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Research Article

Integrating Intuitionistic Fuzzy Diamond-Shaped Sets with Combinative Distance-Based Assessment Method for Multi-Criteria Decision-Making

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Abstract: In diverse decision values may find it difficult to offer a coherent and truthful viewpoint in situations with multiple decision makers. In order to overcome the limitation, this work introduces the Diamond Intuitionistic Fuzzy Set (Dia-IFS). The Dia-IFS model, an extended version of the Intuitionistic Fuzzy Set (IFS) model, provides better performance by extending intuitionistic fuzzy sets to Interval-Valued Intuitionistic Fuzzy Set (IVIFS). By describing its components using Diamond Intuitionistic Fuzzy Values (Dia-IFVs), the Dia-IFS framework improves the IFS. The main features of fundamental algebraic and arithmetic operations, including union, intersection, addition, multiplication, and scalar multiplication, are examined and codified for Dia-IFVs. Additionally, new Diamond Intuitionistic Fuzzy (Dia-IF) weighted average and geometric aggregation operators are presented, and their special features are thoroughly examined. Using t-norms and t-conorms, algebraic procedures between Dia-IFVs are devised. Specialized weighted aggregation operators are suggested in order to combine input values represented by Dia-IFVs into a single output. Additionally, by utilizing both Euclidean and Hamming distances, the Dia-IF framework incorporates the "Combinative Distance-based Assessment" (CODAS) methodology. Through comparison analysis and instructive instances, the Dia-IF model's usefulness is illustrated. The outcomes demonstrate the framework's viability and efficiency in assessing and choosing the best options, expanding its possible uses in challenging decision-making situations.

***Keywords*:** diamond intuitionistic fuzzy set, weighted aggregation operators, euclidean distance, hamming distance, combinative distance-based assessment method, multi-criteria decision-making

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1. Introduction

Fuzzy sets, which extend the notion of the characteristic function to membership functions that assign degrees of membership between 0 and 1 to elements within a set, were first proposed by Zadeh [1] as a way to describe ambiguous and partial data. However, ambiguous data may not always be well modeled by membership functions alone. Alternative fuzzy set theories have been put out to better capture uncertain information in order to get around this restriction. Atanassov [2] proposed the idea of Intuitionistic Fuzzy Sets (IFS), which incorporates a non-membership function. The membership and non-membership functions in an IFS combine to create a pair of values between 0 and 1, with the restriction that their aggregate cannot be more than 1. This gives more information that is unclear and ambiguous a more thorough depiction. Feng et al. [3] introduced the concept of Minkowski weighted score functions of intuitionistic fuzzy values. Atanassov [4] proposed the idear of interval-valued intuitionistic fuzzy sets. Additional, they proposed some operations with non-trivial examples. Alcantud and Santos-García [5] presented the some transformation techniques for interval-valued intuitionistic fuzzy sets with applications. Li and Zhang [7, 6] discussed Threshold-based value-driven method to support consensus reaching in multi-criteria group sorting problems for a minimum adjustment perspective. Additionally, they have also proposed the modeling personalized individual semantics in Multi-Criteria Decision-Making (MCDM) with incomplete linguistic preference relations for a preference disaggregation perspective. Moreover, Liang [8] introduced the constructive preference elicitation for MCDM using an estimate-then-select strategy. Atanassov [9] extended the idea to Circular Intuitionistic Fuzzy Sets (C-IFSs), further developing the IFS notion. Each element in a C-IFS is represented by a circle, whose radius ranges from 0 to 1 and whose center is established by the membership and non-membership degrees. Compared to point-based representations, the usage of circles in C-IFS provides a more flexible and expressive way to portray ambiguous and inconsistent information. MCDM is one of the fields where the C-IFS idea has been implemented. The circular membership features in C-IFS enable decision-makers to define degrees and preferences, facilitating a more comprehensive assessment of options throughout the decision-making process. As a result, C-IFS has been used in a number of research, including Yazdani et al. [10], to solve MCDM obstacles and problems resulting from imprecise and unclear data. To sum up, decision-makers now have a strong tool for modeling and evaluating complicated and uncertain data thanks to the advent of C-IFS as an extension of fuzzy set theory. C-IFSs are a useful framework for addressing MCDM issues because they employ circular membership functions, which provide a more reliable and efficient method of collecting and expressing ambiguity and inconsistency.

The well-established field of MCDM entails selecting the best option based on expert assessments and conflicting criteria. The literature describes a wide range of MCDM techniques, each specifically designed to handle different fuzzy settings. The works of Bilal et al. [11], Chu et al. [12], Garg and Atef [13], Haque et al. [14], Seikh and Mandal [15], Seikh and Chatterjee [16], Kumar and Chen [17], Lu et al. [18], Mishra et al. [19], Liu et al. [20], Jiang et al. [21], Ocampo et al. [22], and Unver et al. [23, 24] can be consulted for a comprehensive examination of these techniques. A number of researchers have made significant contributions to the field of C-IFSs. While Kahraman and Otay [25] devised a scoring system and applied it to MCDM tasks, Atanassov [26] created distance measures for C-IFSs. In the framework of C-IFSs, Bolturk and Kahraman [27] focused on present worth analysis. Additionally, C-IFSs have been effectively used in a number of MCDM techniques, such as VIseKriterijum-ska Optimizacija I Kompromisno Resenje (VIKOR) and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). For instance, Kahraman and Otay [25] developed a VIKOR technique, Kahraman and Alkan [28] presented a TOPSIS method for supplier selection based on C-IFSs, and Alkan and Kahraman [29] used TOPSIS for hospital location selection during a pandemic using C-IFSs. Furthermore, divergence measurements for C-IFSs were proposed by Khan et al. [30]. The works of Otay and Kahraman [31], Cakir and Tas [32], Cakir et al. [33], Caloglu Büyükselçuk and Sari [34], Imanov and Aliyev [35], and Chen [36, 37] are suggested for a more thorough examination of C-IFSs. The comprehension and utilization of C-IFSs in various decision-making scenarios are improved by these studies.

Ghorabaee et al. [38] introduced the Evaluation based on Distance from Average Solution (EDAS) method, a new MCDM methodology that is related to distance-based decision-making techniques like VIKOR and TOPSIS. The EDAS technique selects the best option based on how close it is to the average answer, while TOPSIS and VIKOR identify the best option based on ideal solutions (both positive and negative).

The intricacy of choosing positive or negative ideal solutions, which can occasionally be difficult, is avoided with this method. The application of the Combinative Distance-based Assessment (CODAS) approach to Diamond Intuitionistic Fuzzy Sets (Dia-IFSs) has not yet been investigated, despite the fact that it has been extensively studied in the literature. We suggest a modification of the CODAS approach for Dia-IFSs in order to close this gap. With this improved method, we use t-norms and t-conorms to introduce subtraction and division operations for Dia-IFSs and Circular Intuitionistic Fuzzy Values (C-IFVs). The CODAS approach may now be used in fuzzy environments thanks to this addition, which makes it possible to analyze Dia-IFS decision-making problems in greater detail. It should be emphasized that the original CODAS approach uses simple algebraic operations with scalars and depends on the scores of average solutions. Nevertheless, we overcome this constraint and extend the CODAS method's reach to efficiently handle Dia-IFSs by adding subtraction and division operations for Dia-IFSs. We give decision-makers a strong tool to evaluate options and make informed choices in situations with ambiguous and insufficient facts by fusing the CODAS approach with Dia-IFSs. The suggested strategy opens up new possibilities for using the CODAS method to solve a wide range of Dia-IFS decision-making problems. The CODAS approach is thoroughly such that Liao et al. [39], Liu and Yang [40], Su et al. [41], Kahraman [42], Ozcelik and Nalkiran [43], Gul [44], Liang [45], Zhang et al. [46], Li et al. [47], Stevic et al. [48], Ghorabaee et al. [49], Kahraman et al. [50], Khan et al. [51], Peng et al. [52], Ghorabaee et al. [49], Stanujkic et al. [53], Fan et al. [54], Han and Wei [55], Huang et al. [56], Chinram et al. [57], Khan et al. [51], Jiang et al. [58], Imran et al. [59], Ijaz et al. [60] and Wei et al. [61]. It highlights the various uses and expansions discussed in the literature while summarizing the research done on the CODAS approach. The CODAS and EDAS method's citation trend over time is depicted in Table 1 (in Section 5.2). The amount of citations on the graph clearly demonstrates an upward trajectory, indicating its increasing recognition and impact within the academic community. The CODAS method's significance and influence in the field of MCDM are underscored by the increase in citations.

In the context of fuzzy set theory, aggregation operators must be defined using algebraic operations. Inverse operations like subtraction and division have also been investigated in a variety of fuzzy contexts, despite the fact that addition and multiplication have received the most attention in the literature. For example, Liao and Xu [62] introduced subtraction and division operations for hesitant fuzzy sets, while Du [63] established these operations for IFSs based on the Hamming distance. Recetly, Khan et al. [64] and Shi et al. [65] intoducrd the concept of Dia-IFSs and Diamond Intuitionistic Fuzzy Values (Dia-IFVs) as well proped some new oparations using Dia-IFVs. By suggesting subtraction and division operations for Dia-IFVs using t-norms (triangular norms) and t-conorms (triangular conorms), we expand the use of these algebraic operations in this study. When using the CODAS approach to solve fuzzy MCDM problems, the absence of certain inverse operators has led to difficulties and limitations. We design an updated CODAS approach that incorporates these aggregation operators and inverse operations, and we offer weighted arithmetic and geometric operators for Dia-IFVs to address this problem. We use an MCDM problem from the body of existing research to illustrate the effectiveness of the suggested expanded CODAS approach. We validate the usefulness and relevance of the extended CODAS approach in fuzzy environments by demonstrating its performance and efficacy in handling actual decision-making problems. We propose the extended CODAS method for MCDM issues in fuzzy contexts and conduct a thorough study to evaluate its applicability and performance. Through these assessments, we thoroughly evaluate the extended CODAS method's efficacy and efficiency, highlighting its benefits over current techniques and demonstrating its resilience in managing challenging decision-making tasks.

Motivated and inspired by ongoing research, our work's effective handling of uncertainty in the context of MCDM is one of its main contributions. Despite the fact that uncertainty has been the subject of earlier studies, our method has the following special advantages:

(1) The techniques presented in this research are especially intended to address Dia-IFS-based decision-making problems. A more accurate depiction and management of uncertainty is made possible by our method, which integrates the diamond character of Dia-IFSs into the decision-making process.

(2) For Dia-IFSs, we introduce a novel algebraic framework that incorporates addition, multiplication, subtraction, and division. These techniques, which are specifically designed for Dia-IFSs, make it easier to combine and manipulate uncertain data in a logical and cogent manner. We can better understand and analyze ambiguous information in scenarios involving decision-making by specifying these operations.

(3) We suggest using weighted aggregation procedures that were created especially for Dia-IFVs. These operators make it possible to combine several Dia-IFVs—each of which represents a different opinion or assessment—into a single consensus rating. The weighted aggregate method lessens the impact of personal prejudices or uncertainty while guaranteeing the inclusion of varied viewpoints. This feature distinguishes our work from alternative approaches that might not use these weighted aggregation techniques.

(4) The expanded CODAS approach combines weighted aggregation and algebraic operations to produce a thorough framework for MCDM with Dia-IFS. It successfully manages uncertainty and gives decision-makers a strong and reliable instrument for decision-making by using these unique characteristics.

The Dia-IFSs framework is adopted in this study due to its superior ability to represent uncertainty, hesitation, and expert judgment in complex MCDM environments. While traditional fuzzy sets consider only membership degrees, and intuitionistic fuzzy sets incorporate both membership and non-membership information, Dia-IFSs further enhance this representation through a structured geometric formulation. Compared with other fuzzy extensions such as interval-valued, circular, and intuitionistic fuzzy sets, the Dia-IFSs framework provides improved discrimination among alternatives, clearer interpretability, and stable aggregation of decision information without excessive computational burden. These characteristics make it particularly suitable for practical decision-making problems involving ambiguous and conflicting criteria.

The paper is organized as follows. A succinct summary of the main ideas used in this study is provided in Section 2. In addition to introducing the weighted arithmetic and weighted geometric aggregation operators, Section 3 describes the algebraic operations for Dia-IFVs, such as division and subtraction. We present the enhanced CODAS approach in Section 4 and show how to use it to solve a real-world MCDM problem that was taken from the literature. An internal comparison, sensitivity, and complexity analysis of the enhanced CODAS approach is presented in Section 5. The paper is finally concluded with a discussion of the findings in Section 6.

2. Preliminaries

This section presents several classical definitions, results, and concepts that will facilitate the discussion of the main findings.

