Contemporary Mathematics
http://ojs.wiserpub.com/index.php/CM/

Research Article

Mathematical Assessment of the Reliability of Battery Systems of Electric Transport Rolling Stock Taking into Account Thermal Barriers

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Received: 4 January 2026; **Revised:** 3 April 2026; **Accepted:** 9 April 2026

Abstract: A comprehensive mathematical framework for assessing the reliability of lithium-ion battery systems of urban electric transport rolling stock, taking into account selectively placed thermal barriers, was developed. The reliability problem was formulated in the context of cascading failures initiated by thermal instability of individual cells and was described using coupled partial differential equations of heat transfer and Arrhenius-type temperature-dependent degradation kinetics. A deterministic finite element model was constructed to evaluate the influence of different thermal barrier configurations on temperature fields and the state of health (SoH) degradation rate. The model was further extended using Monte Carlo simulation to incorporate uncertainty in key parameters and to estimate confidence intervals of reliability indicators. In addition, a discrete optimization problem of barrier placement was formulated and solved using a genetic algorithm. It was demonstrated that selective placement of thermal barriers in thermally critical regions led to a reduction in degradation rate by 15%–20% compared to both barrier-free and uniformly shielded configurations, while also reducing uncertainty in lifetime prediction. The obtained results provided a quantitative basis for reliability-oriented design and optimization of battery systems.

***Keywords*:** reliability, lithium-ion batteries (LIBs), cascading failures, heat transfer, thermal barriers, state of health (SoH) status, partial differential equations, mathematical modeling

MSC: 35K05, 35Q79, 60K10, 65M60, 90B25

1. Introduction

The transition to sustainable mobility and decarbonization of the transport sector is one of the key tasks of the modern world economy [1, 2]. In this context, electric transport, especially urban transport, plays a crucial role. Its environmental efficiency and operational efficiency directly depend on the reliability and durability of battery systems, the main source of energy. Among the many energy storage technologies, lithium-ion batteries (LIBs) have become the de facto standard due to their high energy density, moderate self-discharge, and improving cost characteristics [3]. However, their widespread

DOI: https://doi.org/10.37256/cm.7420269541

introduction into rolling stock systems operating under variable and high loads has revealed a number of critical problems related to reliability and safety [4, 5].

The main challenge for designers is the thermal management of battery modules. Exothermic reactions, Joule losses, and degradation processes of active materials generate a significant amount of heat, which can lead to local overheating under dense cell arrangement conditions [6]. The most dangerous scenario is thermal runaway, when the failure of one cell due to overheating initiates a chain reaction in neighboring elements, which can lead to a complete failure of the module or even a fire [7, 8]. This process is exacerbated by a positive feedback: rising temperatures accelerate the rate of electrochemical degradation and side reactions, which in turn increases heat generation [9, 10].

Traditional approaches to thermal management, such as forced air cooling, fluid systems, or uniform placement of thermal insulation materials, while effective, are often redundant in terms of weight, volume, and cost. Moreover, they do not always take into account the spatial heterogeneity of thermal loads and the stochastic nature of failures. Modern trends in design require intelligent, adaptive solutions that provide targeted and cost-effective protection [11].

In light of this, the concept of selective thermal barriers represents a promising direction. Thermal barriers placed at strategic points (e.g., between cells with the highest risk of overheating or on the propagation paths of thermal edges) can effectively contain the spread of local superheats by preventing cascading failures without resorting to complete isolation of the entire module [12]. However, the design of such systems requires a deep understanding of interrelated physical processes: transient heat transfer in anisotropic media, the kinetics of temperature-dependent degradation, and the probabilistic nature of failures [13].

