathscr{H}^{sf}}{}^{2}\varphi_{\mathfrak{R}^{sf}}{}^{2}\right)\right)^{\eta}\\ -\left(\begin{array}{l}1-\left(\varphi_{\mathscr{H}^{sf}}{}^{2}+\varphi_{\mathfrak{R}^{sf}}{}^{2}-\varphi_{\mathscr{H}^{sf}}{}^{2}\varphi_{\mathfrak{R}^{sf}}{}^{2}\right)-\left(1-\varphi_{\mathfrak{R}^{sf}}{}^{2}\right)\delta_{\mathscr{H}^{sf}}{}^{2}\\ -\left(1-\varphi_{\mathscr{H}^{sf}}{}^{2}\right)\delta_{\mathfrak{R}^{sf}}{}^{2}+\delta_{\mathscr{H}^{sf}}{}^{2}\delta_{\mathfrak{R}^{sf}}{}^{2}\end{array}\right)^{\eta}\end{array}}\end{array}\right\}$$

And so, we can say that, $(\mathscr{H}^{sf}\bigotimes\mathfrak{R}^{sf})^{\mu}=\mathscr{H}^{sf\mu}\bigotimes\mathfrak{R}^{sf\mu}$.

6. Exponentiation of single SFS:

$$\mathscr{H}^{sf\mu_1} \bigotimes \mathscr{H}^{sf\mu_2} = \mathscr{H}^{sf\mu_1+\mu_2}$$

Definition 2.11 The score and accuracy functions of SFS:
Score function defined as:

$$Score\left(\mathscr{H}^{sf}\right) = \left(\frac{\theta_{\mathscr{H}^{sf}} + 1 - \varphi_{\mathscr{H}^{sf}} + 1 - \delta_{\mathscr{H}^{sf}}}{3}\right)$$

$$Score\left(\mathscr{H}^{sf}\right) = \frac{1}{3}(2 + \theta_{\mathscr{H}^{sf}} - \varphi_{\mathscr{H}^{sf}} - \delta_{\mathscr{H}^{sf}})$$

Accuracy function defined as:

$$Accuracy\left(\mathscr{H}^{sf}\right) = \theta_{\mathscr{H}^{sf}}{}^2 + \varphi_{\mathscr{H}^{sf}}{}^2 + \delta_{\mathscr{H}^{sf}}{}^2$$

Two SFS $\mathscr{H}^{sf}$ and $\mathfrak{R}^{sf}$ is determined as follows:
$\mathscr{H}^{sf} < \mathfrak{R}^{sf}$ iff:
• $Score\left(\mathscr{H}^{sf}\right) < Score\left(\mathfrak{R}^{sf}\right)$ or
• $Score\left(\mathscr{H}^{sf}\right) < Score\left(\mathfrak{R}^{sf}\right)$ and $Accuracy\left(\mathscr{H}^{sf}\right) < Accuracy\left(\mathfrak{R}^{sf}\right)$.

These functions provide an organized method to evaluate and organize SFS according to them indeterminacy degree (δ), non-membership degree (φ), and the membership degree (θ).

Spherical Fuzzy Programming:

Defined as an optimization framework where parameters and/or decision variables are represented using spherical fuzzy sets, characterized by membership, non-membership, and hesitancy degrees satisfying spherical constraints (Table 3).

Table 3. Mathematical notation

Notation	
c_{ij}	Transportation cost per unit from source i to destination j
c_{ij}^k	Transportation cost per unit from source i to destination j for the k^{th} objective
Z_k	Objective function value for the k^{th} objective
θ	Membership function
φ	Non-membership function
δ	Indeterminacy function
$\theta_{\mathscr{H}^{sf}}(x)$	Membership degree of x in $\mathscr{H}^{sf}$
$\varphi_{\mathscr{H}^{sf}}(x)$	Non-membership degree of x in $\mathscr{H}^{sf}$
$\delta_{\mathscr{H}^{sf}}(x)$	Indeterminacy (hesitancy) degree of x in $\mathscr{H}^{sf}$
$\mathscr{H}^{sf}$	SFS I
$\mathfrak{R}^{sf}$	SFS II
$\theta_{\mathfrak{R}^{sf}}$	Membership degree of x in $\mathfrak{R}^{sf}$

Table 3. (cont.)

Notation	
$\varphi_{\mathfrak{R}^{sf}}$	Non-membership degree of x in $\mathfrak{R}^{sf}$
$\delta_{\mathfrak{R}^{sf}}$	Indeterminacy (hesitancy) degree of x in $\mathfrak{R}^{sf}$
c^k_{ijs}	Transportation cost per unit for conveyances p_s from source i to destination j for the k^{th} objective
$\mathscr{S}_{ij}$	Fixed charge associated with the transportation link between source i and destination j
x_{ij}	Amount of first commodity transported from source i to destination j
y_{ij}	Amount of second commodity transported from source i to destination j
w_k	Weight assigned to the k^{th} objective in a MOTP
τ	Deviations variable
μ	Degree of satisfaction

3. Mathematical model

The traditional TP is extended by the MOTP, in which the distribution and allocation of resources from supply $(\sum_{i=1}^{m} a_i)$ locations to demand points $(\sum_{j=1}^{n} b_j)$ takes into account multiple competing objectives $(\sum_{k=1}^{K} Z_k)$, where Z_k is the k^{th} objective function, at the same time. Unlike the single objective TP, which only considers minimize transportation costs, MOTP incorporates additional goals $(\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij})$ such as reducing delivery time, increasing customer satisfaction, reducing environmental impact, or balancing resource utilization. Since these objectives often conflict, a compromise is required to find a solution that meets all these criteria at an acceptable level. The presence of multiple objectives and the necessity of balancing them significantly increase the complexity of decision-making in MOTP.

MOTP Model I:

Minimize/Maximize $Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$.

Subject to: $\sum_{j=1}^{n} x_{ij} = a_i$.

$\sum_{i=1}^{m} x_{ij} = b_j$.

$x_{ij} \geq 0 \; \forall \; i = 1, 2 \ldots m, \; j = 1, 2 \ldots n, \; k = 1, 2 \ldots K$.

The feasibility condition is $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. It is necessary to strike a balance between multiple competing goals while making decisions across a variety of businesses, which is where the MOTP comes into play as an essential component. The use of MOTP in urban transportation planning helps to ensure that public transportation systems are efficient by lowering costs, shortening travel times, and minimizing the negative effects on the environment. When it comes to the distribution of energy and utilities, it provides assistance in the establishment of power networks that optimize reliability while simultaneously minimizing costs and energy losses. Through the balance of coverage dependencies, network latency, and operating expenses, MOTP contributes to the optimization of telecommunications transmission networks. MOTP is used in e-commerce and retail logistics to optimize delivery routes in order to accomplish the goals of lowering carbon emissions, enhancing speed, and reducing costs. During times of emergency or disaster, it makes it easier to send relief supplies to areas that have been impacted in a way that is both efficient and cost savings. In a similar manner, the management of supply chains makes use of MOTP in order to enhance the reliability of delivery, reduce costs, and streamline the movement of commodities. MOTP strikes a balance between cost, environmental impact, and recycling efficiency in the field of waste management. When it comes to agricultural logistics, MOTP helps reduce the amount of perishable items that are wasted and ensures that they are distributed fairly. MOTP is relied upon by the healthcare industry to ensure that necessary medical supplies are transported to remote areas in a timely manner and at an affordable cost. Last but not least, in the field of logistics, it guarantees efficient resource deployment while maintaining a balance between safety, time, and financial concerns. In order to handle complicated and multi-objective problems that are encountered in the real world, these examples demonstrate the versatility of MOTP. The STP is generalized as the MOSTP, which must account for capacity constraints $(\sum_{s=1}^{S} p_s)$ on sources (S_i), destinations (D_j), and modes of transportation, while

simultaneously optimizing multiple competing objectives. This model is particularly useful in real-world situations where decision makers must evaluate trade-offs.

MOSTP Model II:

Minimize/Maximize $Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S} c_{ijs}x_{ij}$.

Subject to: $\sum_{j=1}^{n}\sum_{s=1}^{S} x_{ijs} = a_i$.

$\sum_{i=1}^{m}\sum_{s=1}^{S} x_{ijs} = b_j$.

$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijs} = p_s$.

$x_{ij} \geq 0 \; \forall \; i = 1, 2 \ldots m$, $j = 1, 2 \ldots n$, $k = 1, 2 \ldots K$, and $s = 1, 2 \ldots S$.