Definition 1 [2] Let be E a fixed universe and its sub-set T. The set

$$T = \{\langle \omega, \zeta_T(\omega), \gamma_T(\omega)\rangle : \text{ for all } \ \omega \in E\}, \tag{1}$$

where $0 \leq \zeta_T(\omega) + \gamma_T(\omega) \leq 1$, is called IFS and functions $\zeta_T, \gamma_T : E \to [0, 1]$ indicate the degree of membership (validity, etc.) and non-membership (non-validity, etc.) of element $\omega \in E$ to a fixed set $T \subseteq E$. Now, we can define also function $\pi_T : E \to [0, 1]$ by means of

$$\pi_T(\omega) = 1 - \zeta_T(\omega) - \gamma_T(\omega), \tag{2}$$

and it corresponds to degree of indeterminacy (uncertainty, etc.). An IFV is the pair "$\langle \zeta_T(\omega), \gamma_T(\omega)\rangle$" given an element ω of X. To make things easier to understand, we can write $T = \langle \zeta_T, \gamma_T\rangle$, where $\zeta_T \in [0, 1]$, $\gamma_T \in [0, 1]$ and $0 \leq \zeta_T + \gamma_T \leq 1$. The degree of indeterminacy is represented by π_T, subject to the constraints that $\pi_T \in [0, 1]$ and $\pi_T = 1 - \zeta_T - \gamma_T$.

The definition of the complement of an IFV $T = \langle \gamma_T, \zeta_T, \pi_T\rangle$ is as follows:

$$T^C = \langle \gamma_T, \zeta_T, \pi_T\rangle. \tag{3}$$

Definition 2 [4] Let $D[0, 1]$ be the set of all closed subintervals of $[0, 1]$. An Interval-Valued Intuitionistic Fuzzy Set (IVIFS) $\mathscr{A}$ in X is defined as $\mathscr{A} = \{\langle \omega, u_{\mathscr{A}}(\omega), v_{\mathscr{A}}(\omega) \rangle \, \omega \in X\}$ where $u_{\mathscr{A}} : X \to D[0, 1]$ and "$v_{\mathscr{A}} : X \to D[0, 1]$", with the condition "$0 \leq \sup u_{\mathscr{A}}(\omega) + \sup v_{\mathscr{A}}(\omega) \leq 1, \, \omega \in X$". The membership and non-membership degrees of X to $\mathscr{A}$ are represented by the intervals $u_{\mathscr{A}}(\omega)$ and $v_{\mathscr{A}}(\omega)$, respectively.

An IVIFV is the pair $\langle u_{\mathscr{A}}(\omega), v_{\mathscr{A}}(\omega) \rangle$ for any $\omega \in X$ [4]. In this study, $\widetilde{\mathscr{A}} = \left(\left[u^-_{\widetilde{\mathscr{A}}}, u^+_{\widetilde{\mathscr{A}}} \right], \left[v^-_{\widetilde{\mathscr{A}}}, v^+_{\widetilde{\mathscr{A}}} \right] \right)$ is used to conveniently denote an IVIFV. Here, $\left[u^-_{\widetilde{\mathscr{A}}}, u^+_{\widetilde{\mathscr{A}}} \right] \in D[0, 1]$, $\left[v^-_{\widetilde{\mathscr{A}}}, v^+_{\widetilde{\mathscr{A}}} \right] \in D[0, 1]$ and $u^+_{\widetilde{\mathscr{A}}} + v^+_{\widetilde{\mathscr{A}}} \leq 1$.

2.1 *Diamond intuitionistic fuzzy sets*

Definition 3 [66] Let us have a fixed universe E and its sub-set T. The set

$$\mathscr{W}_{\aleph} = \{\langle \omega, \mathfrak{z}(\omega), \gamma(\omega); \aleph \rangle \mid \omega \in E\}, \tag{4}$$

where $0 \leq \mathfrak{z}(\omega) + \gamma(\omega) \leq 1$ and $\aleph \in [0, 2]$ is called Dia-IFS and functions $\mathfrak{z}, \gamma : E \to [0, 1]$ indicate the degree of membership (validity, etc.) and non-membership (non-validity, etc.) of element $\omega \in E$ to a fixed set $T \subseteq E$. Now, we can define also function $\pi : E \to [0, 1]$ by means of

$$\pi(\omega) = 1 - \mathfrak{z}(\omega) - \gamma(\omega),$$

and it corresponds to degree of indeterminacy (uncertainty, etc.), see Figures 1 and 2.

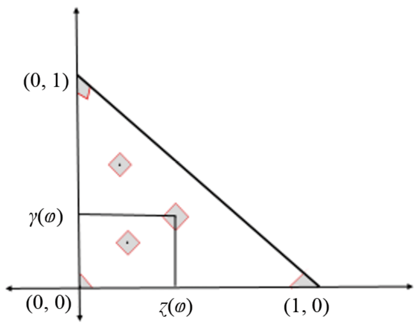

Figure 1. Geometrical presentation of diamond-IFS

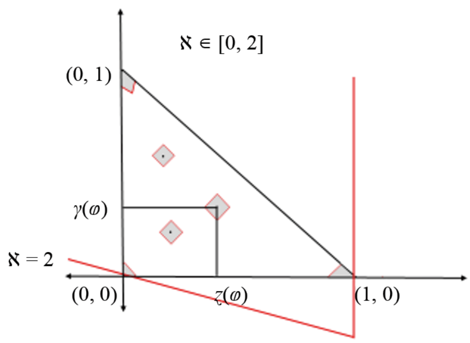

Figure 2. Iriangular coverage of different $\aleph$ values of diamond-IFSs

On the other hand $\mathscr{W}_{\aleph}$ can also be defined by using following approach such that.
Let $\mathscr{W}_1 = \{\langle h, m\rangle : h, m \in [0, 1], \text{ and } h+m \leq 1\}$. Then,

$$\mathscr{W}_{\aleph} = \{\langle \omega, \aleph^1(\zeta(\omega), \sigma(\omega))\rangle : \omega \in E\}. \tag{5}$$

Where

$$\aleph^1(\zeta(\omega), \gamma(\omega)) = \{\langle h, m\rangle : h, m \in [0,\ 1] \text{ and } (|\zeta(\omega)-h|+|\gamma(\omega)-m|) \leq \aleph\} \cap \mathscr{W}_1,$$

$$= \{\langle h, m\rangle : h, m \in [0,\ 1],\ (|\zeta(\omega)-h|+|\gamma(\omega)-m|) \leq \aleph \text{ and } h+m \leq 1\}.$$

3. Diamond intuitionistic fuzzy number and distance measures

In this section, we start with the definition of diamond intuitionistic fuzzy value such that:
Definition 4 A diamond intuitionistic fuzzy set is a collection of

$$\mathscr{W}_{\aleph} = (\zeta, \gamma; \aleph), \tag{6}$$

where $\mathscr{W}_{\aleph}$ represent the Dia-IFVs with conditions;
(i) $0 \leq \zeta(\varpi)+\gamma(\varpi) \leq 1$.
(ii) $0 \leq \zeta(\varpi), \gamma(\varpi), \aleph \leq 1$.
(iii) $0 \leq \aleph \leq 2$, (or $0 \leq \aleph \leq 1$).
In order to facilitate collective decision-making, we now create a mechanism for converting collections of IFSs into a Dia-IFVs.
Proposition 1 Let a set of IFVs be denoted as

$$\left\{\mathscr{W}_{\aleph_1} = (\zeta_1, \gamma_1; \aleph_1), \mathscr{W}_{\aleph_2} = (\zeta_2, \gamma_2; \aleph_2), \ldots, \mathscr{W}_{\aleph_n} = (\zeta_n, \gamma_n; \aleph_n)\right\}.$$

Then

$$\mathscr{W}_{\aleph} = \langle \zeta, \gamma; \aleph\rangle,$$

is a Dia-IFV with

$$\zeta = \sum_{j=1}^{n_i} \frac{\zeta_{i,j}}{n_i} \text{ and } \gamma = \sum_{j=1}^{n_i} \frac{\gamma_{i,j}}{n_i},$$

$$\aleph = \min\left(\max_{1 \leq j \leq n_i} \left(\left|\zeta - \zeta_{i,j}\right| + \left|\gamma - \gamma_{i,j}\right|\right), 2\right).$$

Proof. Since $\zeta = \sum_{j=1}^{n_i} \frac{\zeta_{i,\,j}}{n_i}$ and $\gamma = \sum_{j=1}^{n_i} \frac{\gamma_{i,\,j}}{n_i}$, then we have

$$\begin{aligned}\zeta + \gamma &= \sum_{j=1}^{n_i} \frac{\zeta_{i,\,j}}{n_i} + \sum_{j=1}^{n_i} \frac{\gamma_{i,\,j}}{n_i} \\ &= \frac{\sum_{j=1}^{n_i} \left(\zeta_{i,\,j} + \gamma_{i,\,j}\right)}{n_i} \\ &\leq \frac{\sum_{j=1}^{n_i} 1}{n_i} \\ &= 1.\end{aligned}$$

Example 1 The following sets of IFSs are represented as:

$$\{(0.3, 0.7),\ (0.2, 0.7),\ (0.6, 0.2)\},$$

$$\{(0.2, 0.5),\ (0.3, 0.4),\ (0.9, 0.1)\},$$

and

$$\{(0.1, 0.6),\ (0.5, 0.5),\ (0.1, 0.8)\}.$$

With the help of Proposition 1, we find the corresponding Dia-IFSs, and we have

$$(0.37, 0.53; 0.63),\ (0.47, 0.33; 0.93),\ (0.23, 0.63; 0.53).$$

3.1 *Distance measures via Dia-IFNs*

The next outcomes are introduced for representing different distances over Dia-IFSs via Subsection 3 approach.

Definition 5 Let d be a cardinality of E. Then normalized Euclidean distance for two Dia-IFSs $\mathbb{W}_{\aleph_1}$ and $\mathbb{W}_{\aleph_2}$ is defined as

$$H_2^1(\text{FIS, A}_i) = \frac{1}{2} \times \left(\frac{1}{d} \sum_{\rho \in E} \frac{|\aleph_1 - \aleph_2|}{2} + \frac{1}{2d} \sum_{\rho \in E} \left(\left| z_{\ell_{ij}} - z_{\ell_j} \right| + \left| \gamma_{\ell_{ij}} - \gamma_{\ell_j} \right| \right) \right), \tag{7}$$

$$H_2^2(\text{FIS, A}_i) = \frac{1}{2} \times \left(\frac{1}{d} \sum_{\rho \in E} \frac{|\aleph_1 - \aleph_2|}{2} + \sqrt{\frac{1}{2d} \sum_{\rho \in E} \left(\left| z_{\ell_{ij}} - z_{\ell_j} \right|^2 + \left| \gamma_{\ell_{ij}} - \gamma_{\ell_j} \right|^2 \right)} \right). \tag{8}$$

Here, the distance $H_2^1\left(\mathcal{W}_{\aleph_1}, \mathcal{W}_{\aleph_2}\right)$ and $H_2^2\left(\mathcal{W}_{\aleph_1}, \mathcal{W}_{\aleph_2}\right)$ are known as Hamming distance and Euclidean distance for Dia-IFSs, respectively.

Definition 6 Let d be a cardinality of E. Then normalized Euclidean distance for two Dia-IFSs $\mathcal{W}_{\aleph_1}$ and $\mathcal{W}_{\aleph_2}$ is defined as

$$H_3^1\left(\text{FIS}, \text{A}_i\right) = \frac{1}{2} \times \left(\frac{1}{d}\sum_{\rho\in E}\frac{|\aleph_1 - \aleph_2|}{2} + \frac{1}{2d}\sum_{\rho\in E}\left(\left|z_{\ell_{ij}} - z_{\ell_j}\right| + \left|\gamma_{\ell_{ij}} - \gamma_{\ell_j}\right| + \left|\pi_{\ell_{ij}} - \pi_{\ell_j}\right|\right)\right), \tag{9}$$

and

$$H_3^2\left(\text{FIS}, \text{A}_i\right) = \frac{1}{2} \times \left(\frac{1}{d}\sum_{\rho\in E}\frac{|\aleph_1 - \aleph_2|}{2} + \sqrt{\frac{1}{2d}\sum_{\rho\in E}\left(\left|z_{\ell_{ij}} - z_{\ell_j}\right|^2 + \left|\gamma_{\ell_{ij}} - \gamma_{\ell_j}\right|^2 + \left|\pi_{\ell_{ij}} - \pi_{\ell_j}\right|^2\right)}\right). \tag{10}$$

Here, the distance $H_3^1(\mathcal{W}_{\aleph_1}, \mathcal{W}_{\aleph_2})$ and $H_3^2(\mathcal{W}_{\aleph_1}, \mathcal{W}_{\aleph_2})$ are known as Szmidt and Kacprzyk's form of Hamming distance and, Szmidt and Kacprzyk's form of Euclidean distance for Dia-IFSs, respectively.

4. Aggregation operators via Dia-IFVs

Aggregation operators play a crucial role in converting input values expressed as fuzzy values into a single output value. In this section, we present a weighted arithmetic aggregation operator and a weighted geometric aggregation operator for Dia-IFVs, utilizing the algebraic operations outlined in Section 3. Note that Dia-Intuitionistic Fuzzy Numbers (Dia-IFNs) on E is denoted by Dia-IFV(E).