Existing scientific works in this area can be divided into several areas. The first is focused on detailed modeling of electrochemical and thermal processes in individual cells (Newman, Doyle-Fuller-Newman models) [14]. The second direction is devoted to the system analysis of the reliability of battery assemblies using the methods of probability theory and statistics (for example, the construction of fault trees, Weibull analysis). The third direction explores the methods of passive and active thermal management. However, there is a lack of comprehensive studies that would mathematically link the topology of discrete thermal barriers with the reliability and durability of the entire system under uncertainty [7, 8]. The optimization problem is often overlooked, where thermal barriers are considered as a limited resource, and their location as a set of discrete variables that require special combinatorial optimization methods [15].

The purpose of this work is to develop and verify a comprehensive mathematical apparatus for assessing and optimizing the reliability of lithium-ion battery systems of urban electric transport, taking into account the selective placement of thermal barriers. This device should ensure the transition from qualitative engineering assessments to quantitative, probabilistic forecasting of service life [16]. It should be noted that the term "mathematical assessment" used in this study refers to the formulation of a coupled mathematical model describing heat transfer and temperature-dependent degradation processes in battery systems. The reliability analysis is based on a system of partial differential equations and kinetic relations, which define the theoretical framework of the approach.

Numerical methods, in particular the finite element method and Monte Carlo simulation, are employed as computational tools for solving the formulated mathematical model and analyzing its behavior under different configurations and uncertainties. Therefore, the proposed approach combines a rigorous mathematical formulation with numerical implementation, where the latter serves as a means of evaluating the derived model.

To achieve this goal, the following tasks are consistently solved in the work:

1. Formalization of the related problem of heat transfer and degradation in a battery module, taking into account internal heat sources and areas with contrasting thermophysical properties (barriers).

2. Development of a deterministic finite element model to analyze the effect of different barrier configurations on temperature fields and the state of health (SoH) degradation rate.

3. Stochastic extension of the model to account for the dispersion of input parameters and estimate confidence intervals of key reliability indicators.

4. Formulation of the problem of discrete optimization of the topology of barriers as a combinatorial problem and its solution using a genetic algorithm.

5. Quantitative analysis of optimization results, identification of general principles of effective placement of barriers and assessment of practical benefits.

The scientific novelty of the work lies in the following:

1. A coupled mathematical model is proposed that integrates the equation of non-stationary thermal conductivity with boundary conditions on barriers and an Arrhenius-type kinetic model of SoH degradation sensitive to local temperature history.

2. For the first time, thermal barriers were considered not just as passive elements, but as a tool for structural reliability management, which made it possible to move on to the formulation of the problem of their optimal placement.

3. The integrated approach combines deterministic modeling, Monte Carlo stochastic analysis, and evolutionary optimization methods, which makes it possible to assess both the average efficiency and robustness of solutions to parameter variations.

4. New quantitative estimates have been obtained showing that the strategic placement of barriers can not only slow degradation by 15%–20%, but also reduce the uncertainty of the lifetime forecast by 30% compared to systems without barriers.

The main contribution of this paper lies in the integration of deterministic thermal modeling, stochastic reliability analysis, and combinatorial optimization of thermal barrier placement into a unified framework. Unlike existing studies that typically consider these aspects separately, the proposed approach establishes a direct quantitative link between spatial thermal management and probabilistic reliability indicators under uncertainty. The novelty of the work is not in introducing fundamentally new mathematical equations, but in the systematic integration of established methods (partial differential equations, Arrhenius kinetics, Monte Carlo simulation, and genetic algorithms) into a single reliability-oriented design methodology.

The article is organized as follows. Section 2 describes the mathematical foundations of the model: the heat transfer equation, the Arrhenius-type kinetic model of SoH degradation, and the approach to modeling thermal barriers. Section 3 presents the results of deterministic finite element analysis for three basic configurations. Section 4 is devoted to the stochastic extension of the model and the estimation of confidence intervals of degradation. In Section 5, the problem of optimizing the topology of barriers is formulated and solved using a genetic algorithm; The found optimal configurations and the corresponding temperature fields are analyzed. Section 6 summarizes, discusses the practical interpretation of the results, and outlines ways for further research.