The feasibility condition is $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{s=1}^{S} p_s$. It is possible to make use of MOSTP in circumstances when the planning of logistics incorporates the fulfilment of competing goals. In the context of supply chain management, for instance, MOSTP is utilized to strike a balance between available capacity constraints, transportation expenses, delivery timeframes, and carbon emissions. Additionally, this assists in the management of the trade-off between transmission losses, cost reduction, and supply reliability in the energy distribution industry. The applications of waste management include preserving the capacity of vehicles and facilities while striking a balance between cost, the efficiency of recycling, and the influence on the environment. In the context of health care logistics, MOSTP ensures the timely delivery of medical supplies while minimizing the impact on both the environment and the economy, particularly in the event of an emergency. In order to maximize resource deployment, it helps strike a balance between cost, speed, and security. The aforementioned applications demonstrate how MOSTP can be utilized to address difficult and multidimensional decision-making problem situations.

A fixed cost is charged whenever a shipment is made from a source to a destination, in addition to variable costs dependent on the quantity delivered. This is known as the FCTP which is an extension of the classical transportation problem. Compared to the classical transportation problem, this makes the problem nonlinear and more difficult to solve. In real-world situations were establishing or using a transportation link involves heavy overhead expense, such as opening a shipping lane or activating a facility, this approach is particularly relevant. Reducing the total cost of transportation, including:

• Fixed cost: An expense that always occurs when a route is opened or a transportation link is used, regardless of the quantity delivered.

• Variable cost: Price that depends on how many goods are being transported.

FCTP Model III:

Minimize $Z = Z_{Var.\ cost} + Z_{Fixed\ cost} = \sum_{i=1}^{m}\sum_{j=1}^{n} (c_{ij}x_{ij} + \mathscr{S}_{ij}\wp_{ij})$.

Subject to: $\sum_{j=1}^{n} x_{ij} = a_i$, $\sum_{i=1}^{m} x_{ij} = b_j$, $x_{ij} \geq 0 \; \forall \; i = 1, 2 \ldots m$, $j = 1, 2 \ldots n$.

$$\begin{cases} \wp_{ij} = 0, \text{ if } x_{ij} = 0 \\ \wp_{ij} = 1, \text{ if } x_{ij} > 0 \end{cases}$$

The feasibility condition is $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

The FCTP comment by Balinsky [3] is an important piece of information to resolve this issue. According to observation, there is an ideal solution for the relaxed form of FCTP, where integer bounds on $\wp_{ij}$ are relaxed with the following property:

$$\wp_{ij} = c_{ij} + \frac{\mathscr{S}_{ij}}{\min(a_i, b_j)}$$

In addition to variable cost based on volume of transport MOFCTP add fixed cost for each route used, expanding the MOTP. The quantity of the product is represented by x_{ij} the factor coefficient corresponding to the objective Z_k for transporting one unit of the product is represented by c_{ij} and the fixed cost $\mathscr{S}_{ij}$ associated with using the route corresponds

to the objective Z_k is vehicle, transportation cost and damage cost upon arrival from source i to destination j. The supply, or what is available at source S_i and the demand D_j required at destination j are represented by a_i and b_j respectively. This problem, which has several goals including minimizing transportation time and cost, is modelled using binary variables (0 or 1) for $\wp_{ij}$ which represent continuous decision variables for route usage and transportation volume. In this case, $\wp_{ij}$ is defined as a function with $\wp_{ij} = 1$ if $x_{ij} > 0$ and otherwise $\wp_{ij} = 0$.

MOFCTP Model IV:

Minimize $Z_k = Z_{Var.\ cost} + Z_{Fixed\ cost} = \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}x_{ij} + \mathscr{S}_{ij}\wp_{ij})$.

Subject to: $\sum_{j=1}^{n} x_{ij} = a_i$, $\sum_{i=1}^{m} x_{ij} = b_j$, $x_{ij} \geq 0\ \forall\ i = 1, 2\ldots m$, $j = 1, 2\ldots n$, $k = 1, 2\ldots K$.

$$\begin{cases} \wp_{ij} = 0, & \text{if } x_{ij} = 0 \\ \wp_{ij} = 1, & \text{if } x_{ij} > 0 \text{ (if goods are transported from } i \text{ to } j) \end{cases}$$

Those in charge of making decisions in a variety of different industries are required to strike a balance between many conflicting objectives while also taking into consideration the fixed costs connected with new transportation routes. This issue is commonly referred to as the MOFCTP. Within the realm of supply chain management, it assists firms in determining the most cost-effective routes and warehouse activations, all while maintaining a balance between efficiency and sustainability. In order to minimize the expenses of operation and the negative effects on the environment, MOFCTP optimizes the layouts of power plants and transmission lines located within energy distribution networks. MOFCTP, in a similar fashion, helps enhance latency and quality of service in the field of telecommunications while also decreasing costs associated with network setup and maintenance. In the context of urban transportation systems, MOFCTP provides assistance in the design of optimal bus or metro networks, the reduction of travel times and operation costs, and the promotion the sustainability of the environment. This strategy is also utilized in the field of disaster relief logistics, where it is utilized to rapidly establish warehouses and transportation routes. This helps to guarantee the prompt delivery of necessary products while simultaneously lowering the costs of both setup and operation. MOFCTP is utilized by waste management companies in order to optimize collection routes and facility activations, which ultimately results in a reduction in fees and emissions associated with transportation. Within the realm of e-commerce logistics, it assists firms in determining efficient delivery routes and warehouse arrangements, which in turn helps to reduce the amount of time and money spent on delivery, as well as the influence that they have on the environment. In the end, MOFCTP optimizes the transportation of perishable items in agricultural product distribution, hence lowering the negative effects on the environment, financial costs associated with transportation, and the amount of time that perishable goods are wasted. Due to the fact that MOFCTP takes into account fixed setup costs and optimizes several performance measurements, it becomes possible to allocate resources in a more efficient manner across all of these applications.

TOPSIS is a MCDM technique that ranks alternatives based on their proximity to the best alternative, or ideal solution, and the worst alternative, or negative-ideal solution. The process begins by creating a decision matrix containing criteria and options. To ensure comparability, the metrics are then normalized. Each normalized value is subsequently weighted according to the importance of the corresponding criterion. Then, the negative-ideal solution is identified as the worst value, while the ideal solution is identified as the best value for each criterion (minimum for the cost criterion and maximum for the benefit criterion). The Euclidean distance between each alternative and both the ideal $\int_i^+$ and negative-ideal $\int_i^-$. The solutions are then calculated. The relative proximity of each alternative to the ideal alternative is computed, and these values are used to rank the options, with larger values indicating better choices. TOPSIS is widely preferred due to it is simplicity and ability to account for both positive and negative aspects of the selection criteria; however, it is sensitive to the normalization process and requires precise weighting for criteria.

$$Rank\ (\mathscr{R}) = \frac{\int_i^-}{\int_i^+ + \int_i^-}$$

In order to address the MOTP problem while taking into account a number of different items or commodities, the approach that the authors have proposed features a comprehensive framework. A number of aspects of both the STP and the FCTP are incorporated into this framework. The authors hope that by utilizing this strategy, they will be able to address the complexity that exists in the actual world by optimizing numerous objectives while also taking into consideration practical limits such as fixed costs, various item distributions, and routing capabilities. The authors have used a spherical fuzzy optimization framework with a ranking technique to steer decision makers toward optimal solutions by the process of balancing various objectives, which are frequently in conflict with one another. This allows us to tackle the inherent complexities that are present in real transportation systems. When it comes to dealing with transportation features that are unpredictable or imprecise, such as variable costs, uncertain demand, or varying capacity, this technique is very useful.