4.1 *Diamond intuitionistic fuzzy weighted averaging and geometric aggregation operators*

Definition 7 Let $\left\{\mathcal{W}_i = \left\langle z_{\mathcal{W}_i}, \gamma_{\mathcal{W}_i}; \aleph_{\mathcal{W}_i}\right\rangle : i = 1, \ldots, n\right\}$ be the set of Dia-IFVs. Then, Dia-Intuitionistic Fuzzy Weighted Averaging Aggregation (Dia-IFWAA) operator with mapping Dia-IFWAA: Dia-IFV(E) → Dia-IFV(E) is computed as follows

$$\text{Dia-IFWA}_{\max}\left(\mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3, \ldots, \mathcal{W}_n\right) = \bigotimes_{i_{\max}}^{n} \varpi_i \mathcal{W}_i, \tag{11}$$

$$\text{Dia-IFWA}_{\min}\left(\mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3, \ldots, \mathcal{W}_n\right) = \bigotimes_{i_{\min}}^{n} \varpi_i \mathcal{W}_i, \tag{12}$$

with weight vector $\varpi = (\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n)^T$ with $0 \le \varpi_j \le 1$ and $\sum_{j=1}^{n} \varpi_j = 1$.

Theorem 1 Let $\left\{\mathcal{W}_i = \left\langle z_{\mathcal{W}_i}, \gamma_{\mathcal{W}_i}; \aleph_{\mathcal{W}_i}\right\rangle : i = 1, \ldots, n\right\}$ be the set of Dia-IFVs. If Dia-IFWAA operator is defined with the help of this transformation Dia-IFWAA: Dia-IFV(E) → Dia-IFV(E), then Dia-IFWAA$_q\left(\mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3, \ldots, \mathcal{W}_i\right)$ is Dia-IFV and we have

$$\text{Dia-IFWAA}_{\max}\left(\mathcal{W}_1, \ldots, \mathcal{W}_n\right) = \left\langle 1 - \prod_{i=1}^{n}\left(1 - z_{\mathcal{W}_i}\right)^{\varpi_i}, \prod_{i=1}^{n}\gamma_{\mathcal{W}_i}^{\varpi_i}; \prod_{i=1}^{n}\aleph_{\mathcal{W}_i}^{\varpi_i}\right\rangle, \tag{13}$$

and

$$\text{Dia-IFWAA}_{\min}\left(\mathcal{W}_1, \ldots, \mathcal{W}_n\right) = \left\langle 1 - \prod_{i=1}^{n}\left(1 - z_{\mathcal{W}_i}\right)^{\varpi_i}, \prod_{i=1}^{n}\gamma_{\mathcal{W}_i}^{\varpi_i}; 1 - \prod_{i=1}^{n}\left(1 - \aleph_{\mathcal{W}_i}\right)^{\varpi_i}\right\rangle, \tag{14}$$

with weight vector $\varpi = (\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n)^T$ with $0 \leq \varpi_j \leq 1$ and $\sum_{j=1}^{n}\varpi_j = 1$.

Proof. As evident from Definition 1, $\text{Dia-IFWAA}_{\max}\left(\mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3, \ldots, \mathcal{W}_n\right)$ is a Dia-IFV. By utilizing mathematical induction, it can be seen that the second part is also true. If $n = 2$, then we have

$$\text{Dia-IFWAA}_{\max}\left(\mathcal{W}_1, \mathcal{W}_2\right) = \varpi_1\mathcal{W}_1 \oplus_{\max} \varpi_2\mathcal{W}_2$$

$$= \left(1 - (1 - z_1)^{\varpi_1}, \gamma_1^{\varpi_1}; \aleph_1\right) \oplus_{\max} \left(1 - (1 - z_2)^{\varpi_2}, \gamma_2^{\varpi_2}; \aleph_2\right)$$

$$= \left(1 - (1 - z_1)^{\varpi_1} + 1 - (1 - z_2)^{\varpi_2} - \left(1 - (1 - z_1)^{\varpi_1}\right)\left(1 - (1 - z_2)^{\varpi_2}\right), \gamma_1^{\varpi_1}\gamma_2^{\varpi_2}; \aleph_1^{\varpi_1}\aleph_2^{\varpi_2}\right)$$

$$= \left(1 - \prod_{i}^{2}(1 - z_i)^{\varpi_i}, \prod_{i}^{2}\gamma_i^{\varpi_i}; \prod_{i}^{2}\aleph_i^{\varpi_i}\right).$$

Suppose (6) is true for $n = k$, that is,

$$\text{Dia-IFWAA}_{\max}\left(\mathcal{W}_1, \ldots, \mathcal{W}_k\right) = \left(1 - \prod_{i}^{k}(1 - z_i)^{\varpi_i}, \prod_{i}^{k}\gamma_i^{\varpi_i}; \prod_{i}^{k}\aleph_i^{\varpi_i}\right).$$

We need to prove true for $n = k + 1$,

$$\text{Dia-IFWAA}_{\max}\left(\mathcal{W}_1, \ldots, \mathcal{W}_{k+1}\right)$$

$$= \left(1 - \prod_{i}^{k}(1 - z_i)^{\varpi_i}, \prod_{i}^{k}\gamma_i^{\varpi_i}; \prod_{i}^{k}\aleph_i^{\varpi_i}\right) \oplus_{\max} \left(1 - \left(1 - z_{k+1}\right)^{\varpi_{k+1}}, \gamma_{k+1}^{\varpi_{k+1}}; \aleph_{k+1}^{\varpi_{k+1}}\right)$$

$$= \left(1 - \prod_{i}^{k+1}(1 - z_i)^{\varpi_i}, \prod_{i}^{k+1}\gamma_i^{\varpi_i}; \prod_{i}^{k+1}\aleph_i^{\varpi_i}\right).$$

This implies (5) is true. Similarly, we can prove (6).

Definition 8 Let $\left\{\mathcal{W}_i = \left\langle \mathfrak{z}_{\mathcal{W}_i}, \gamma_{\mathcal{W}_i}; \aleph_{\mathcal{W}_i} \right\rangle : i = 1, \ldots, n\right\}$ be the set of Dia-IFVs. Then, Dia-Intuitionistic Fuzzy Weighted Geometric Aggregation (Dia-IFWGA) operator with mapping Dia-IFWGA: Dia-IFV(E) $\to$ Dia-IFV(E) is computed as follows

$$\text{Dia-IFWGA}_{\max}(\mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3, \ldots, \mathcal{W}_n) = \bigoplus_{i=1_{\max}}^{n} \varpi_i \mathcal{W}_i, \tag{15}$$

$$\text{Dia-IFWGA}_{\min}(\mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3, \ldots, \mathcal{W}_n) = \bigoplus_{i=1_{\min}}^{n} \varpi_i \mathcal{W}_i, \tag{16}$$

with weight vector $\varpi = (\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n)^T$ with $0 \leq \varpi_j \leq 1$ and $\sum_{j=1}^{n} \varpi_j = 1$.

Theorem 2 Let $\left\{\mathcal{W}_i = \left\langle \mathfrak{z}_{\mathcal{W}_i}, \gamma_{\mathcal{W}_i}; \aleph_{\mathcal{W}_i} \right\rangle : i = 1, \ldots, n\right\}$ be the set of Dia-IFVs. If Dia-IFWGA operator is defined with the help of this transformation Dia-IFWGA: Dia-IFV(E) $\to$ Dia-IFV(E), then $\text{Dia-IFWGA}_{\max}(\mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3, \ldots, \mathcal{W}_n)$ and $\text{Dia-IFWGA}_{\min}(\mathcal{W}_1, \ldots, \mathcal{W}_n)$ are Dia-IFV and we have

$$\text{Dia-IFWGA}_{\max}(\mathcal{W}_1, \ldots, \mathcal{W}_n) = \left\langle \prod_{i=1}^{n} \mathfrak{z}_{\mathcal{W}_i}^{\varpi_i}, 1 - \prod_{i=1}^{n} \left(1 - \gamma_{\mathcal{W}_i}\right)^{\varpi_i}; \prod_{i=1}^{n} \aleph_{\mathcal{W}_i}^{\varpi_i} \right\rangle, \tag{17}$$

and

$$\text{Dia-IFWGA}_{\min}(\mathcal{W}_1, \ldots, \mathcal{W}_n) = \left\langle \prod_{i=1}^{n} \mathfrak{z}_{\mathcal{W}_i}^{\varpi_i}, 1 - \prod_{i=1}^{n} \left(1 - \gamma_{\mathcal{W}_i}\right)^{\varpi_i}; 1 - \prod_{i=1}^{n} \left(1 - \aleph_{\mathcal{W}_i}\right)^{\varpi_i} \right\rangle, \tag{18}$$

with weight vector $\varpi = (\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_n)^T$ with $0 \leq \varpi_j \leq 1$ and $\sum_{j=1}^{n} \varpi_j = 1$.

Proof. As evident from Definition 1, $\text{Dia-IFWAA}_{\max}(\mathcal{W}_1, \mathcal{W}_2, \mathcal{W}_3, \ldots, \mathcal{W}_n)$ is a Dia-IFV. By utilizing mathematical induction, it can be seen that the second part is also true. If $n = 2$, then we have

$$\begin{aligned}
\text{Dia-IFWGA}_{\max}(\mathcal{W}_1, \mathcal{W}_2) &= \varpi_1 \mathcal{W}_1 \oplus_{\max} \varpi_2 \mathcal{W}_2 \\
&= \left(\mathfrak{z}_1^{\varpi_1}, 1 - (1 - \gamma_1)^{\varpi_1}; \aleph_1\right) \oplus_{\max} \left(\mathfrak{z}_2^{\varpi_2}, 1 - (1 - \gamma_2)^{\varpi_2}; \aleph_2\right) \\
&= \left(\mathfrak{z}_1^{\varpi_1} \mathfrak{z}_2^{\varpi_2}, 1 - (1 - \gamma_1)^{\varpi_1} + 1 - (1 - \gamma_2)^{\varpi_2} - \left(1 - (1 - \gamma_1)^{\varpi_1}\right)\left(1 - (1 - \gamma_2)^{\varpi_2}\right); \aleph_1^{\varpi_1} \aleph_2^{\varpi_2}\right) \\
&= \left(\prod_{i}^{2} \mathfrak{z}_i^{\varpi_i}, 1 - \prod_{i}^{2} (1 - \gamma_i)^{\varpi_i}; \prod_{i}^{2} \aleph_i^{\varpi_i}\right).
\end{aligned}$$

Suppose (6) is true for $n = k$, that is,

$$\text{Dia-IFWGA}_{\max}\left(\mathscr{W}_1, \ldots, \mathscr{W}_k\right)=\left(\prod_{i}^{k} z_i^{\varpi_i},\, 1-\prod_{i}^{k}\left(1-\gamma_i\right)^{\varpi_i};\, \prod_{i}^{k} \aleph_i^{\varpi_i}\right).$$

We need to prove true for $n=k+1$,

$$\text{Dia-IFWGA}_{\max}\left(\mathscr{W}_1, \ldots, \mathscr{W}_{k+1}\right)$$

$$=\left(\prod_{i}^{k} z_i^{\varpi_i},\, 1-\prod_{i}^{k}\left(1-\gamma_i\right)^{\varpi_i};\, \prod_{i}^{k} \aleph_i^{\varpi_i}\right) \oplus_{\max}\left(z_{k+1}^{\varpi_{k+1}},\, 1-\left(1-\gamma_{k+1}\right)^{\varpi_{k+1}};\, \aleph_{k+1}^{\varpi_{k+1}}\right)$$

$$=\left(\prod_{i}^{k+1} z_i^{\varpi_i},\, 1-\prod_{i}^{k+1}\left(1-\gamma_i\right)^{\varpi_i};\, \prod_{i}^{k+1} \aleph\right).$$

This implies (11) is true. Similarly, we can prove (12).

The following three theorems focus on demonstrating three fundamental characteristics of aggregation operators within this framework: monotonicity, boundedness, and idempotency. These technical findings ensure the effective functioning of the proposed operators. Notably, both monotonicity and idempotency are intuitive properties, and their demonstrations are consequently omitted for brevity.