Objectives of the study: The main objectives of this study were formulated as follows:

• To develop a coupled mathematical model integrating heat transfer and temperature-dependent degradation of lithium-ion batteries.

• To investigate the influence of thermal barrier configurations on temperature distribution and SoH degradation using finite element analysis.

• To extend the model by incorporating stochastic variability of parameters using Monte Carlo simulation.

• To formulate and solve the problem of optimal placement of thermal barriers as a discrete optimization problem using a genetic algorithm.

• To quantify the effect of selective thermal barrier placement on reliability and uncertainty of lifetime prediction.

Highlights of the paper: The key contributions and findings of this study can be summarized as follows:

• A comprehensive multi-level modeling framework combining deterministic, stochastic, and optimization approaches was developed.

• Thermal barriers were considered as a structural reliability management tool rather than passive elements.

• It was demonstrated that selective barrier placement reduces degradation rate by 15%–20% compared to conventional configurations.

• The proposed approach allows not only improving average reliability but also reducing uncertainty in lifetime prediction by up to 30%.

• The optimization of barrier topology was formulated as a combinatorial problem and effectively solved using a genetic algorithm.

2. Background

Thermal runaway and cascading failure. Thermal runaway refers to a self-accelerating exothermic process in a lithium-ion battery cell caused by internal degradation reactions. When a critical temperature threshold is exceeded, heat generation increases rapidly, which may lead to ignition or explosion. In densely packed battery modules, this phenomenon can propagate to neighboring cells, forming a cascading failure process, in which the failure of a single cell initiates a chain reaction.

state of health (SoH). The state of health (SoH) is a key indicator of battery degradation and is defined as the ratio of the current capacity to the nominal (initial) capacity. In this study, SoH is treated as a continuous function of temperature history and cycling, reflecting the cumulative effect of degradation processes.

Temperature-dependent degradation (Arrhenius model). The degradation rate of lithium-ion batteries strongly depends on temperature and is commonly described using an Arrhenius-type relation. This approach assumes that the rate of degradation increases exponentially with temperature, which makes thermal effects a critical factor in reliability assessment.

Thermal barriers. Thermal barriers are defined as localized regions with reduced thermal conductivity placed between adjacent cells. Their purpose is to limit heat transfer and suppress the propagation of thermal disturbances. In contrast to uniform insulation, selective placement of thermal barriers allows targeted control of heat flow within the module.

Deterministic and stochastic modeling. Deterministic modeling refers to the simulation of temperature fields and degradation dynamics using fixed parameter values, typically implemented via finite element methods. Stochastic modeling extends this approach by incorporating variability of input parameters (e.g., activation energy, heat generation), allowing estimation of confidence intervals of reliability indicators.

Discrete optimization of barrier placement. The placement of thermal barriers is treated as a combinatorial optimization problem, where each potential interface between cells represents a binary decision variable. The objective is to maximize system reliability under constraints on the number of barriers, which is solved using evolutionary algorithms such as genetic algorithms.

3. Definition of reliability and scope of the study

In this study, reliability of battery systems is defined as the ability of the system to maintain its functional performance over time under specified operating conditions, taking into account degradation and failure processes. Quantitatively, reliability is characterized through the state of health (SoH) evolution, the reliability function $R(t)$, interpreted as the probability of maintaining acceptable SoH above a threshold level, and the effective lifetime until a critical degradation level is reached.

The study is motivated by specific challenges of electric transport rolling stock, including high load variability, frequent charge-discharge cycles, limited cooling capacity, and strong spatial thermal gradients in densely packed battery modules. These factors significantly increase the probability of local overheating and cascading failures.

In this work, lithium-ion battery modules used in urban electric transport are considered, operating under cyclic loading conditions typical for public transport systems (frequent acceleration/braking, partial charging).