Proposed Model V

Pseudo code of proposed algorithm

! Decision variables;

MAXIMIZE:

$$\mu/\omega_k \left[\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K}(c_{ijs}x_{ijs}+s_{ijs}\wp_{ijs}) - \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K}(c_{ijs}x_{ijs}+s_{ijs}\wp_{ijs})^{Min}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K}(c_{ijs}x_{ijs}+s_{ijs}\wp_{ijs})^{Max} - \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K}(c_{ijs}x_{ijs}+s_{ijs}\wp_{ijs})^{Min}} \right];$$

! Constraints;

@FOR (SOURCES(i):
  @FOR (CAPACITY (s):
    @SUM (DESTINATIONS(j), WEIGHTS(k): x_{ijs}=a_i;
  );
);

@FOR (SOURCES(i):
  @FOR (CAPACITY (s):
    @SUM (DESTINATIONS(j), WEIGHTS(k): y_{ijs}=a_i;
  );
);

@FOR (DESTINATIONS(j):
  @FOR (CAPACITY (s):
    @SUM (SOURCES(**i**), WEIGHTS(k): x_{ijs}=b_j;
  );
);

@FOR (DESTINATIONS(**j**):
  @FOR (CAPACITY (**s**):
    @SUM (SOURCES(**i**), WEIGHTS(k): y_{ijs}=b_j;
  );
);

@FOR (CAPACITY (s):
  @SUM (SOURCES(i), DESTINATIONS(j), WEIGHTS(k): x_{ijs}=p_s;
);

@FOR (CAPACITY (s):
  @SUM (SOURCES(i), DESTINATIONS(j), WEIGHTS(k): y_{ijs}=p_s;
);

@FOR (SOURCES(i):
  @FOR (DESTINATIONS(j):
    @FOR (CAPACITY (s):
      $\wp_{ij}$= 1 @BND (@SUM (WEIGHTS(k): x_{ij}> 0);
    );
  );

);
! Weight constraints;
@SUM (WEIGHTS(k): $w_k = 11$;
@FOR (WEIGHTS(k): $0 \leq w_k \leq 1$);
! Fixed charge constraints;
SUM (SOURCES, DESTINATIONS, CAPACITIES, WEIGHTS:
$(c^k_{ijs} x_{ijs} + \mathscr{S}_{ijs} \wp_{ijs}) \leq Z^* + \tau(1 - w_k)$;
SUM (SOURCES, DESTINATIONS, CAPACITIES, WEIGHTS:
$(c^k_{ijs} x_{ijs} + \mathscr{S}_{ijs} \wp_{ijs}) + (Z^{\mathscr{U}} - Z^{\mathscr{L}}) >= Z^{\mathscr{U}}$;
! Bounds;
@FOR (SOURCES(i):
@FOR (DESTINATIONS(j):
@FOR (CAPACITY (s):
@FOR (WEIGHTS(k):
$x_{ijs,}\, y_{ijs} \geq 0$;
);
);
);
);
! μ constraints;
$0 \leq \mu \leq 1$;
END
Maximize

$$= \frac{\mu}{w_k \left[\dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K} (c^k_{ijs} x_{ijs} + \mathscr{S}_{ijs} \wp_{ijs}) - \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K} (c^k_{ijs} x_{ijs} + \mathscr{S}_{ijs} \wp_{ijs})^{Min}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K} (c^k_{ijs} x_{ijs} + \mathscr{S}_{ijs} \wp_{ijs})^{Max} - \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K} (c^k_{ijs} x_{ijs} + \mathscr{S}_{ijs} \wp_{ijs})^{Min}} \right]}$$

Subject to:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K} (c^k_{ijs} x_{ijs} + \mathscr{S}_{ijs} \wp_{ijs}) \leq Z^* + \tau(1 - w_k)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K} (c^k_{ijs} x_{ijs} + \mathscr{S}_{ijs} \wp_{ijs}) + \left(Z^{\mathscr{U}} - Z^{\mathscr{L}}\right) \leq Z^{\mathscr{U}}$$

$$\sum_{j=1}^{n}\sum_{s=1}^{S} x_{ijs} = a_i,\ \sum_{i=1}^{m}\sum_{s=1}^{S} x_{ijs} = b_j,\ \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijs} = p_s$$

$$\sum_{j=1}^{n}\sum_{s=1}^{S} y_{ijs} = a_i,\ \sum_{i=1}^{m}\sum_{s=1}^{S} y_{ijs} = b_j,\ \sum_{i=1}^{m}\sum_{j=1}^{n} y_{ijs} = p_s$$

$$\sum_{k=1}^{K} w_k = 1,\ 0 \le w_k \le 1\ \forall\ k = 1, 2 \ldots K,\ 0 \le \mu \le 1,\ \tau > 0$$

$$x_{ijs,}\, y_{ijs,} \ge 0\ \ \forall\ i = 1, 2 \ldots m,\ \ j = 1, 2 \ldots n,\ \ k = 1, 2 \ldots K\ \text{ and }\ s = 1, 2 \ldots S$$

$$\begin{cases} \wp_{ij} = 0, \text{ if } x_{ij} = 0 \\ \wp_{ij} = 1, \text{ if } x_{ij} > 0 \end{cases}$$

By utilizing this strategy, strategic planning can be accomplished under settings that are unpredictable and dynamic, while simultaneously improving operational efficiency and cost-effectiveness. There are numerous real-world applications for the strategy that was described, which span a variety of fields. It is possible to use it to supply chain optimization in order to strike a balance between delivery time, cost, and environmental concerns, which will result in the creation of effective logistics networks for many items. In addition, it helps with the distribution of resources in urban transportation planning by taking into consideration route capacity and fixed costs. These resources include things like food, gasoline, and medical supplies. This is especially important in the field of humanitarian logistics, where it ensures the effective distribution of essential products such as food, water, and medical supplies to areas that have been devastated by a disaster. It also maximizes coverage and decreases delivery delays. This strategy is also well-suited for retail and online commerce, since it allows for the efficient distribution of various items at a low cost, while simultaneously boosting customer satisfaction and ensuring that deliveries are made on time. In the context of energy distribution networks, it optimizes the transmission of various energy sources, such as gas, coal, and renewable energy, across areas while taking into consideration the impact on the environment and the fixed costs of setting up the network. By taking into account the complexities and uncertainties of these scenarios, the proposed methodologies provide decision makers with assistance in locating solutions for transportation systems that are not only feasible but also cost-effective and environmentally friendly.

3.1 *Solution process for solving the proposed approach (MOSFCTP using SFN (shown in Figure 6))*

Step 1: Start the Process.

Start with the initialization of the problem and the parameters needed to solve it.

Step 2: Solve the SFNs.

Solve SFNs using appropriate methods or techniques to handle implicit uncertainty. This will give us a representation of the uncertainty in the decision-making process.

Step 3: Convert the SFNs to a crisp value.

Once the SFNs is resolved, convert it to a crisp value using appropriate defuzzification methods (score function).