Theorem 3 (Monotonicity) Let $\{\mathscr{W}_i=(z_i,\, \gamma_i;\, \aleph_i)\}_{i=1,\, \ldots,\, n}$ and $\left\{\mathscr{L}_i=\left(z_{\mathscr{L}_i},\, \gamma_{\mathscr{L}_i};\, \aleph_{\mathscr{L}_i}\right)\right\}_{i=1,\, \ldots,\, n}$ be two collections of n Dia-IFVs. If $z_{\mathscr{W}_i} \leq z_{\mathscr{L}_i}$, $\gamma_{\mathscr{W}_i} \geq \gamma_{\mathscr{L}_i}$, and $\aleph_{\mathscr{W}_i} \geq \aleph_{\mathscr{L}_i}$, then

$$\text{Dia-IFWAA}_{\max}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \leq \text{Dia-IFWAA}_{\max}\left(\mathscr{L}_1, \ldots, \mathscr{L}_n\right); \tag{19}$$

$$\text{Dia-IFWAA}_{\min}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \leq \text{Dia-IFWAA}_{\min}\left(\mathscr{L}_1, \ldots, \mathscr{L}_n\right); \tag{20}$$

$$\text{Dia-IFWGA}_{\max}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \leq \text{Dia-IFWGA}_{\max}\left(\mathscr{L}_1, \ldots, \mathscr{L}_n\right); \tag{21}$$

$$\text{Dia-IFWGA}_{\min}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \leq \text{Dia-IFWGA}_{\min}\left(\mathscr{L}_1, \ldots, \mathscr{L}_n\right). \tag{22}$$

Theorem 4 (Boundedness) Let $\{\mathscr{W}_i=(z_i,\, \gamma_i;\, \aleph_i)\}_{i=1,\, \ldots,\, n}$ be a list of n Dia-IFVs. If $\underline{\mathfrak{p}}$ and $\bar{\mathfrak{p}}$ are two Dia-IFVs such that $\underline{\mathfrak{p}}=(\underline{z},\, \bar{\gamma};\, \underline{\aleph})=(\min(\bar{z}_i),\, \max(\gamma_i),\, \min(\aleph_i))$ and $\bar{\mathfrak{p}}=(\bar{z},\, \underline{\gamma},\, \bar{\aleph})=(\max(z_i),\, \min(\gamma_i),\, \max(\aleph_i))$, then

$$(\underline{z},\, \bar{\gamma};\, \underline{\aleph}) \leq \text{Dia-IFWAA}_{\max}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \leq (\bar{z},\, \underline{\gamma},\, \bar{\aleph}); \tag{23}$$

$$(\underline{z},\, \bar{\gamma};\, \underline{\aleph}) \leq \text{Dia-IFWAA}_{\min}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \leq (\bar{z},\, \underline{\gamma},\, \bar{\aleph}); \tag{24}$$

$$(\underline{z},\, \bar{\gamma};\, \underline{\aleph}) \leq \text{Dia-IFWGA}_{\max}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \leq (\bar{z},\, \underline{\gamma},\, \bar{\aleph}); \tag{25}$$

$$(\underline{z},\ \bar{\gamma};\ \underline{\aleph}) \leq \text{Dia-IFWGA}_{\min}(\mathcal{W}_1, \ldots, \mathcal{W}_n) \leq (\bar{z},\ \underline{\gamma},\ \bar{\aleph}). \tag{26}$$

Proof. We establish the initial segment of the theorem. The other sections can be demonstrated in a comparable manner. To finalize the first segment, we need to show that $\underline{z} \leq 1 - \prod_i^n (1 - z_i)^{\varpi_i} \leq \bar{z}$, $\bar{\gamma} \geq \prod_i^n \gamma_i^{\varpi_i} \geq \underline{\gamma}$, and $\bar{\aleph} \geq \prod_i^n \aleph_i^{\varpi_i} \geq \underline{\aleph}$.

Since $\underline{z} \leq z_i \leq \bar{z}$, we have

$$\prod_i^n (1 - \underline{z})^{\varpi_i} \geq \prod_i^n (1 - z_i)^{\varpi_i} \geq \prod_i^n (1 - \bar{z})^{\varpi_i},$$

$$(1 - \underline{z})^{\sum_i^n \varpi_i} \geq \prod_i^n (1 - z_i)^{\varpi_i} \geq (1 - \bar{z})^{\sum_i^n \varpi_i},$$

$$\underline{z} \leq 1 - \prod_i^n (1 - z_i)^{\varpi_i} \leq \bar{z}.$$

Similarly, we can prove $\bar{\gamma} \geq \prod_i^n \gamma_i^{\varpi_i} \geq \underline{\gamma}$, and $\bar{\aleph} \geq \prod_i^n \aleph_i^{\varpi_i} \geq \underline{\aleph}$.

Theorem 5 (Idempotency) Let $\{\mathcal{W}_i = (z_i,\ \gamma_i;\ \aleph_i)\}_{i=1,\ldots,n}$ be a series of n Dia-IFVs such that $\mathcal{W}_i = \mathcal{W} = (z,\ \gamma;\ \aleph)$. If $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_n)$ is a weight vector with $\sum_{i=1}^n \varpi_i = 1$, then

$$\text{Dia-IFWAA}_{\max}(\mathcal{W}_1, \ldots, \mathcal{W}_n) = \mathcal{W}; \tag{27}$$

$$\text{Dia-IFWAA}_{\min}(\mathcal{W}_1, \ldots, \mathcal{W}_n) = \mathcal{W}; \tag{28}$$

$$\text{Dia-IFWGA}_{\max}(\mathcal{W}_1, \ldots, \mathcal{W}_n) = \mathcal{W}; \tag{29}$$

$$\text{Dia-IFWGA}_{\min}(\mathcal{W}_1, \ldots, \mathcal{W}_n) = \mathcal{W}. \tag{30}$$

Theorem 6 Let $\{\mathcal{W}_i = (z_i,\ \gamma_i;\ \aleph_i)\}_{i=1,\ldots,n}$ be a collection of n Dia-IFVs and $\mathcal{W} = (z,\ \gamma;\ \aleph)$ be any Dia-IFV. If $\varpi = (\varpi_1, \ldots, \varpi_n)$ is a weight vector with $\sum_{i=1}^n \varpi_i = 1$, then

$$\text{Dia-IFWAA}_{\mathcal{M}}(\mathcal{W}_1 \oplus_{\mathcal{M}} \mathcal{W}, \ldots, \mathcal{W}_n \oplus_{\mathcal{M}} \mathcal{W}) \geq \text{Dia-IFWAA}_{\mathcal{M}}(\mathcal{W}_1 \otimes_{\mathcal{M}} \mathcal{W}, \ldots, \mathcal{W}_n \otimes_{\mathcal{M}} \mathcal{W}); \tag{31}$$

$$\text{Dia-IFWAA}_{\mathcal{N}}(\mathcal{W}_1 \oplus_{\mathcal{N}} \mathcal{W}, \ldots, \mathcal{W}_n \oplus_{\mathcal{N}} \mathcal{W}) \geq \text{Dia-IFWAA}_{\mathcal{N}}(\mathcal{W}_1 \otimes_{\mathcal{N}} \mathcal{W}, \ldots, \mathcal{W}_n \otimes_{\mathcal{N}} \mathcal{W}); \tag{32}$$

$$\text{Dia-IFWAA}_{\mathcal{M}}(\mathcal{W}_1 \oplus_{\mathcal{M}} \mathcal{W}, \ldots, \mathcal{W}_n \oplus_{\mathcal{M}} \mathcal{W}) \geq \text{Dia-IFWAA}_{\mathcal{M}}(\mathcal{W}_1 \otimes_{\mathcal{M}} \mathcal{W}, \ldots, \mathcal{W}_n \otimes_{\mathcal{M}} \mathcal{W}); \tag{33}$$

$$\text{Dia-IFWAA}_{\mathcal{N}}(\mathcal{W}_1 \oplus_{\mathcal{N}} \mathcal{W}, \ldots, \mathcal{W}_n \oplus_{\mathcal{N}} \mathcal{W}) \geq \text{Dia-IFWAA}_{\mathcal{N}}(\mathcal{W}_1 \otimes_{\mathcal{N}} \mathcal{W}, \ldots, \mathcal{W}_n \otimes_{\mathcal{N}} \mathcal{W}); \tag{34}$$

$$\text{Dia-IFWGA}_{\mathscr{M}}\left(\mathscr{W}_1 \oplus_{\mathscr{M}} \mathscr{W}, \ldots, \mathscr{W}_n \oplus_{\mathscr{M}} \mathscr{W}\right) \geq \text{Dia-IFWGA}_{\mathscr{M}}\left(\mathscr{W}_1 \otimes_{\mathscr{M}} \mathscr{W}, \ldots, \mathscr{W}_n \otimes_{\mathscr{M}} \mathscr{W}\right); \tag{35}$$

$$\text{Dia-IFWGA}_{\mathscr{N}}\left(\mathscr{W}_1 \oplus_{\mathscr{N}} \mathscr{W}, \ldots, \mathscr{W}_n \oplus_{\mathscr{N}} \mathscr{W}\right) \geq \text{Dia-IFWGA}_{\mathscr{N}}\left(\mathscr{W}_1 \otimes_{\mathscr{N}} \mathscr{W}, \ldots, \mathscr{W}_n \otimes_{\mathscr{N}} \mathscr{W}\right); \tag{36}$$

$$\text{Dia-IFWGA}_{\mathscr{N}}\left(\mathscr{W}_1 \oplus_{\mathscr{M}} \mathscr{W}, \ldots, \mathscr{W}_n \oplus_{\mathscr{M}} \mathscr{W}\right) \geq \text{Dia-IFWGA}_{\mathscr{N}}\left(\mathscr{W}_1 \otimes_{\mathscr{M}} \mathscr{W}, \ldots, \mathscr{W}_n \otimes_{\mathscr{M}} \mathscr{W}\right); \tag{37}$$

$$\text{Dia-IFWGA}_{\mathscr{N}}\left(\mathscr{W}_1 \oplus_{\mathscr{N}} \mathscr{W}, \ldots, \mathscr{W}_n \oplus_{\mathscr{N}} \mathscr{W}\right) \geq \text{Dia-IFWGA}_{\mathscr{N}}\left(\mathscr{W}_1 \otimes_{\mathscr{N}} \mathscr{W}, \ldots, \mathscr{W}_n \otimes_{\mathscr{N}} \mathscr{W}\right), \tag{38}$$

where the subscripts $\mathscr{M}$ and $\mathscr{N}$ have been replaced with max and min, respectively.

Proof. For any $\mathscr{W}_i = \left(z_{\mathscr{W}_i}, \gamma_{\mathscr{W}_i}; \aleph_{\mathscr{W}_i}\right)$ and $\mathfrak{p} = (z, \gamma; \aleph)$, we have $\mathscr{W}_i \oplus_{\mathscr{M}} \mathfrak{p} \geq \mathscr{W}_i \otimes_{\mathscr{M}} \mathfrak{p}$ and $\mathscr{W}_i \oplus_{\mathscr{N}} \mathfrak{p} \geq \mathscr{W}_i \otimes_{\mathscr{N}} \mathfrak{p}$. The derivations for all components follow directly from the monotonic properties of Dia-IFWA$A_{\max}$, Dia-IFWA$A_{\min}$, Dia-IFWG$A_{\max}$, and Dia-IFWG$A_{\min}$. □

Theorem 7 Let $\{\mathscr{W}_i = (z_i, \gamma_i; \aleph_i)\}_{i=1, \ldots, n}$ be a collection of n Dia-IFVs and $\mathscr{W} = (z, \gamma; \aleph)$ be any Dia-IFV. If $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_n)$ is a weight vector with $\sum_{i=1}^{n} \varpi_i = 1$, then

$$\text{Dia-IFWAA}_{\max}\left(\mathscr{W}_1 \oplus_{\max} \mathscr{W}, \ldots, \mathscr{W}_n \oplus_{\max} \mathscr{W}\right) \geq \text{Dia-IFWAA}_{\max}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \otimes_{\max} \mathscr{W}; \tag{39}$$

$$\text{Dia-IFWAA}_{\min}\left(\mathscr{W}_1 \oplus_{\min} \mathscr{W}, \ldots, \mathscr{W}_n \oplus_{\min} \mathscr{W}\right) \geq \text{Dia-IFWAA}_{\min}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \otimes_{\min} \mathscr{W}; \tag{40}$$

$$\text{Dia-IFWAA}_{\max}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \oplus_{\max} \mathscr{W} \geq \text{Dia-IFWAA}_{\max}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \otimes_{\max} \mathscr{W}; \tag{41}$$

$$\text{Dia-IFWAA}_{\min}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \oplus_{\min} \mathscr{W} \geq \text{Dia-IFWAA}_{\min}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \otimes_{\min} \mathscr{W}; \tag{42}$$

$$\text{Dia-IFWGA}_{\max}\left(\mathscr{W}_1 \oplus_{\max} \mathscr{W}, \ldots, \mathscr{W}_n \oplus_{\max} \mathscr{W}\right) \geq \text{Dia-IFWGA}_{\max}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \otimes_{\max} \mathscr{W}; \tag{43}$$

$$\text{Dia-IFWGA}_{\min}\left(\mathscr{W}_1 \oplus_{\min} \mathscr{W}, \ldots, \mathscr{W}_n \oplus_{\min} \mathscr{W}\right) \geq \text{Dia-IFWGA}_{\min}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \otimes_{\min} \mathscr{W}; \tag{44}$$

$$\text{Dia-IFWGA}_{\max}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \oplus_{\max} \mathscr{W} \geq \text{Dia-IFWGA}_{\max}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \otimes_{\max} \mathscr{W}; \tag{45}$$

$$\text{Dia-IFWGA}_{\min}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \oplus_{\min} \mathscr{W} \geq \text{Dia-IFWGA}_{\min}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \otimes_{\min} \mathscr{W}. \tag{46}$$

Proof. In this theorem, we demonstrate only the initial portion. The remaining sections can be established in a similar manner.

Let $\mathscr{W}_i \oplus_{\max} \mathscr{W} = (T_i, S_i, N_i) = (z_i + z - z_i z \gamma_i \gamma; \aleph_i \aleph)$. Then,

$$\text{Dia-IFWAA}_{\max}\left(\mathscr{W}_1 \oplus_{\max} \mathscr{W}, \ldots, \mathscr{W}_n \oplus_{\max} \mathscr{W}\right) = \left(1 - \prod_{i}^{n} (1 - T_i)^{\varpi_i}, \prod_{i}^{n} S_i^{\varpi_i}; \prod_{i}^{n} N_i^{\varpi_i}\right).$$

On the left-hand side, we obtain

$$\text{Dia-IFWAA}_{\max}(\mathcal{W}_1, \mathcal{W}_2, \ldots, \mathcal{W}_n) \otimes_{\max} \mathcal{W}$$

$$= \left(1 - \prod_i^n (1 - z_i)^{\varpi_i}, \prod_i^n \gamma_i^{\varpi_i}; \prod_i^n \aleph_i^{\varpi_i}\right) \otimes_{\max} (z, \gamma; \aleph)$$

$$= \left(z\left(1 - \prod_i^n (1 - z_i)^{\varpi_i}\right), \prod_i^n \gamma_i^{\varpi_i} + \gamma - \gamma \prod_i^n \gamma_i^{\varpi_i}; \prod_i^n \aleph_i^{\varpi_i} + \aleph - \aleph \cdot \prod_i^n \aleph_i^{\varpi_i}\right).$$

It is necessary to demonstrate that

$$1 - \prod_i^n (1 - T_i)^{\varpi_i} \geq z \cdot 1 - \prod_i^n (1 - z_i)^{\varpi_i},$$

$$\prod_i^n \gamma_i^{\varpi_i} + \gamma - \gamma \cdot \prod_i^n \gamma_i^{\varpi_i} \geq \prod_i^n S_i^{\varpi_i}.$$

It is essential to confirm the validity of the above expressions.

$$1 - \prod_i^n (1 - (z_i + z - z_i z))^{\varpi_i} \geq z\left(1 - \prod_i^n (1 - z_i)^{\varpi_i}\right),$$

$$\prod_i^n (1 - (z_i + z - z_i z))^{\varpi_i} \leq \prod_i^n (1 - z_i)^{\varpi_i},$$

$$1 - (z_i + z - z_i z) \leq 1 - z_i,$$

$$z_i z \leq z,$$

which holds true in general. If $\prod_i^n \gamma_i^{\varpi_i} = \delta$ and $\prod_i^n S_i^{\varpi_i} = \prod_i^n \gamma_i^{\varpi_i} \cdot \gamma$, then

$$\left(\prod_i^n \gamma_i^{\varpi_i}\right) + \gamma - \left(\prod_i^n \gamma_i^{\varpi_i}\right)\gamma \geq \prod_i^n S_i^{\varpi_i}$$

$$= \prod_i^n (\gamma_i \gamma)^{\varpi_i} = \prod_i^n \gamma_i^{\varpi_i} \gamma (\delta + \gamma - \delta\gamma) \geq \delta\gamma.$$

In a similar manner, it can be demonstrated for $\aleph$, which holds true, thereby establishing the first part.

Theorem 8 Let $\{\mathcal{W}_i = (\zeta_i, \gamma_i; \aleph_i)\}_{i=1,\ldots,n}$ be a series of n Dia-IFVs. If $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_n)$ be the weight vector of $\mathcal{W}_i$ with $\sum_{i=1}^{n} \varpi_i = 1$ and $\lambda > 0$, then

$$\text{Dia-IFWAA}_{\max}(\lambda \mathcal{W}_1, \ldots, \lambda \mathcal{W}_n) \geq \text{Dia-IFWAA}_{\max}\left(\mathcal{W}_1^{\lambda}, \ldots, \mathcal{W}_n^{\lambda}\right); \tag{47}$$

$$\text{Dia-IFWAA}_{\min}(\lambda \mathcal{W}_1, \ldots, \lambda \mathcal{W}_n) \geq \text{Dia-IFWAA}_{\min}\left(\mathcal{W}_1^{\lambda}, \ldots, \mathcal{W}_n^{\lambda}\right); \tag{48}$$

$$\lambda \text{Dia-IFWAA}_{\max}(\mathcal{W}_1, \ldots, \mathcal{W}_n) \geq (\text{Dia-IFWAA}_{\max}(\mathcal{W}_1, \ldots, \mathcal{W}_n))^{\lambda}; \tag{49}$$

$$\lambda \text{Dia-IFWAA}_{\min}(\mathcal{W}_1, \ldots, \mathcal{W}_n) \geq (\text{Dia-IFWAA}_{\min}(\mathcal{W}_1, \ldots, \mathcal{W}_n))^{\lambda}; \tag{50}$$

$$\text{Dia-IFWGA}_{\max}(\lambda \mathcal{W}_1, \ldots, \lambda \mathcal{W}_n) \geq \text{Dia-IFWGA}_{\max}\left(\mathcal{W}_1^{\lambda}, \ldots, \mathcal{W}_n^{\lambda}\right); \tag{51}$$

$$\text{Dia-IFWGA}_{\min}(\lambda \mathcal{W}_1, \ldots, \lambda \mathcal{W}_n) \geq \text{Dia-IFWGA}_{\min}\left(\mathcal{W}_1^{\lambda}, \ldots, \mathcal{W}_n^{\lambda}\right); \tag{52}$$

$$\lambda \text{Dia-IFWGA}_{\max}(\mathcal{W}_1, \ldots, \mathcal{W}_n) \geq (\text{Dia-IFWGA}_{\max}(\mathcal{W}_1, \ldots, \mathcal{W}_n))^{\lambda}; \tag{53}$$

$$\lambda \text{Dia-IFWGA}_{\min}(\mathcal{W}_1, \ldots, \mathcal{W}_n) \geq (\text{Dia-IFWGA}_{\min}(\mathcal{W}_1, \ldots, \mathcal{W}_n))^{\lambda}. \tag{54}$$

Proof. The proof of this theorem is straightforward from the fact $\lambda \mathcal{W}_i \geq \mathcal{W}_i^{\lambda}$.

Theorem 9 Let $\{\mathcal{W}_i = (\zeta_i, \gamma_i; \aleph_i)\}_{i=1,\ldots,n}$ and $\left\{\mathcal{L}_i = \left(\zeta_{\mathcal{L}_i}, \gamma_{\mathcal{L}_i}; \aleph_{\mathcal{L}_i}\right)\right\}_{i=1,\ldots,n}$ be two collections of n Dia-IFVs. If $\varpi = (\varpi_1, \ldots, \varpi_n)$ is a weight vector with $\sum_{i=1}^{n} \varpi_i = 1$, then

$$\text{Dia-IFWAA}_{\max}(\mathcal{W}_1 \oplus_{\max} \mathcal{L}_1, \ldots, \mathcal{W}_n \oplus_{\max} \mathcal{L}_n) \geq \text{Dia-IFWAA}_{\max}(\mathcal{W}_1 \otimes_{\max} \mathcal{L}_1, \ldots, \mathcal{W}_n \otimes_{\max} \mathcal{L}_1); \tag{55}$$

$$\text{Dia-IFWAA}_{\max}(\mathcal{W}_1 \oplus_{\min} \mathcal{L}_1, \ldots, \mathcal{W}_n \oplus_{\min} \mathcal{L}_n) \geq \text{Dia-IFWAA}_{\max}(\mathcal{W}_1 \otimes_{\min} \mathcal{L}_1, \ldots, \mathcal{W}_n \otimes_{\min} \mathcal{L}_1); \tag{56}$$

$$\text{Dia-IFWAA}_{\min}(\mathcal{W}_1 \oplus_{\max} \mathcal{L}_1, \ldots, \mathcal{W}_n \oplus_{\max} \mathcal{L}_n) \geq \text{Dia-IFWAA}_{\min}(\mathcal{W}_1 \otimes_{\max} \mathcal{L}_1, \ldots, \mathcal{W}_n \otimes_{\max} \mathcal{L}_1); \tag{57}$$

$$\text{Dia-IFWAA}_{\min}(\mathcal{W}_1 \oplus_{\min} \mathcal{L}_1, \ldots, \mathcal{W}_n \oplus_{\min} \mathcal{L}_n) \geq \text{Dia-IFWAA}_{\min}(\mathcal{W}_1 \otimes_{\min} \mathcal{L}_1, \ldots, \mathcal{W}_n \otimes_{\min} \mathcal{L}_1); \tag{58}$$

$$\text{Dia-IFWAA}_{\max}(\mathcal{W}_1, \ldots, \mathcal{W}_n) \oplus_{\max} \text{Dia-IFWAA}_{\max}(\mathcal{L}_1, \ldots, \mathcal{L}_n)$$

$$\geq \text{Dia-IFWAA}_{\max}(\mathcal{W}_1, \ldots, \mathcal{W}_n) \otimes_{\max} \text{Dia-IFWAA}_{\max}(\mathcal{L}_1, \ldots, \mathcal{L}_n); \tag{59}$$

$$\text{Dia-IFWAA}_{\min}(\mathcal{W}_1, \ldots, \mathcal{W}_n) \oplus_{\max} \text{Dia-IFWAA}_{\min}(\mathcal{L}_1, \ldots, \mathcal{L}_n)$$

$$\geq \text{Dia-IFWAA}_{\min}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \otimes_{\max} \text{Dia-IFWAA}_{\min}\left(\mathscr{L}_1, \ldots, \mathscr{L}_n\right); \tag{60}$$

$$\text{Dia-IFWGA}_{\max}\left(\mathscr{W}_1 \oplus_{\max} \mathscr{L}_1, \ldots, \mathscr{W}_n \oplus_{\max} \mathscr{L}_n\right) \geq \text{Dia-IFWGA}_{\max}\left(\mathscr{W}_1 \otimes_{\max} \mathscr{L}_1, \ldots, \mathscr{W}_n \otimes_{\max} \mathscr{L}_1\right); \tag{61}$$

$$\text{Dia-IFWGA}_{\min}\left(\mathscr{W}_1 \oplus_{\min} \mathscr{L}_1, \ldots, \mathscr{W}_n \oplus_{\min} \mathscr{L}_n\right) \geq \text{Dia-IFWGA}_{\min}\left(\mathscr{W}_1 \otimes_{\min} \mathscr{L}_1, \ldots, \mathscr{W}_n \otimes_{\min} \mathscr{L}_1\right); \tag{62}$$

$$\text{Dia-IFWGA}_{\min}\left(\mathscr{W}_1 \oplus_{\max} \mathscr{L}_1, \ldots, \mathscr{W}_n \oplus_{\max} \mathscr{L}_n\right) \geq \text{Dia-IFWGA}_{\min}\left(\mathscr{W}_1 \otimes_{\max} \mathscr{L}_1, \ldots, \mathscr{W}_n \otimes_{\max} \mathscr{L}_1\right); \tag{63}$$

$$\text{Dia-IFWGA}_{\min}\left(\mathscr{W}_1 \oplus_{\min} \mathscr{L}_1, \ldots, \mathscr{W}_n \oplus_{\min} \mathscr{L}_n\right) \geq \text{Dia-IFWGA}_{\min}\left(\mathscr{W}_1 \otimes_{\min} \mathscr{L}_1, \ldots, \mathscr{W}_n \otimes_{\min} \mathscr{L}_1\right); \tag{64}$$

$$\text{Dia-IFWGA}_{\max}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \oplus_{\max} \text{Dia-IFWGA}_{\max}\left(\mathscr{L}_1, \ldots, \mathscr{L}_n\right)$$

$$\geq \text{Dia-IFWGA}_{\max}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \otimes_{\max} \text{Dia-IFWGA}_{\max}\left(\mathscr{L}_1, \ldots, \mathscr{L}_n\right); \tag{65}$$

$$\text{Dia-IFWGA}_{\min}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \oplus_{\max} \text{Dia-IFWGA}_{\min}\left(\mathscr{L}_1, \ldots, \mathscr{L}_n\right)$$

$$\geq \text{Dia-IFWGA}_{\min}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right) \otimes_{\max} \text{Dia-IFWGA}_{\min}\left(\mathscr{L}_1, \ldots, \mathscr{L}_n\right). \tag{66}$$

Proof. The derivation of this theorem follows directly from the observations that for any Dia-IFVs $\mathscr{W}_i$ and $\mathscr{L}_i$, we have $\mathscr{W}_i \oplus_{\max} \mathscr{L}_i \geq \mathscr{W}_i \otimes_{\max} \mathscr{L}_i$, $\mathscr{W}_i \oplus_{\min} \mathscr{L}_i \geq \mathscr{W}_i \otimes_{\min} \mathscr{L}_i$, and monotonicity of Dia-IFWWA$_{\max}$, Dia-IFWAA$_{\min}$, Dia-IFWGA$_{\max}$, and Dia-IFWGA$_{\min}$.