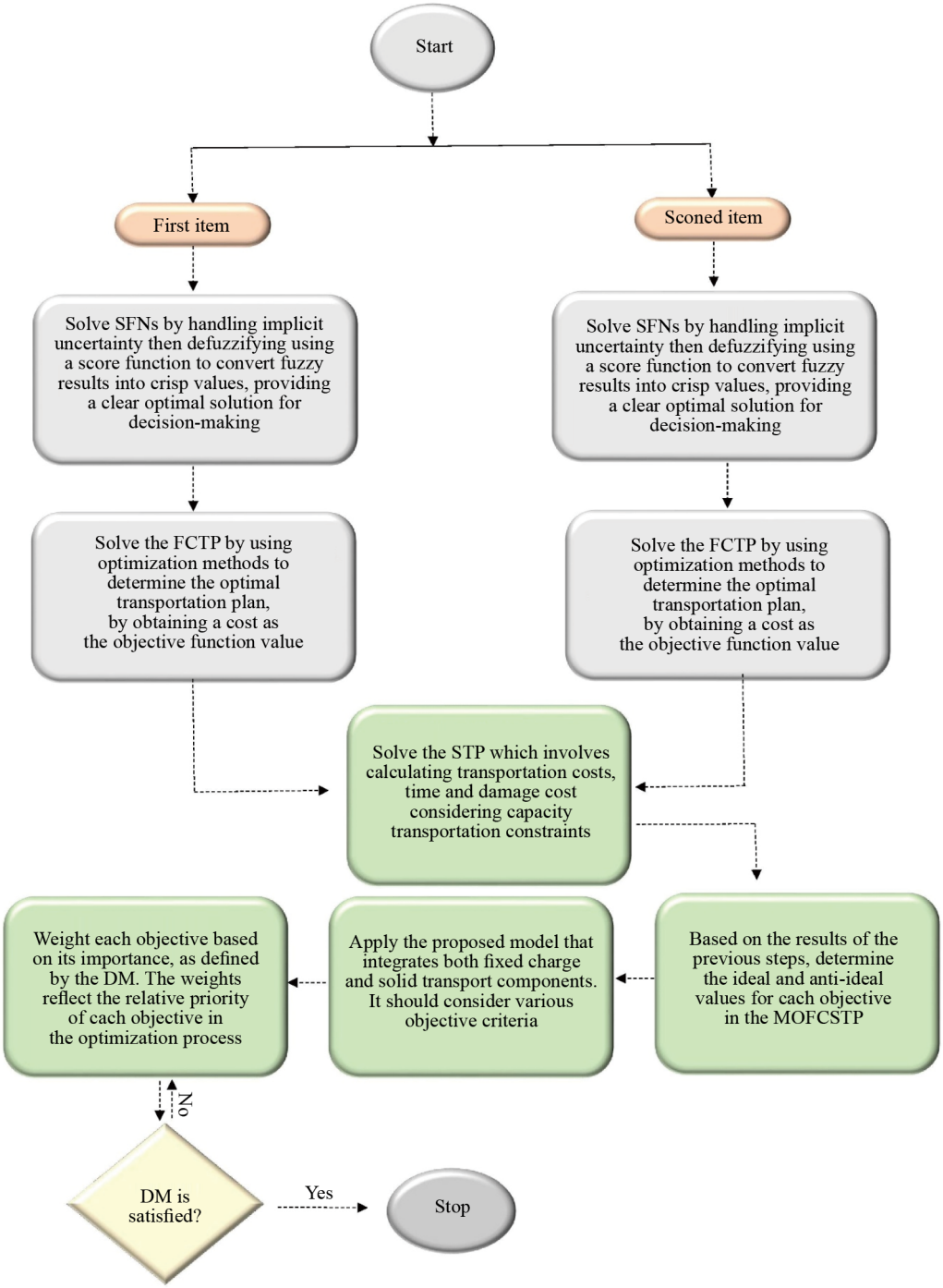


Figure 6. Solution framework for multi-item MOFCSTP

Step 4: Resolve the FCTP.

Solve the FCTP by using optimization methods to determine the optimal transportation plan, by obtaining a cost as the objective function value.

Step 5: Solve the STP.

Next, solve the STP which involves calculating transportation costs considering capacity transportation constraints.

Step 6: Find ideal and anti-ideal values for each objective.

Based on the results of the previous steps, determine the ideal (best possible) and anti-ideal (worst possible) values for each objective in the MOFCSTP.

Step 7: Apply the Proposed Model.

Apply the proposed model that integrates both fixed charge and solid transport components. It should consider various objective criteria.

Step 8: Assign a weight to each objective.

Weight each objective based on its importance, as defined by the Decision Maker (DM). The weights reflect the relative priority of each objective in the optimization process.

Step 9: Check the DM's Satisfaction.

Evaluate the results and present them to the decision maker. If the DM is satisfied with the results, proceed to step 10 and stop the process.

Step 10: If not satisfied, return to previous steps.

If the decision maker is not satisfied, revisit the previous steps, adjusting the model, weights, or other parameters to refine the solution until satisfaction is achieved.

Step 11: Stop the Process.

Once the DM is satisfied with the result, terminate the algorithm and finalize the solution.

4. Illustrative scenario

Take into consideration a multi-objective transportation problem that involves the transfer of two different commodities, x_{ij} and y_{ij} (for example, food and pharmaceuticals), from three different sources (S_1, S_2, S_3) to four different destinations (D_1, D_2, D_3, D_4). According to the information presented in the Appendix (Tables A1-A6), the shipment is comprised of two different goods and employs two distinct conviction levels. From the three separate sources, the supply amounts of the first item are in the following order: 85, 45, and 95 units, respectively. Seventy-two, fifty, sixty, and forty-three units are the required quantity of this item across all four locations, respectively. In a similar manner, the second item is supplied by three different sources, with 78, 62, and 50 units available, but the demand for it across the four different destinations is 55, 70, 30, and 35 units, respectively. For this particular situation, the objective is to minimize the three competing goals, which are the cost of transportation (Z_1), the amount of time (Z_2), and the amount of damage (Z_3). In order to take into consideration, the ambiguity and inaccuracy that are present in the data, the model utilizes spherical fuzzy numbers. Depending on the source-destination pair and the type of object, the prices of transportation, the amount of time required, and the potential damage costs for each item are different. In addition, there are two capacity restrictions $(p_1 = 225$ and $p_2 = 190)$ included in the transportation routes in order to guarantee that there is a connection between the origins and the destinations of the transportation routes. In order to achieve the goal of finding the ideal transportation strategy that effectively balances these three objectives, the purpose is to ensure that demand at each place is met while simultaneously minimizing overall costs, time, and damage across all routes.

Now, the score function is used to solve SFS and provide a crisp value from SFNs:

$$Score\left(\mathscr{H}^{sf}\right) = \left(\frac{\theta_{\mathscr{H}^{sf}} + 1 - \varphi_{\mathscr{H}^{sf}} + 1 - \delta_{\mathscr{H}^{sf}}}{3}\right)$$

$$Score\left(\mathscr{H}^{sf}\right) = \frac{1}{3}(2 + \theta_{\mathscr{H}^{sf}} - \varphi_{\mathscr{H}^{sf}} - \delta_{\mathscr{H}^{sf}})$$

This approach combines the indeterminacy degree (δ), non-membership degree (φ), and the membership degree (θ) to produce a crisp value (as shown in Appendix (Tables A7-A12)) for informed decision-making.

To solve the FCTP formulation (presented in Tables 4-6), the objective is to minimize the total cost incurred, which includes both fixed charges and variable costs.

Table 4. Data on crisp transportation cost Z_1 for item 1 and item 2 under different conviction levels

Conveyance = 1	D_1		D_2		D_3		D_4	
	Item = 1	Item = 2	Item = 1	Item = 2	Item = 1	Item = 2	Item = 1	Item = 2
S_1	0.677	0.607	0.84	0.559	0.613	0.822	0.565	0.749
S_2	0.413	0.508	0.547	0.678	0.812	0.548	0.410	0.613
S_3	0.506	0.482	0.741	0.41	0.67	0.517	0.539	0.681
Conveyance = 2	D_1		D_2		D_3		D_4	
	Item = 1	Item = 2	Item = 1	Item = 2	Item = 1	Item = 2	Item = 1	Item = 2
S_1	0.43	0.55	0.63	0.5	0.67	0.83	0.67	0.67
S_2	0.47	0.6	0.53	0.8	0.55	0.57	0.47	0.47
S_3	0.6	0.47	0.5	0.57	0.53	0.6	0.4	0.4

Table 5. Data on crisp transportation time Z_2 for item 1 and item 2 under different conviction levels

Conveyance = 1	D_1		D_2		D_3		D_4	
	Item = 1	Item = 2	Item = 1	Item = 2	Item = 1	Item = 2	Item = 1	Item = 2
S_1	0.836	0.484	0.511	0.6118	0.61	0.85	0.715	0.545
S_2	0.541	0.408	0.610	0.511	0.411	0.692	0.56	0.543
S_3	0.805	0.742	0.679	0.646	0.408	0.497	0.813	0.411
Conveyance = 2	D_1		D_2		D_3		D_4	
	Item = 1	Item = 2	Item = 1	Item = 2	Item = 1	Item = 2	Item = 1	Item = 2
S_1	0.6	0.67	0.55	0.83	0.8	0.6	0.73	0.55
S_2	0.5	0.4	0.67	0.53	0.53	0.8	0.6	0.4
S_3	0.47	0.5	0.4	0.73	0.5	0.66	0.67	0.53

Table 6. Data on crisp damage cost Z_3 for item 1 and item 2 under different conviction levels

Conveyance = 1	D_1		D_2		D_3		D_4	
	Item = 1	Item = 2	Item = 1	Item = 2	Item = 1	Item = 2	Item = 1	Item = 2
S_1	0.481	0.810	0.81	0.539	0.678	0.743	0.416	0.849
S_2	0.383	0.540	0.687	0.510	0.414	0.69	0.543	0.492
S_3	0.407	0.613	0.54	0.482	0.611	0.424	0.713	0.385
Conveyance = 2	D_1		D_2		D_3		D_4	
	Item = 1	Item = 2	Item = 1	Item = 2	Item = 1	Item = 2	Item = 1	Item = 2
S_1	0.8	0.83	0.83	0.5	0.6	0.6	0.53	0.7
S_2	0.47	0.53	0.73	0.6	0.67	0.4	0.47	0.55
S_3	0.6	0.8	0.8	0.67	0.83	0.4	0.4	0.8