Theorem 10 Let $\{\mathscr{W}_i = (z_i, \gamma_i; \aleph_i)\}_{i=1, \ldots, n}$ be a list of n Dia-IFVs. If $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_n)$ is a weight vector with $\sum_{i=1}^{n} \varpi_i = 1$, then

$$\text{Dia-IFWAA}_{\max}\left(\mathscr{W}_1^c, \ldots, \mathscr{W}_n^c\right) = \left(\text{Dia-IFWGA}_{\max}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right)\right)^c; \tag{67}$$

$$\text{Dia-IFWAA}_{\min}\left(\mathscr{W}_1^c, \ldots, \mathscr{W}_n^c\right) = \left(\text{Dia-IFWGA}_{\min}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right)\right)^c; \tag{68}$$

$$\text{Dia-IFWGA}_{\max}\left(\mathscr{W}_1^c, \ldots, \mathscr{W}_n^c\right) = \left(\text{Dia-IFWWA}_{\max}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right)\right)^c; \tag{69}$$

$$\text{Dia-IFWGA}_{\min}\left(\mathscr{W}_1^c, \ldots, \mathscr{W}_n^c\right) = \left(\text{Dia-IFWWA}_{\min}\left(\mathscr{W}_1, \ldots, \mathscr{W}_n\right)\right)^c. \tag{70}$$

Proof. The proof is obvious.

5. Applications of Dia-IFSs in MCDM scenarios via CODAS technique

In this study, the CODAS method via Dia-IFSs is employed to address the MCDM due to its strong discrimination capability and robustness when evaluating closely competing alternatives. CODAS determines the relative performance of alternatives by simultaneously considering the Euclidean distance and the Taxicab distance from the negative-ideal solution. This dual-distance mechanism enhances the sensitivity of the ranking process, particularly in situations where alternatives exhibit similar performance levels across criteria.

Input:

Step 1: A Dia-IFS based decision-maker group is formed for the given alternatives and attributes, denoted as ℓ_j ($j \in \mathbb{N}$). Here, $\mathring{E} = \{\mathring{E}_1, \mathring{E}_2, ..., \mathring{E}_u, \mathring{E}_l\}$ represents a group of experts. The preferences of each expert are evaluated using Dia-IFVs. Thus, the decision data, expressed as Dia-IFVs, are arranged in the decision matrix M_1, M_2, and M_3.

Step 2: Next, we calculate the combined Dia-IF information using the Proposition 1 or find collective decision matrix (M).

Step 3: To obtain precise and meaningful results, it is necessary to normalize the Dia-IF input data before performing subsequent calculations. Therefore, the Dia-IF analysis is standardized by the following procedure:

$$\ell_j = \begin{cases} \left(\left\langle z_{\ell l_i}, \gamma_{\ell l_i} \right\rangle\right), & \text{same type input data} \\ \left(\left\langle \gamma_{\ell l_i}, z_{\ell l_i} \right\rangle\right), & \text{different type input data.} \end{cases} \tag{71}$$

Since the input data associated with the attributes are not uniform, normalization is necessary. Moreover, the alternatives and criteria in the considered problem differ in nature.

Step 4: Using Proposition 1, we construct the Dia-IF decision matrix (M) based on the decision data provided in matrices M_k ($k = 1, 2, 3$). With the help of Proposition 1, we then determine the overall preference values A_i for the alternatives.

Step 5: To finalize the dataset, experts initially assign weights to each criterion, ensuring that the final decision incorporates the collective judgments of all professional experts.

$$\varpi = \left(\frac{1}{r} \sum_{\zeta=1}^{r} \varpi_1^{\zeta}, \frac{1}{r} \sum_{\zeta=1}^{r} \varpi_2^{\zeta}, \ldots, \frac{1}{r} \sum_{\zeta=1}^{r} \varpi_n^{\zeta} \right)^T. \tag{72}$$

Step 6: The calculation of the weighted Dia-IF Decision Matrix (DM) is carried out by applying the following formulation with the criteria weight vector ϖ_j ($j = 1, 2, 3, 4, 5$), as presented below:

$$\varpi_j \ell_{ij} = \left\langle \left(1 - \left(1 - z_{\ell l_i}\right)^{\varpi_j}\right)^{\frac{1}{2}}, \gamma_{\ell_i}^{\varpi_j}; \left(1 - \left(1 - \aleph_{\ell_i}\right)^{\varpi_j}\right)^{\frac{1}{2}} \right\rangle. \tag{73}$$

Step 7: To identify the diamond intuitionistic fuzzy Negative Ideal Solution (NIS), the maximum criterion value across all alternatives is selected for each attribute, as defined below:

$$NIS = \left\{ \left\langle \mathbb{u}_{\ell_j}, \mathbb{v}_{\ell_j}; \aleph_{\ell_j} \right\rangle, j = 1, 2, 3, \ldots, m \right\},$$

$$= \left\{ \left\langle \min_{1 \le i \le n} z_{\ell_{ij}}, \max_{1 \le i \le n} \gamma_{\ell_{ij}}; \min_{1 \le i \le n} \aleph_{\ell_{ij}} \right\rangle, j = 1, 2, 3, \ldots, m \right\}. \tag{74}$$

Step 8: The Dia-IF Hamming and Dia-IF Euclidean distances between each alternative and the FIS are evaluated.

$$\Omega_i = H_2^1\left(\mathrm{FIS},\, \mathrm{A}_i\right) = \frac{1}{2} \times \left(\frac{1}{d} \sum_{\rho \in E} \frac{|\aleph_1 - \aleph_2|}{2} + \frac{1}{2d} \sum_{\rho \in E} \left(\left| z_{\ell_{ij}} - z_{\ell_j} \right| + \left| \gamma_{\ell_{ij}} - \gamma_{\ell_j} \right| \right) \right), \tag{75}$$

and

$$\mathring{\hat{\mathrm{E}}}_i = H_2^2\left(\mathrm{FIS},\, \mathrm{A}_i\right) = \frac{1}{2} \times \left(\frac{1}{d} \sum_{\rho \in E} \frac{|\aleph_1 - \aleph_2|}{2} + \sqrt{\frac{1}{2d} \sum_{\rho \in E} \left(\left| z_{\ell_{ij}} - z_{\ell_j} \right|^2 + \left| \gamma_{\ell_{ij}} - \gamma_{\ell_j} \right|^2 \right)} \right). \tag{76}$$

Or

$$\Omega_i = H_3^1\left(\mathrm{FIS},\, \mathrm{A}_i\right) = \frac{1}{2} \times \left(\frac{1}{d} \sum_{\rho \in E} \frac{|\aleph_1 - \aleph_2|}{2} + \frac{1}{2d} \sum_{\rho \in E} \left(\left| z_{\ell_{ij}} - z_{\ell_j} \right| + \left| \gamma_{\ell_{ij}} - \gamma_{\ell_j} \right| + \left| \pi_{\ell_{ij}} - \pi_{\ell_j} \right| \right) \right), \tag{77}$$

and

$$\mathring{\hat{\mathrm{E}}}_i = H_3^2\left(\mathrm{FIS},\, \mathrm{A}_i\right) = \frac{1}{2} \times \left(\frac{1}{d} \sum_{\rho \in E} \frac{|\aleph_1 - \aleph_2|}{2} + \sqrt{\frac{1}{2d} \sum_{\rho \in E} \left(\left| z_{\ell_{ij}} - z_{\ell_j} \right|^2 + \left| \gamma_{\ell_{ij}} - \gamma_{\ell_j} \right|^2 + \left| \pi_{\ell_{ij}} - \pi_{\ell_j} \right|^2 \right)} \right). \tag{78}$$

Step 9: Using the computed values of H_2^q and H_3^q from the preceding step, the Relative Assessment Matrix (RAM) is constructed as follows:

$$\mathscr{R} = [\mathscr{R}_{ik}]_{m \times m},$$

$$\mathscr{R}_{ik} = \left(\mathring{\hat{\mathrm{E}}}_i - \mathring{\hat{\mathrm{E}}}_k \right) + \left(\Lambda_{ik} \left(\mathring{\hat{\mathrm{E}}}_i - \mathring{\hat{\mathrm{E}}}_k \right) + \left(\Omega_i - \Omega_k \right) \right). \tag{79}$$

The parameter Λ_{ik} is introduced as a threshold function to evaluate whether the Euclidean distances of two alternatives can be considered equivalent. Its formulation is given as follows

$$\Lambda_{ik} = \begin{cases} 1 \ \text{if} & \left| \mathring{\hat{\mathrm{E}}}_i - \mathring{\hat{\mathrm{E}}}_k \right| \leq \gamma \\ 0 \ \text{if} & \left| \mathring{\hat{\mathrm{E}}}_i - \mathring{\hat{\mathrm{E}}}_k \right| > \gamma. \end{cases} \tag{80}$$

The threshold parameter serves as a control mechanism in the CODAS procedure. When the Euclidean distance between two alternatives falls below this threshold, the Hamming distance is applied as a supplementary criterion to ensure a more reliable distinction between the alternatives.

Step 10: Based on the RAM evaluation, the alternative achieving the highest score is ranked first and identified as the optimal choice, as shown below:

$$\mathscr{R}_i = \sum_{k=1}^{m} \mathscr{R}_{ik}. \tag{81}$$

5.1 *Selection of airways via Dia-IFSs*

In this section, a practical case study is presented to select the best airline worldwide. The Dia-IFS-CODAS method is applied to rank four alternatives based on a combination of quantitative and qualitative, and potentially conflicting, criteria.

Airways, or airlines, are a vital mode of transportation that connect cities, countries, and continents, facilitating global trade, tourism, and personal mobility. They operate on established flight routes using aircraft of varying capacities to serve domestic, regional, and international destinations. Airlines are broadly categorized into full-service carriers, which provide comprehensive amenities such as in-flight meals and entertainment, and low-cost carriers, which prioritize affordability with minimal services. Regional airlines cater to shorter routes, often linking smaller towns to larger hubs. The airline industry is a cornerstone of the global economy, supporting millions of jobs and enabling rapid connectivity. Technological advancements have transformed the sector, leading to fuel-efficient planes, enhanced safety measures, and digital systems for booking and customer service. However, challenges such as volatile fuel prices, environmental concerns, and economic fluctuations often impact operations. Despite these challenges, airways continue to adapt and evolve, playing a critical role in connecting people and facilitating commerce worldwide.

Four international airlines (A_1, A_2, A_3, and A_4) were considered by three experts (DM_1, DM_2, and DM_3) based on five criteria ($\hat{\mathrm{C}}_1$, $\hat{\mathrm{C}}_2$, $\hat{\mathrm{C}}_3$, $\hat{\mathrm{C}}_4$, and $\hat{\mathrm{C}}_5$) such that:

Qatar Airways ($\mathbf{A}_1$), established in 1993 and headquartered in Doha, Qatar, is one of the world's leading airlines known for its exceptional service and extensive global network. Operating from its hub at Hamad International Airport, it connects over 160 destinations across six continents. Renowned for luxury and innovation, the airline frequently earns accolades, including multiple awards for being the "World's Best Airline" by Skytrax. Qatar Airways is a member of the oneworld alliance, enhancing connectivity and convenience for travelers. It offers modern fleet options, including state-of-the-art aircraft like the Airbus A350 and Boeing 787, emphasizing passenger comfort, cutting-edge technology, and sustainability.

Singapore Airlines ($\mathbf{A}_2$), founded in 1947, is a globally recognized premium airline headquartered in Singapore. Known for its exceptional service and innovation, it operates a modern fleet and connects travelers to over 130 destinations worldwide. Its hub at Changi Airport is renowned for efficiency and luxury. The airline has consistently earned accolades for its cabin service, first-class amenities, and operational excellence. Singapore Airlines was also a pioneer in introducing the Airbus A380 for commercial flights and continues to focus on sustainability and passenger comfort, making it one of the most respected names in the aviation industry.

Emirates ($\mathbf{A}_3$), founded in 1985 and based in Dubai, is one of the world's largest and most successful airlines. Known for its luxury services and modern fleet, Emirates connects over 150 destinations across six continents. The airline has consistently been a leader in offering innovative in-flight services, including private suites in first class and its high-tech entertainment system. With a focus on customer satisfaction and operational efficiency, Emirates has earned a reputation for excellence in global air travel. It is also recognized for its extensive network, which makes Dubai a central hub for international travel.