5. Results and discussion

Maximize

$$= \frac{\mu}{w_k \left[\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K}(c_{ijs}x_{ijs}+\mathscr{S}_{ijs}\wp_{ijs}) - \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K}(c_{ijs}x_{ijs}+\mathscr{S}_{ijs}\wp_{ijs})^{Min}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K}(c_{ijs}x_{ijs}+\mathscr{S}_{ijs}\wp_{ijs})^{Max} - \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K}(c_{ijs}x_{ijs}+\mathscr{S}_{ijs}\wp_{ijs})^{Min}} \right]}$$

Subject to:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K}(c_{ijs}x_{ijs}+\mathscr{S}_{ijs}\wp_{ijs}) \leq \left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K}(c_{ijs}x_{ijs}+\mathscr{S}_{ijs}\wp_{ijs})\right)^{*} + \tau(1-w_k)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K}(c_{ijs}x_{ijs}+s_{ijs}\wp_{ijs}) + \left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K}(c_{ijs}x_{ijs}+s_{ijs}\wp_{ijs})^{u}\right)$$

$$-\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K}(c_{ijs}x_{ijs}+s_{ijs}\wp_{ijs})^{\mathscr{L}}\right)$$

$$\leq \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K}(c_{ijs}x_{ijs}+s_{ijs}\wp_{ijs})^{u}$$

$$\sum_{j=1}^{n}\sum_{s=1}^{S}x_{ijs}, y_{ijs} = a_i, \ \sum_{i=1}^{m}\sum_{s=1}^{S}x_{ijs}, y_{ijs} = b_j, \ \sum_{i=1}^{m}\sum_{j=1}^{n}x_{ijs}, y_{ijs} = p_s$$

$$\sum_{k=1}^{K} w_k = 1, 0 \leq w_k \leq 1 \ \forall \ k = 1, 2 \ldots K, 0 \leq \mu \leq 1, \tau > 0$$

$$x_{ijs,} y_{ijs} \geq 0 \ \forall \ i = 1, 2 \ldots m, \ j = 1, 2 \ldots n, \ k = 1, 2 \ldots K \ \text{and} \ s = 1, 2 \ldots S$$

$$\begin{cases} \wp_{ij} = 0, & \text{if } x_{ij} = 0 \\ \wp_{ij} = 1, & \text{if } x_{ij} > 0 \end{cases}$$

The performance of the three models (Proposed, Joshi, and WSM) varies significantly depending on how the objectives are weighted. Different weight parameters (w_1, w_2, and w_3 for Z_1, Z_2, and Z_3, respectively) were applied to evaluate each model. Table 7 presents the results, showing the objective function output at different weight settings for each model. This

presentation facilitates understanding of which objective is prioritized and how effectively each model performs in various decision-making scenarios.

Table 7. Comparison of objective function for different weight combinations and decision scenarios in different models

Weight	Proposed Model			Joshi's Model			WSM Model		
	Z_1	Z_2	Z_3	Z_1	Z_2	Z_3	Z_1	Z_2	Z_3
0.1, 0.9, 0.0	222.12	208.76	237.37	227.22	210.7	255.6	222.66	208.15	251.26
0.2, 0.8, 0.0	222.12	208.76	232.11	223.78	213.87	255.86	219.93	208.66	251.26
0.3, 0.7, 0.0	223.02	213.74	230.4	222.66	217.05	264.84	212.51	210.96	232.64
0.4, 0.0, 0.6	218.99	226.94	214.93	229.27	236.87	202.07	233.52	271.03	225.69
0.5, 0.0, 0.5	218.99	226.94	219.92	224.37	231.85	216.12	218.72	271.84	225.69
0.6, 0.0, 0.4	210.78	231.43	225.18	219.61	226.98	227.3	209.93	253.46	232.38
0.0, 0.3, 0.7	219.43	226.79	210.23	233.97	241.68	199.41	266.03	245.58	225.46
0.0, 0.2, 0.8	220.61	228.00	205.57	240.33	248.19	199.41	266.03	245.58	223.75
0.0, 0.1, 0.9	233.59	241.29	200.6	247.6	255.63	195.04	260.58	268.92	210.54
0.3, 0.3, 0.4	219.43	226.79	218.29	225.97	233.49	223.84	223.58	224.73	225.46
0.3, 0.4, 0.3	223.65	214.14	222.14	222.19	229.62	232.64	223.02	213.74	231.9
0.4, 0.3, 0.3	218.99	226.94	221.85	222.48	229.91	234.84	213.47	220.49	225.6
Average	220.977	223.3767	219.8825	228.2875	231.32	225.58	230.83	236.93	230.1433
Ideal	199.59	206.48	195.14	199.59	206.48	195.14	199.59	206.48	195.14
Anti-Ideal	260.58	268.91	288.33	260.58	268.91	288.33	260.58	268.91	288.33

Z_1 (primary objective): With an average of 220.98, the proposed model consistently provided values that were closest to the optimal value for Z_1. This outcome shows that the model can achieve this goal while showing comparatively strong performance over the other two across a variety of weight combinations. In instances where Z_1 is weighted adequately ($w_1 = 0.1$, $w_2 = 0.9$, and $w_3 = 0.0$) the model performed remarkably well, providing one of the lowest Z_1 values compared to other models. There is still room for growth to get closer to the ideal value. Z_1 (primary objective): With an average of 228.29, Joshi's model performed somewhat worse than the proposed model for Z_1. This suggests that in situations where minimizing Z_1 is important, the model may not be as appropriate. For most weight combinations, the model produces greater values for Z_1, especially when Z_1 is assigned a lower weight. For different optimization goals, the Joshi model remains a good choice, despite its poor performance for Z_1. Z_2 (secondary objective): The average performance of the proposed model for Z_2 is 223.38, which is reasonably consistent. This model, while not the greatest, performs competitively in most weight combinations. This shows that although the proposed model effectively prioritizes Z_1, it performs less well for Z_2, especially when Z_2 is given high importance in the decision-making process ($w_1 = 0.2$, $w_2 = 0.8$, and $w_3 = 0.0$) (show in Figure 7). In terms of stability and preventing significant deviations from the norm, it outperforms Joshi's model.

Z_2 (secondary objective): Joshi's model had the best performance for this objective, minimizing Z_2 with an average value of 231.32. This is especially noticeable when Z_2 was assigned high weights ($w_1 = 0.0$, $w_2 = 0.3$, and $w_3 = 0.7$) (show in Figure 8). A compromise between balancing all objectives and optimizing for one is highlighted by the fact that this success comes at the cost of reduced performance for primary and secondary objectives.

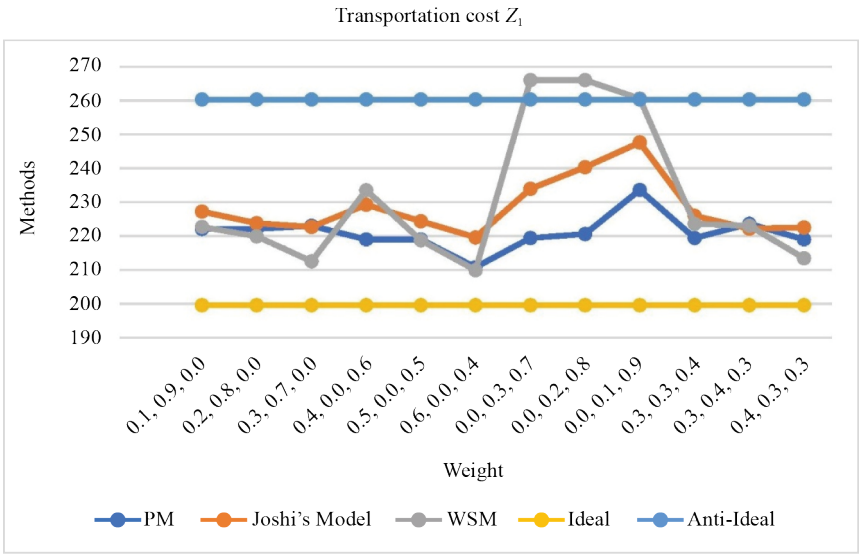


Figure 7. Impact of weight distribution on transportation cost Z_1 across different methods (proposed model, Joshi's model, WSM, optimal and anti-ideal value)

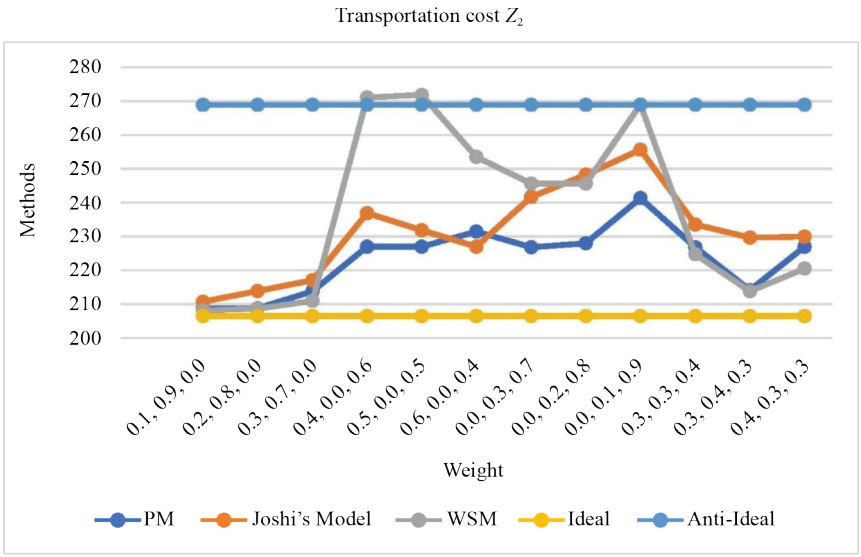


Figure 8. Impact of weight distribution on transportation time Z_2 across different methods (proposed model, Joshi's model, WSM, optimal, and anti-ideal value)

Z_3 (third objective): The proposed model had the best balance between different weights of Z_3, averaging 219.88. The results were quite close to the optimal value of 195.14, especially for weight combinations in which Z_3 was either medium or low weight. For this purpose, the proposed model performed most consistently compared to other models (shown in Figure 9). Z_3 (third objective): Joshi's model performed the worst for Z_3 with an average score of 225.58. The model's performance for Z_3 was consistently better than the other models, suggesting that Joshi's model biases the findings in favour of Z_3, resulting in inferior performance for Z_3 when all three objectives are considered. This is still a good choice when Z_3 optimization is more important.

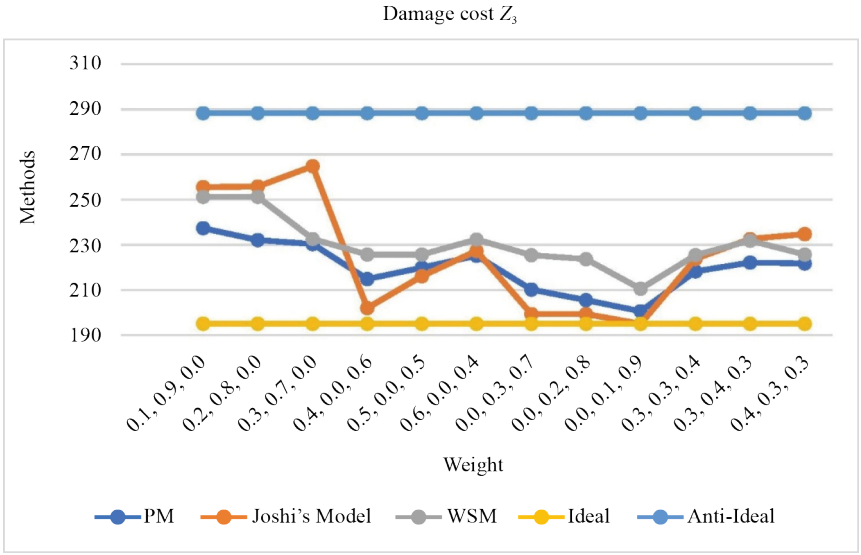


Figure 9. Impact of weight distribution on damage cost Z_3 across different methods (proposed model, Joshi's model, WSM, optimal, and anti-ideal value)

When compared to earlier models, the suggested model has a superior performance for Z_1 with average values of 220.98, which is relatively near to the number that is considered to be optimal, which is 199.59. Although the WSM model also performs well, Joshi's model produces somewhat greater numbers, which indicates that it performs less perfect in terms of the primary purpose. The WSM model (236.93) and the model given for Z_2 (223.38) are both outperformed by Joshi's model, which demonstrates the best average value of 231.32 among comparable models. On the other hand, the WSM model generates results that are more consistent across a variety of weights, which makes it an appealing option in circumstances when decision consistency is of utmost importance. The proposed model has a good overall performance for Z_3, with an average of 219.88, which is relatively near to the value that is considered to be ideal, which is 195.14. The WSM model and the Joshi model both have high numbers; the Joshi model consistently generates values around 230.14, which indicates that it has a tendency to target upside to a higher extent than other models. Particularly with regard to Z_1 and Z_3, the performance of the suggested model was significantly closer to the values that were ideal. Since this is the case, it suggests that it might produce superior results in optimization problems where it is necessary to minimize Z_1 and Z_3. Additionally, its performance across all load combinations demonstrates that it is capable of achieving an acceptable balance between the three goals. Joshi's model performed extraordinarily well for Z_2, but it drifted much further from the ideal for Z_1 and Z_3 than it did for Z_2. It is possible that the performance of the model could be improved provided that the objectives were more evenly distributed. On the other hand, when this objective is of greater significance, the Z_2 is an outstanding alternative due to the superb performance it offers. It was proved that the WSM model is effective in restricting non-ideal results, particularly for Z_2, by the fact that it maintains a consistent distance from anti-ideal values. The stability of the WSM model is advantageous, particularly in practical applications that demand the capability to make decisions in a reliable manner. When all factors are taken into account, the suggested model has the best overall performance, taking into account the equilibrium of all three objectives, particularly for Z_1 and Z_3. Despite the fact that Joshi's model is quite good at maximizing Z_2, it is not as well balanced when other goals are taken into consideration. Although it is somewhat less efficient for Z_1 and Z_3, the WSM model may be more dependable in certain decision-making circumstances due to its stable and consistent performance across all objective functions. This is despite the fact that it is slightly less efficient for Z_1 and Z_3. In subsequent study, it may be possible to examine making tweaks to the model in order to further improve its equilibrium for specific weight combinations. Model selection in real-world applications will be heavily influenced by the relative relevance of each target, which will play a significant role. For instance, the proposed model would be most appropriate for use in the optimization of transportation or logistics when time and cost (represented by Z_1 and Z_3) are

essential needs. As an alternative, Joshi's model might be a more suitable option for supply chain optimization in situations where resource allocation Z_2 is of significant importance.

6. Conclusion

The study comes to a conclusion by resolving the difficulties associated with DM in dynamic and complex logistics networks. This is accomplished by establishing a robust MOSTP that takes into account the adaptability of multi-commodity flows, fixed charges, and SFNs. The approach that has been offered provides enhanced customer happiness and reliable service by efficiently balancing any competing goals, such as minimizing the costs of transportation, the amount of time it takes to deliver the product, and fixed costs. The versatility and practical utility of this method is demonstrated by its application to real-world scenarios, such as the allocation of resources, the distribution of products in metropolitan areas, and the management of supply chains under instances of uncertainty. This methodology enhances both the operational efficiency and the outcomes of decision-making by providing decision-makers with the ability to systematically prioritize and ensure that objectives are balanced through the utilization of ranking tools. The suggested model exceeds the current technique, as demonstrated by the comparison study, which reveals that it achieves 91% efficiency in ranking, beating Joshi's model by 70% and the WSM model by 57%, and coming close to the optimal performance of 100% (as shown in Table 8 and Figure 10). Through the generation of solutions that are extremely close to the ideal value, the MOFCSTP that has been proposed has proven to be quite advantageous across a wide range of sectors. Through the incorporation of fixed charges, multi-commodity flows, and uncertainties, this model offers decision makers options that are dependable, cost-effective, and flexible in order to address actual logistics difficulties.