All Nippon Airways (ANA) ($\mathbf{A}_4$), founded in 1952, is Japan's largest airline and a key player in international air travel. Headquartered in Tokyo, ANA operates both domestic and international flights, serving a network that spans over 80 destinations across Asia, Europe, North America, and beyond. Known for its commitment to service excellence, ANA has earned recognition for its punctuality, customer service, and the quality of its in-flight experience. The airline is a member of the Star Alliance, enabling it to offer a wide array of connections globally. ANA has consistently ranked as one of the top airlines in terms of safety and passenger satisfaction.

The criteria considered for determining the optimal alternative are as follows:

Booking and ticketing ($\hat{\mathbf{C}}_1$), services of airlines are essential components that enable passengers to reserve seats for flights and manage their travel plans. These services are typically offered through multiple channels, including airline websites, mobile apps, travel agencies, and direct airline counters. Online booking systems allow passengers to search for flights, compare prices, select seats, and pay for their tickets conveniently. Modern ticketing also involves electronic tickets (e-tickets), which eliminate the need for paper tickets and provide more flexibility for managing bookings.

Airlines offer a range of services associated with booking and ticketing, such as options for seat selection, meal preferences, baggage allowance, and flight upgrades. Additionally, passengers can modify or cancel their bookings, request refunds, and receive real-time flight updates. Many airlines also provide loyalty programs and frequent flyer benefits that can be integrated into the booking and ticketing process. The efficiency and user-friendliness of booking platforms have significantly improved over the years, making travel more accessible and manageable for customers worldwide (see Figure 3).

Check-in and boarding process ($\hat{\mathbf{C}}_2$), is a crucial step in air travel, ensuring that passengers are properly registered for their flights and can board the aircraft smoothly. The process typically starts with check-in, which can be done either online (via an airline's website or mobile app) or at the airport. Online check-in allows passengers to select their seats, confirm their details, and sometimes even print their boarding pass or download it digitally. For those who check in at the airport, the process usually involves presenting identification and flight details, with an airline agent verifying your booking and issuing a physical boarding pass.

After check-in, passengers proceed to security checks where they must clear customs and security protocols. Once cleared, they proceed to the designated boarding gate. Boarding involves passengers lining up in groups (as per their seat rows or class of service), and presenting their boarding pass to airline staff for validation. Upon receiving clearance, passengers then board the aircraft in an orderly fashion. The efficiency and ease of this process are essential for timely departures and a stress-free experience for passengers.

Cabin service ($\hat{\mathbf{C}}_3$), refers to the range of services provided to passengers during their flight to ensure comfort and satisfaction. This includes assistance with seating, meal and beverage offerings, entertainment, and addressing any special requests or concerns. Flight attendants are responsible for delivering these services, ensuring that passengers are well taken care of throughout the journey. Cabin service also encompasses safety instructions, including the demonstration of emergency protocols, and helping passengers settle into their seats. High-quality cabin service is integral to a positive flying experience, with airlines often striving to provide a welcoming and efficient atmosphere. Additionally, service can vary depending on the class of travel, with premium classes often receiving more personalized and luxurious attention compared to economy class.

Responsiveness ($\hat{\mathbf{C}}_4$), in the context of airways refers to how quickly and efficiently airlines respond to customer needs, issues, and feedback. This includes addressing concerns during booking, check-in, flight delays, or cancellations, as well as handling complaints or inquiries through customer service channels. A responsive airline ensures that passengers' concerns are acknowledged and resolved promptly, contributing to overall customer satisfaction. Timely updates on flight status, the ability to adapt to changing circumstances like delays or emergencies, and the swift provision of assistance or compensation in case of service disruption are key indicators of an airline's responsiveness. It also involves seamless communication across multiple platforms, such as phone, email, and social media, to ensure passengers have a positive experience. Airlines with strong responsiveness typically foster higher customer loyalty and brand reputation.

Network and connectivity ($\hat{\mathbf{C}}_5$), are essential factors for airlines, as they determine the scope and efficiency of an airline's operations and its ability to serve a wide range of destinations. A well-established network allows airlines to offer a variety of routes, ensuring convenience and flexibility for passengers. Airlines with extensive networks typically serve major global hubs and provide direct flights to a large number of international destinations. Connectivity, on the other hand, involves the ease with which passengers can connect between flights, either through well-timed layovers or seamless connections to other destinations within the airline's network.

Airlines that have strong connectivity offer passengers more choices in terms of direct flights or short transit times, helping to reduce travel time and improve convenience. For instance, major international airlines like Emirates, Singapore Airlines, and Qatar Airways have robust global networks that link passengers from regional locations to key international cities, often via their well-connected hubs. A wider network not only increases the competitiveness of airlines but also enhances customer satisfaction by providing greater flexibility and more flight options. Additionally, interline agreements and alliances such as the Star Alliance, SkyTeam, and Oneworld further enhance connectivity by allowing airlines to cooperate, expanding their reach while maintaining efficiency in service delivery.

In summary, a strong network and connectivity play a vital role in shaping an airline's success, as they directly impact customer satisfaction, operational efficiency, and market competitiveness.

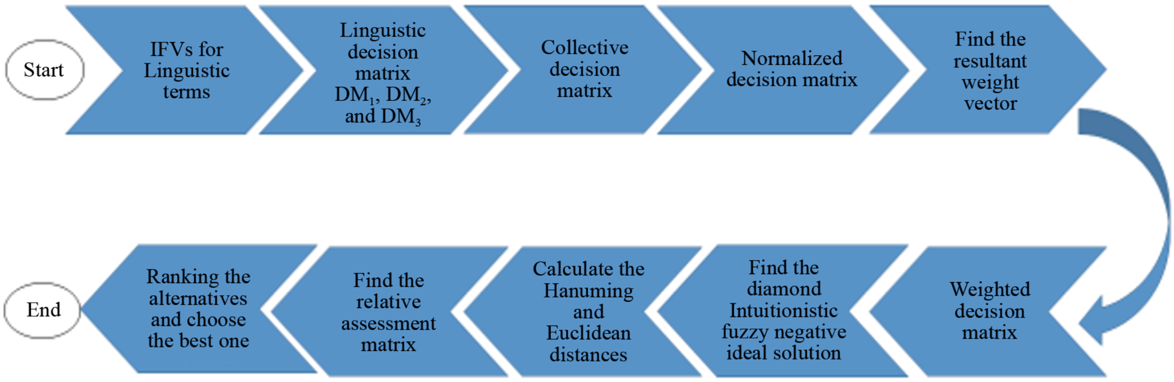

Figure 3. Flow chart of the proposed algorithm

5.2 *Using algorithm to solve*

Step 1: The proposed algorithm is applied to the input data, in which three medical experts evaluate four emergency alternatives A_i ($i = 1, 2, 3, 4$) with respect to five criteria $\hat{C}_i$ ($i = 1, 2, 3, 4, 5$). To construct the decision matrices for the three experts, DM_ζ ($\zeta = 1, 2, 3$), IFVs corresponding to linguistic terms are adopted as specified in Table 1.

Table 1. IFVs for linguistic terms

Linguistic terms	Acronym	IFVs
Extremely Good	EG	$\langle 0.8, 0.2 \rangle$
Very Very Good	VVG	$\langle 0.6, 0.3 \rangle$
Very Good	VG	$\langle 0.5, 0.5 \rangle$
Good	G	$\langle 0.4, 0.5 \rangle$
Medium Good	MG	$\langle 0.4, 0.5 \rangle$
Medium	M	$\langle 0.4, 0.5 \rangle$
Medium Bad	MB	$\langle 0.5, 0.4 \rangle$
Bad	B	$\langle 0.3, 0.7 \rangle$
Very Bad	VB	$\langle 0.3, 0.6 \rangle$
Very Very Bad	VVB	$\langle 0.1, 0.8 \rangle$
Extremely Bad	EB	$\langle 0.1, 0.9 \rangle$

Subsequently, expert evaluations for the decision-making process are gathered and expressed using IF linguistic terms, as presented in Tables 2–4. Based on these assessments, the corresponding Intuitionistic Fuzzy Decision Making (IFDM), shown in Tables 5–7, are constructed from Tables 2–4, respectively, by applying the IFVs defined in Table 1.

Table 2. Linguistic decision matrix DM_1 via IFVs

	$\hat{C}_1$	$\hat{C}_2$	$\hat{C}_3$	$\hat{C}_4$	$\hat{C}_5$
$\underset{\sim}{A}_1$	G	M	B	VVG	VG
$\underset{\sim}{A}_2$	MG	EB	VVG	VG	VB
$\underset{\sim}{A}_3$	VG	G	VVB	EB	MG
$\underset{\sim}{A}_4$	MB	B	VG	VB	G

Table 3. Linguistic decision matrix DM_2 via IFVs

	$\hat{C}_1$	$\hat{C}_2$	$\hat{C}_3$	$\hat{C}_4$	$\hat{C}_5$
$\underset{\sim}{A}_1$	MB	B	M	G	VVB
$\underset{\sim}{A}_2$	VB	VVG	EB	MG	VG
$\underset{\sim}{A}_3$	MG	VVB	G	VVG	EB
$\underset{\sim}{A}_4$	M	VG	B	VVB	VB

Table 4. Linguistic decision matrix DM_3 via IFVs

	$\hat{C}_1$	$\hat{C}_2$	$\hat{C}_3$	$\hat{C}_4$	$\hat{C}_5$
$\underset{\sim}{A}_1$	M	VVG	MB	G	B
$\underset{\sim}{A}_2$	EG	VG	VB	MG	VVG
$\underset{\sim}{A}_3$	G	EB	MG	VVG	VVB
$\underset{\sim}{A}_4$	B	VB	M	VVB	VG

Table 5. Decision matrix DM_1 via IFVs

	$\hat{C}_1$	$\hat{C}_2$	$\hat{C}_3$	$\hat{C}_4$	$\hat{C}_5$
$\underset{\sim}{A}_1$	$\langle 0.4, 0.5\rangle$	$\langle 0.4, 0.5\rangle$	$\langle 0.3, 0.7\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0.5, 0.5\rangle$
$\underset{\sim}{A}_2$	$\langle 0.4, 0.5\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.3, 0.6\rangle$
$\underset{\sim}{A}_3$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.5\rangle$	$\langle 0.1, 0.8\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.4, 0.5\rangle$
$\underset{\sim}{A}_4$	$\langle 0.5, 0.4\rangle$	$\langle 0.3, 0.7\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.3, 0.6\rangle$	$\langle 0.4, 0.5\rangle$

Table 6. Decision matrix DM_2 via IFVs

	$\hat{C}_1$	$\hat{C}_2$	$\hat{C}_3$	$\hat{C}_4$	$\hat{C}_5$
$\underset{\sim}{A}_1$	$\langle 0.5, 0.4\rangle$	$\langle 0.3, 0.7\rangle$	$\langle 0.4, 0.5\rangle$	$\langle 0.4, 0.5\rangle$	$\langle 0.1, 0.8\rangle$
$\underset{\sim}{A}_2$	$\langle 0.3, 0.6\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.4, 0.5\rangle$	$\langle 0.5, 0.5\rangle$
$\underset{\sim}{A}_3$	$\langle 0.4, 0.5\rangle$	$\langle 0.1, 0.8\rangle$	$\langle 0.4, 0.5\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0.1, 0.9\rangle$
$\underset{\sim}{A}_4$	$\langle 0.4, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.3, 0.7\rangle$	$\langle 0.1, 0.8\rangle$	$\langle 0.3, 0.6\rangle$

Table 7. Decision matrix DM_3 via IFVs

	$\hat{C}_1$	$\hat{C}_2$	$\hat{C}_3$	$\hat{C}_4$	$\hat{C}_5$
A_1	$\langle 0.4, 0.5\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.4, 0.5\rangle$	$\langle 0.3, 0.7\rangle$
A_2	$\langle 0.8, 0.2\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.3, 0.6\rangle$	$\langle 0.4, 0.5\rangle$	$\langle 0.6, 0.3\rangle$
A_3	$\langle 0.4, 0.5\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.4, 0.5\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0.1, 0.8\rangle$
A_4	$\langle 0.3, 0.7\rangle$	$\langle 0.3, 0.6\rangle$	$\langle 0.4, 0.5\rangle$	$\langle 0.1, 0.8\rangle$	$\langle 0.5, 0.5\rangle$

Step 2: The collective Dia-IF information is then derived using Proposition 1, as summarized in Table 8.