Table 8. Comparative performance analysis of objective function models based on ideal, anti-ideal values and ranking

Method	Z_1	Z_2	Z_3	%	Rank
WSM Model	230.83	236.92	230.14	57%	4
Joshi's Model	228.28	231.32	225.58	70%	3
Proposed Model	220.97	223.37	219.88	91%	2
Ideal-Value	199.59	206.48	195.14	100%	1
Anti Ideal-Value	260.58	268.91	288.33	0	5

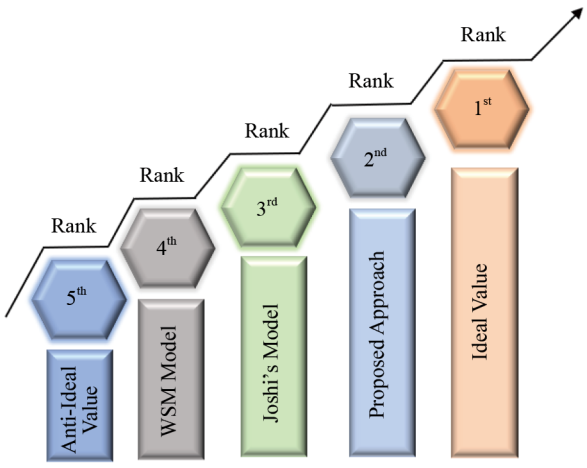


Figure 10. Ranking framework for multi-item MOFCSTP

Author contributions

Vishwas Deep and Priya: conceptualization and writing—original draft. Huda and Lenka: investigation, resources, supervision, validation, and writing—review and editing. B. Swarna: conceptualization, supervision and writing—review and editing, visualization. Ajay: conceptualization, methodology, investigation, supervision, and writing—review and editing.

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Conflict of interest

The authors declare no competing financial interest.

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Appendix

Table A1. Spherical fuzzy transportation cost Z_1 and fixed charge parameters for item 1 under different conviction levels

Conveyance = 1		D_1	D_2	D_3	D_4
S_1	c_{ij}	(0.5, 0.3, 0.2)	(0.75, 0.25, 0)	(0.4, 0.4, 0.2)	(0.33, 0.33, 0.34)
	$\mathscr{S}_{ij}$	(0.33, 0.34, 0.33)	(0.25, 0.5, 0.25)	(0.75, 0.15, 0.1)	(0.5, 0.25, 0.25)
S_2	c_{ij}	(0.1, 0.6, 0.3)	(0.3, 0.5, 0.2)	(0.7, 0.2, 0.1)	(0.1, 0.45, 0.45)
	$\mathscr{S}_{ij}$	(0.4, 0.4, 0.2)	(0.7, 0.2, 0.1)	(0.35, 0.35, 0.3)	(0.2, 0.3, 0.5)
S_3	c_{ij}	(0.25, 0.25, 0.5)	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.3, 0.4, 0.3)
	$\mathscr{S}_{ij}$	(0.2, 0.3, 0.5)	(0.35, 0.35, 0.3)	(0.4, 0.3, 0.3)	(0.1, 0.6, 0.3)
Conveyance = 2		D_1	D_2	D_3	D_4
S_1		(0.15, 0.5, 0.35)	(0.45, 0.45, 0.1)	(0.5, 0.25, 0.25)	(0.5, 0.4, 0.1)
S_2		(0.2, 0.5, 0.3)	(0.3, 0.4, 0.3)	(0.33, 0.34, 0.33)	(0.2, 0.3, 0.5)
S_3		(0.4, 0.5, 0.1)	(0.25, 0.4, 0.35)	(0.3, 0.4, 0.3)	(0.1, 0.45, 0.45)

Table A2. Spherical fuzzy transportation cost Z_1 and fixed charge parameters for item 2 under different conviction levels

Conveyance = 1		D_1	D_2	D_3	D_4
S_1	c_{ij}	(0.4, 0.3, 0.3)	(0.33, 0.33, 0.34)	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)
	$\mathscr{S}_{ij}$	(0.15, 0.5, 0.35)	(0.45, 0.45, 0.1)	(0.5, 0.25, 0.25)	(0.5, 0.4, 0.1)
S_2	c_{ij}	(0.25, 0.25, 0.5)	(0.5, 0.4, 0.1)	(0.3, 0.3, 0.4)	(0.4, 0.35, 0.25)
	$\mathscr{S}_{ij}$	(0.2, 0.5, 0.3)	(0.3, 0.4, 0.3)	(0.33, 0.34, 0.33)	(0.2, 0.3, 0.5)
S_3	c_{ij}	(0.2, 0.7, 0.1)	(0.1, 0.3, 0.6)	(0.25, 0.5, 0.25)	(0.5, 0.25, 0.25)
	$\mathscr{S}_{ij}$	(0.4, 0.5, 0.1)	(0.25, 0.4, 0.35)	(0.3, 0.4, 0.3)	(0.1, 0.45, 0.45)
Conveyance = 2		D_1	D_2	D_3	D_4
S_1		(0.33, 0.34, 0.33)	(0.25, 0.5, 0.25)	(0.75, 0.15, 0.1)	(0.5, 0.25, 0.25)
S_2		(0.4, 0.4, 0.2)	(0.7, 0.2, 0.1)	(0.35, 0.35, 0.3)	(0.2, 0.3, 0.5)
S_3		(0.2, 0.3, 0.5)	(0.35, 0.35, 0.3)	(0.4, 0.3, 0.3)	(0.1, 0.6, 0.3)

Table A3. Spherical fuzzy transportation time Z_2 and fixed charge parameters for item 1 under different conviction levels

Conveyance = 1		D_1	D_2	D_3	D_4
S_1	c_{ij}	(0.75, 0.25, 0)	(0.25, 0.75, 0)	(0.4, 0.25, 0.35)	(0.55, 0.3, 0.15)
	$\mathscr{S}_{ij}$	(0.25, 0.75, 0)	(0.35, 0.35, 0.3)	(0.4, 0.25, 0.35)	(0.5, 0.4, 0.1)
S_2	c_{ij}	(0.3, 0.5, 0.2)	(0.4, 0.3, 0.3)	(0.1, 0.4, 0.5)	(0.33, 0.34, 0.33)
	$\mathscr{S}_{ij}$	(0.3, 0.2, 0.5)	(0.2, 0.7, 0.1)	(0.3, 0.4, 0.3)	(0.15, 0.3, 0.55)
S_3	c_{ij}	(0.7, 0.2, 0.1)	(0.5, 0.3, 0.2)	(0.1, 0.3, 0.6)	(0.7, 0.1, 0.2)
	$\mathscr{S}_{ij}$	(0.1, 0.3, 0.6)	(0.2, 0.4, 0.4)	(0.25, 0.75, 0)	(0.4, 0.35, 0.25)
Conveyance = 2		D_1	D_2	D_3	D_4
S_1		(0.4, 0.3, 0.3)	(0.33, 0.33, 0.34)	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)
S_2		(0.25, 0.25, 0.5)	(0.5, 0.4, 0.1)	(0.3, 0.3, 0.4)	(0.4, 0.35, 0.25)
S_3		(0.2, 0.7, 0.1)	(0.1, 0.3, 0.6)	(0.25, 0.5, 0.25)	(0.5, 0.25, 0.25)

Table A4. Spherical fuzzy transportation time Z_2 and fixed charge parameters for item 2 under different conviction levels

Conveyance = 1		D_1	D_2	D_3	D_4
S_1	c_{ij}	(0.2, 0.3, 0.5)	(0.4, 0.25, 0.35)	(0.75, 0.15, 0.1)	(0.3, 0.3, 0.4)
	$\mathscr{S}_{ij}$	(0.7, 0.2, 0.1)	(0.75, 0.15, 0.1)	(0.4, 0.25, 0.35)	(0.3, 0.5, 0.2)
S_2	c_{ij}	(0.1, 0.45, 0.45)	(0.25, 0.75, 0)	(0.5, 0.25, 0.25)	(0.3, 0.2, 0.5)
	$\mathscr{S}_{ij}$	(0.2, 0.5, 0.3)	(0.6, 0.1, 0.3)	(0.5, 0.4, 0.1)	(0.2, 0.5, 0.3)
S_3	c_{ij}	(0.6, 0.3, 0.1)	(0.45, 0.45, 0.1)	(0.2, 0.3, 0.5)	(0.1, 0.3, 0.6)
	$\mathscr{S}_{ij}$	(0.4, 0.35, 0.25)	(0.7, 0.1, 0.2)	(0.75, 0.1, 0.15)	(0.1, 0.45, 0.45)
Conveyance = 2		D_1	D_2	D_3	D_4
S_1		(0.5, 0.3, 0.2)	(0.75, 0.25, 0)	(0.4, 0.4, 0.2)	(0.33, 0.33, 0.34)
S_2		(0.1, 0.6, 0.3)	(0.3, 0.5, 0.2)	(0.7, 0.2, 0.1)	(0.1, 0.45, 0.45)
S_3		(0.25, 0.25, 0.5)	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.3, 0.4, 0.3)

Table A5. Spherical fuzzy damage cost Z_3 and fixed charge parameters for item 1 under different conviction levels