Table 8. Collective DM via Dia-IFVs

	$\hat{C}_1$	$\hat{C}_2$	$\hat{C}_3$	$\hat{C}_4$	$\hat{C}_5$
A_1	$\langle 0.43, 0.47; 0.14\rangle$	$\langle 0.43, 0.50; 0.37\rangle$	$\langle 0.40, 0.53; 0.27\rangle$	$\langle 0.47, 0.43; 0.26\rangle$	$\langle 0.30, 0.67; 0.37\rangle$
A_2	$\langle 0.50, 0.43; 0.53\rangle$	$\langle 0.40, 0.57; 0.63\rangle$	$\langle 0.33, 0.60; 0.57\rangle$	$\langle 0.43, 0.50; 0.07\rangle$	$\langle 0.47, 0.47; 0.30\rangle$
A_3	$\langle 0.43, 0.50; 0.07\rangle$	$\langle 0.20, 0.73; 0.43\rangle$	$\langle 0.30, 0.60; 0.40\rangle$	$\langle 0.43, 0.50; 0.73\rangle$	$\langle 0.20, 0.73; 0.43\rangle$
A_4	$\langle 0.40, 0.53; 0.27\rangle$	$\langle 0.37, 0.60; 0.23\rangle$	$\langle 0.40, 0.57; 0.23\rangle$	$\langle 0.17, 0.73; 0.26\rangle$	$\langle 0.40, 0.37; 0.33\rangle$

Step 3: To normalize the Dia-IF decision matrix, the complement of the cost criteria is applied. Since criteria such as the Booking and ticketing cost ($\hat{C}_1$) is non-beneficial, adjustments are made by interchanging the membership and non-membership mapping. This yields the normalized Dia-IF decision matrix, as outlined in Table 9.

Table 9. Collective DM via Dia-IFVs

	$\hat{C}_1$	$\hat{C}_2$	$\hat{C}_3$	$\hat{C}_4$	$\hat{C}_5$
A_1	$\langle 0.47, 0.43; 0.14\rangle$	$\langle 0.43, 0.50; 0.37\rangle$	$\langle 0.40, 0.53; 0.27\rangle$	$\langle 0.47, 0.43; 0.26\rangle$	$\langle 0.30, 0.67; 0.37\rangle$
A_2	$\langle 0.43, 0.50; 0.53\rangle$	$\langle 0.40, 0.57; 0.63\rangle$	$\langle 0.33, 0.60; 0.57\rangle$	$\langle 0.43, 0.50; 0.07\rangle$	$\langle 0.47, 0.47; 0.30\rangle$
A_3	$\langle 0.50, 0.43; 0.07\rangle$	$\langle 0.20, 0.73; 0.43\rangle$	$\langle 0.30, 0.60; 0.40\rangle$	$\langle 0.43, 0.50; 0.73\rangle$	$\langle 0.20, 0.73; 0.43\rangle$
A_4	$\langle 0.53, 0.40; 0.27\rangle$	$\langle 0.37, 0.60; 0.23\rangle$	$\langle 0.40, 0.57; 0.23\rangle$	$\langle 0.17, 0.73; 0.26\rangle$	$\langle 0.40, 0.37; 0.33\rangle$

Step 4: The weight vectors associated with the Dia-IF data are summarized as follows

$$w_1 = (0.4, 0.25, 0.1, 0.15, 0.1)^T,$$

$$w_2 = (0.3, 0.3, 0.1, 0.2, 0.1)^T,$$

$$w_3 = (0.4, 0.3, 0.1, 0.1, 0.1)^T,$$

$$w_4 = (0.3, 0.3, 0.2, 0.1, 0.1)^T,$$

$$w_5 = (0.15, 0.45, 0.15, 0.15, 0.1)^T.$$

The criteria weights are computed using Eq. (66), yielding the final weight vector: $\varpi = (0.31, 0.32, 0.13, 0.14, 0.1)^T$. This vector satisfies the normalization condition $\sum_{j=1}^{5} \varpi_j = 1$.

Step 5: Using the obtained weight vector ϖ, Dia-IFDM is constructed. As a result, the corresponding weighted Dia-IFDM is derived and presented in Table 10.

Table 10. Weighted DM via Dia-IFVs

	$\hat{C}_1$	$\hat{C}_2$	$\hat{C}_3$	$\hat{C}_4$	$\hat{C}_5$
A_1	⟨0.16, 0.79; 0.05⟩	⟨0.17, 0.80; 0.14⟩	⟨0.06, 0.92; 0.04⟩	⟨0.08, 0.89; 0.04⟩	⟨0.04, 0.96; 0.05⟩
A_2	⟨0.19, 0.77; 0.21⟩	⟨0.15, 0.83; 0.27⟩	⟨0.05, 0.94; 0.10⟩	⟨0.08, 0.91; 0.01⟩	⟨0.06, 0.93; 0.04⟩
A_3	⟨0.16, 0.81; 0.02⟩	⟨0.07, 0.91; 0.16⟩	⟨0.05, 0.94; 0.06⟩	⟨0.08, 0.91; 0.17⟩	⟨0.02, 0.97; 0.05⟩
A_4	⟨0.15, 0.82; 0.09⟩	⟨0.14, 0.85; 0.08⟩	⟨0.06, 0.93; 0.07⟩	⟨0.03, 0.96; 0.04⟩	⟨0.05, 0.94; 0.04⟩

Step 6: For Dia-IF-negative ideal solution, we have

$$\text{NIS} = \{\langle 0.15, 0.82; 0.02\rangle, \langle 0.07, 0.91; 0.08\rangle, \langle 0.05, 0.94; 0.04\rangle, \langle 0.03, 0.96; 0.01\rangle, \langle 0.02, 0.97; 0.04\rangle\}.$$

Step 7: Following this, the distances of each alternative A_i from the FIS are determined using both Hamming and Euclidean measures, denoted by H_2^1 and H_2^2, and the corresponding results are reported below.

$$H_2^1(A_1) = 0.03, \quad H_2^1(A_2) = 0.04, \quad H_2^1(A_3) = 0.02, \quad H_2^1(A_4) = 0.02,$$

$$H_2^2(A_1) = 0.12, \quad H_2^2(A_2) = 0.12, \quad H_2^2(A_3) = 0.05, \quad H_2^2(A_4) = 0.08,$$

and

$$H_3^1(A_1) = 0.03, \quad H_3^1(A_2) = 0.04, \quad H_3^1(A_3) = 0.02, \quad H_3^1(A_4) = 0.02,$$

$$H_3^2(A_1) = 0.13, \quad H_3^2(A_2) = 0.13, \quad H_3^2(A_3) = 0.06, \quad H_3^2(A_4) = 0.09.$$

Step 8: Based on the data presented above, the relative assessment matrix is generated using Eq. (73), with the threshold parameter assigned a value of 0.07.

For H_2^1 and H_2^2, we have

$$\begin{pmatrix} 0 & -0.010 & 0.085 & 0.053 \\ 0.01 & 0 & 0.095 & -0.085 \\ -0.085 & -0.095 & 0 & -0.032 \\ -0.053 & -0.063 & 0.032 & 0 \end{pmatrix}.$$

Or for H_3^1 and H_3^2, we have

$$\begin{pmatrix} 0 & -0.010 & 0.085 & 0.053 \\ 0.01 & 0 & 0.095 & -0.085 \\ -0.085 & -0.095 & 0 & -0.032 \\ -0.053 & -0.063 & 0.032 & 0 \end{pmatrix}.$$

Step 9: By aggregating the row-wise values, the final scores are calculated as $R_1 = 0.128$, $R_2 = 0.02$, $R_3 = -0.212$, and $R_4 = -0.084$. Based on descending score values, the ranking is $A_1 \succ A_2 \succ A_4 \succ A_3$, indicating that Qatar Airways is the preferred alternative under the Dia-IF CODAS framework.

As this study represents a pioneering effort, no existing methods are currently available for direct comparison within a purely Dia-IF framework. Consequently, this section compares the proposed approaches with well-established MCDM methods implemented in an Dia-IF environment. Since Dia-IFSs constitute an extension of traditional IF sets, a conventional IFS representation can be derived from a Dia-IFS by assigning $\aleph = 0$ for all elements. By applying this transformation to the case study data, the resulting outcomes are summarized in Table 11. The results indicate that the ranking of alternatives varies and is influenced by the choice of the MCDM method. Nevertheless, the proposed methodologies demonstrate a clear advantage by fully exploiting the richer information provided by experts through the broader Dia-IF framework. In particular, the Dia-IF CODAS approach is further strengthened by incorporating effective Hamming and Euclidean distance measures, which enhance its discrimination capability.

Table 11. A systematic comparison of the proposed study with existing classical methods

Authors	$\aleph$	Methods	Applicable or not applicable
Khan et al. [30]	No	VIKOR method	Not calculable
Imran et al. [59]	No	ELimination Et Choix Traduisant la REalite (ELECTRE)-II method	Not calculable
Peng [52]	No	Multi-Attributive Border Approximation Area Comparison (MABAC) method	Not calculable
Khan et al. [30]	No	Choquet Integral Fuzzy Dynamic Weighted Averaging (CIF DWA) operator	Not calculable
Garg [13]	No	Complex q-Rung Orthopair Fuzzy Rough Set (Cq-ROFRS) method	Not calculable
Zhang et al. [46]	No	EDAS method	Not calculable
Proposed	Yes	CODAS method	$A_4 \succ A_3 \succ A_1 \succ A_2$

6. Limitations and discussion of the proposed Dia-IFS-CODAS

Although the proposed Dia-IFS-CODAS framework provides a robust and effective approach for handling uncertainty in MCDM problems, several limitations should be acknowledged to present a comprehensive view of the method.

First, the DIFS representation relies on expert judgments to determine membership, non-membership, and hesitation degrees. As with most fuzzy-based approaches, the quality of the final decision outcomes depends on the reliability and consistency of expert inputs. While the diamond structure enhances expressive power and interpretability, inconsistencies in expert assessments may still influence the results.

Second, the computational complexity of the proposed method may increase with the number of alternatives, criteria, and decision-makers. The use of Dia-IFSs, combined with the dual-distance evaluation mechanism of the CODAS method, can lead to higher computational effort compared to classical fuzzy or crisp MCDM techniques. This limitation may affect scalability in very large-scale decision problems.

Third, the CODAS method assumes independence among decision criteria and evaluates alternatives based on their distances from the negative-ideal solution. In real-world applications, criteria may exhibit interdependencies or causal relationships that are not explicitly captured in the current framework. Incorporating such relationships could further enhance decision accuracy.

Finally, the weighting of criteria in the proposed framework is primarily dependent on subjective assessments. Although this allows flexibility and reflects expert knowledge, integrating objective or hybrid weighting methods could improve robustness and reduce potential bias.

Despite these limitations, the Dia-IFS-CODAS method remains highly suitable for decision-making environments characterized by uncertainty, vagueness, and closely competing alternatives. The identified shortcomings do not reduce the applicability of the proposed approach; rather, they provide valuable directions for future research, such as integrating objective weighting techniques, modeling criteria interdependencies, and developing computationally efficient implementations.

7. Conclusion

A new MCDM method for Dia-IFS decision-making is presented in this research. The usefulness of Dia-IFSs is found in their capacity to depict each alternative's rating inside a norm-parameter-defined diamond range. We outline fundamental algebraic operation rules for Dia-IFSs and suggest weighted aggregation operators to efficiently integrate expert assessments by leveraging the characteristics of Dia-IFSs and the function of Archimedean t-norm operations. These procedures are integrated into our suggested algorithm, an upgraded CODAS technique, which computes defuzzified values for ranking alternatives and aggregates expert preferences. The algorithm supports informed and trustworthy decision-making by providing decision-makers with a comprehensive framework to evaluate the influence of different ratings on the ultimate choice. Using numerical examples, we verify the procedure and compare the outcomes to those of other approaches, such as VIKOR and TOPSIS. Our work has consequences that extend beyond the algorithm. When used in decision-making, Dia-IFSs and their associated operations allow for the explicit management of ambiguity and uncertainty, which makes them especially useful in fields like the management of medical waste disposal. Our method enables local governments and healthcare professionals to choose trash disposal locations more intelligently by combining a variety of criteria and subjective evaluations.

Moreover, by defining important features and examining the applicability of these operations, our study advances the theoretical development of Dia-IFS. This establishes the foundation for further investigation into topics like decision fusion in multi-agent systems, dynamic decision processes, and collective decision-making. Furthermore, our approach has the potential to improve diagnostic accuracy and successfully integrate expert insights in domains such as image processing and medical diagnostics. Our work has wider ramifications for a number of industries where uncertainty and imprecision are prevalent, such as financial decision-making, transportation planning, and environmental management. Our method can help make more dependable and knowledgeable selections in challenging real-world situations by collecting and modeling decision-making ambiguity.

Future studies can apply the proposed method to diverse ambiguous and uncertain situations to reduce confusion and enhance clarity in decision-making. Additionally, this methodology can be used in a number of different domains, including Transformation techniques for interval-valued intuitionistic fuzzy sets [5], data-driven learning [67], evidence theory [68], and intelligent stock investment [69]. Additionally, we want to broaden our scope to investigate a variety of artificial intelligence-related applications, such as multi-objective optimization [70, 71] and neural networks [72].

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Data availability statement

Data sharing is not applicable to this article, as no new data were created or analyzed in this study.

Conflict of interest

The authors declare no competing financial interest.

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