Conveyance = 1		D_1	D_2	D_3	D_4
S_1	c_{ij}	(0.2, 0.3, 0.5)	(0.7, 0.2, 0.1)	(0.5, 0.4, 0.1)	(0.1, 0.45, 0.45)
	$\mathscr{S}_{ij}$	(0.7, 0.2, 0.1)	(0.25, 0.4, 0.35)	(0.3, 0.2, 0.5)	(0.55, 0.15, 0.3)
S_2	c_{ij}	(0.1, 0.25, 0.75)	(0.5, 0.4, 0.1)	(0.1, 0.3, 0.6)	(0.3, 0.5, 0.2)
	$\mathscr{S}_{ij}$	(0.4, 0.3, 0.3)	(0.7, 0.1, 0.2)	(0.5, 0.25, 0.25)	(0.4, 0.25, 0.35)
S_3	c_{ij}	(0.1, 0.6, 0.3)	(0.3, 0.5, 0.2)	(0.4, 0.25, 0.35)	(0.55, 0.3, 0.15)
	$\mathscr{S}_{ij}$	(0.35, 0.3, 0.35)	(0.25, 0.75, 0)	(0.5, 0.1, 0.4)	(0.34, 0.33, 0.33)
Conveyance = 2		D_1	D_2	D_3	D_4
S_1		(0.7, 0.2, 0.1)	(0.75, 0.15, 0.1)	(0.4, 0.25, 0.35)	(0.3, 0.5, 0.2)
S_2		(0.2, 0.5, 0.3)	(0.6, 0.1, 0.3)	(0.5, 0.4, 0.1)	(0.2, 0.5, 0.3)
S_3		(0.4, 0.35, 0.25)	(0.7, 0.1, 0.2)	(0.75, 0.1, 0.15)	(0.1, 0.45, 0.43)

Table A6. Spherical fuzzy damage cost Z_3 and fixed charge parameters for item 3 under different conviction levels

Conveyance = 1		D_1	D_2	D_3	D_4
S_1	c_{ij}	(0.7, 0.2, 0.1)	(0.3, 0.5, 0.2)	(0.6, 0.3, 0.1)	(0.75, 0.1, 0.15)
	$\mathscr{S}_{ij}$	(0.34, 0.33, 0.33)	(0.5, 0.5, 0.0)	(0.1, 0.45, 0.45)	(0.5, 0.4, 0.1)
S_2	c_{ij}	(0.3, 0.2, 0.5)	(0.25, 0.4, 0.35)	(0.5, 0.1, 0.4)	(0.2, 0.3, 0.5)
	$\mathscr{S}_{ij}$	(0.4, 0.3, 0.3)	(0.5, 0.25, 0.25)	(0.4, 0.25, 0.35)	(0.7, 0.2, 0.1)
S_3	c_{ij}	(0.4, 0.5, 0.1)	(0.2, 0.3, 0.5)	(0.1, 0.2, 0.7)	(0.1, 0.25, 0.75)
	$\mathscr{S}_{ij}$	(0.5, 0.4, 0.1)	(0.4, 0.3, 0.3)	(0.6, 0.1, 0.3)	(0.3, 0.4, 0.3)
Conveyance = 2		D_1	D_2	D_3	D_4
S_1		(0.75, 0.25, 0)	(0.25, 0.75, 0)	(0.4, 0.25, 0.35)	(0.55, 0.3, 0.15)
S_2		(0.3, 0.5, 0.2)	(0.4, 0.3, 0.3)	(0.1, 0.4, 0.5)	(0.33, 0.34, 0.33)
S_3		(0.7, 0.2, 0.1)	(0.5, 0.3, 0.2)	(0.1, 0.3, 0.6)	(0.7, 0.1, 0.2)

Table A7. Data on crisp transportation cost Z_1 for item 1 under different conviction levels

Conveyance = 1		D_1	D_2	D_3	D_4
S_1	c_{ij}	0.67	0.83	0.60	0.55
	$\mathscr{S}_{ij}$	0.55	0.50	0.83	0.67
S_2	c_{ij}	0.40	0.53	0.80	0.40
	$\mathscr{S}_{ij}$	0.60	0.80	0.57	0.47
S_3	c_{ij}	0.50	0.73	0.66	0.53
	$\mathscr{S}_{ij}$	0.47	0.57	0.60	0.40
Conveyance = 2		D_1	D_2	D_3	D_4
S_1		0.43	0.63	0.67	0.67
S_2		0.47	0.53	0.55	0.47
S_3		0.60	0.50	0.53	0.40

Table A8. Data on crisp transportation cost Z_1 for item 2 under different conviction levels

Conveyance = 1		D_1	D_2	D_3	D_4
S_1	c_{ij}	0.60	0.55	0.80	0.73
	$\mathscr{S}_{ij}$	0.43	0.63	0.67	0.67
S_2	c_{ij}	0.50	0.67	0.53	0.60
	$\mathscr{S}_{ij}$	0.47	0.53	0.55	0.47
S_3	c_{ij}	0.47	0.40	0.50	0.67
	$\mathscr{S}_{ij}$	0.60	0.50	0.53	0.40
Conveyance = 2		D_1	D_2	D_3	D_4
S_1		0.55	0.50	0.83	0.67
S_2		0.60	0.80	0.57	0.47
S_3		0.47	0.57	0.60	0.40

Table A9. Data on crisp transportation time Z_2 for item 1 under different conviction levels

Conveyance = 1		D_1	D_2	D_3	D_4
S_1	c_{ij}	0.83	0.50	0.60	0.70
	$\mathscr{S}_{ij}$	0.50	0.57	0.60	0.67
S_2	c_{ij}	0.53	0.60	0.40	0.55
	$\mathscr{S}_{ij}$	0.53	0.47	0.53	0.43
S_3	c_{ij}	0.80	0.67	0.40	0.80
	$\mathscr{S}_{ij}$	0.40	0.47	0.50	0.60
Conveyance = 2		D_1	D_2	D_3	D_4
S_1		0.60	0.55	0.80	0.73
S_2		0.50	0.67	0.53	0.60
S_3		0.47	0.40	0.50	0.67

Table A10. Data on crisp transportation time Z_2 for item 2 under different conviction levels

Conveyance = 1		D_1	D_2	D_3	D_4
S_1	c_{ij}	0.47	0.60	0.83	0.53
	$\mathscr{S}_{ij}$	0.80	0.83	0.60	0.53
S_2	c_{ij}	0.40	0.50	0.67	0.53
	$\mathscr{S}_{ij}$	0.47	0.73	0.67	0.47
S_3	c_{ij}	0.73	0.63	0.47	0.40
	$\mathscr{S}_{ij}$	0.60	0.80	0.83	0.40
Conveyance = 2		D_1	D_2	D_3	D_4
S_1		0.67	0.83	0.60	0.55
S_2		0.40	0.53	0.80	0.40
S_3		0.50	0.73	0.66	0.53

Table A11. Data on crisp damage cost Z_3 for item 1 under different conviction levels

Conveyance = 1		D_1	D_2	D_3	D_4
S_1	c_{ij}	0.47	0.80	0.67	0.40
	$\mathscr{S}_{ij}$	0.80	0.50	0.53	0.70
S_2	c_{ij}	0.37	0.67	0.40	0.53
	$\mathscr{S}_{ij}$	0.60	0.80	0.67	0.60
S_3	c_{ij}	0.40	0.53	0.60	0.70
	$\mathscr{S}_{ij}$	0.57	0.50	0.67	0.56
Conveyance = 2		D_1	D_2	D_3	D_4
S_1		0.80	0.83	0.60	0.53
S_2		0.47	0.73	0.67	0.47
S_3		0.60	0.80	0.83	0.40

Table A12. Data on crisp damage cost Z_3 for item 2 under different conviction levels

Conveyance = 1		D_1	D_2	D_3	D_4
S_1	c_{ij}	0.80	0.53	0.73	0.83
	$\mathscr{S}_{ij}$	0.56	0.67	0.40	0.67
S_2	c_{ij}	0.53	0.50	0.67	0.47
	$\mathscr{S}_{ij}$	0.60	0.67	0.60	0.80
S_3	c_{ij}	0.60	0.47	0.40	0.37
	$\mathscr{S}_{ij}$	0.67	0.60	0.73	0.53
Conveyance = 2		D_1	D_2	D_3	D_4
S_1		0.83	0.50	0.60	0.70
S_2		0.53	0.60	0.40	0.55
S_3		0.80	0.67	0.40	0.80