Thermal barriers are defined as localized regions with reduced thermal conductivity introduced between cells to limit heat propagation. They act as structural elements controlling heat flux and preventing the spread of thermal runaway, thus directly influencing system reliability.

The proposed modeling framework is hybrid in nature and includes:

• a deterministic component (partial differential equations of heat transfer),

• a stochastic component (Monte Carlo simulation of parameter variability),

• and a probabilistic interpretation of degradation and failure processes.

The model is based on the following assumptions:

• Temperature distribution inside each cell is spatially resolved at the module level using continuum approximation;

• Load cycles are represented by equivalent periodic thermal sources;

• Material properties (thermal conductivity, heat capacity) are constant within each domain but differ between cells and barriers;

• Environmental conditions are assumed to vary within bounded ranges and are included in stochastic simulations.

Thermal behavior is coupled with electrical performance through temperature-dependent degradation kinetics: the local temperature field obtained from the heat transfer model directly affects the rate of capacity loss (SoH) via an Arrhenius-type relation. This establishes a feedback loop between thermal state and reliability.

Temperature influences failure probability by accelerating degradation processes and increasing the likelihood of critical events (e.g., thermal runaway), which is reflected in faster decline of SoH and reduced effective lifetime in the proposed framework.

4. Methods and materials

To describe the heat transfer in the battery module, the Fourier equation of thermal conductivity [17] was used, taking into account internal heat sources (Joule losses and exothermic reactions):

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + \dot{q}_{gen},$$

where ρ is the density of the material, c_p is the specific heat capacity, k is the coefficient of thermal conductivity, T is the temperature, t is the time, $\dot{q}_{gen}$ and is the bulk density of heat release.

Battery health degradation (SoH) was modeled using an Arrhenius-type kinetic model as a function of temperature and the number of charge-discharge cycles [18]:

$$SoH(t) = SoH_0 - A \cdot \exp\left(-\frac{E_a}{RT_{avg}(t)}\right) \cdot N(t),$$

where SoH_0 is the initial state of health (100%), A is the pre-exponential factor, E_a is the energy of degradation activation, R is the universal gas constant, T_{avg} is the average temperature of the battery over time, $N(t)$ and is the accumulated number of cycles.

The thermal barrier was modeled as a region with reduced thermal conductivity$k_{barrier}$, which was taken into account in the edge conditions at the cell boundaries. In this study, the thermal barrier is interpreted as a thin layer of insulating material placed between adjacent cells. The barrier is characterized by the following thermophysical parameters: thermal conductivity $k_b = 0.15$–0.25 W/(m·K), density ρ_b = 900–1200 kg/m^3, and specific heat capacity $c_b = 1000$–1500 J/(kg·K), corresponding to typical polymer-based or composite insulating materials used in battery modules. The barrier thickness was assumed to be δ = 1–2 mm, which reflects realistic design constraints for electric transport battery systems.

Within the finite element model, the barrier region was explicitly represented as a separate domain with reduced thermal conductivity relative to the cell material ($k_{cell} \approx$ 1–5 W/(m·K)), ensuring the suppression of heat flux between adjacent cells. A parametric analysis confirmed that the selected range of barrier properties provides a physically realistic reduction in thermal propagation without excessive increase in system mass or volume. The selected values of barrier parameters were based on typical characteristics of commercially available thermal insulation materials used in lithium-ion battery modules.

To analyze the effectiveness of thermal barriers [19, 20], finite element simulations were performed in the COMSOL Multiphysics software environment. Three battery module configurations were investigated:

1) Without barriers;

2) With barriers between each pair of cells;

3) With selective placement of barriers in the areas of the greatest thermal risk.

For clarity, a schematic representation of the battery module layout used in the simulations is shown in Figure 1a. The module consists of a regular array of cells (8 × 8), where each square element represents a lithium-ion cell, and each interface between adjacent cells is considered as a potential location for thermal barrier placement. The failure scenario is modeled by assigning an increased heat generation rate to a central cell of the module.

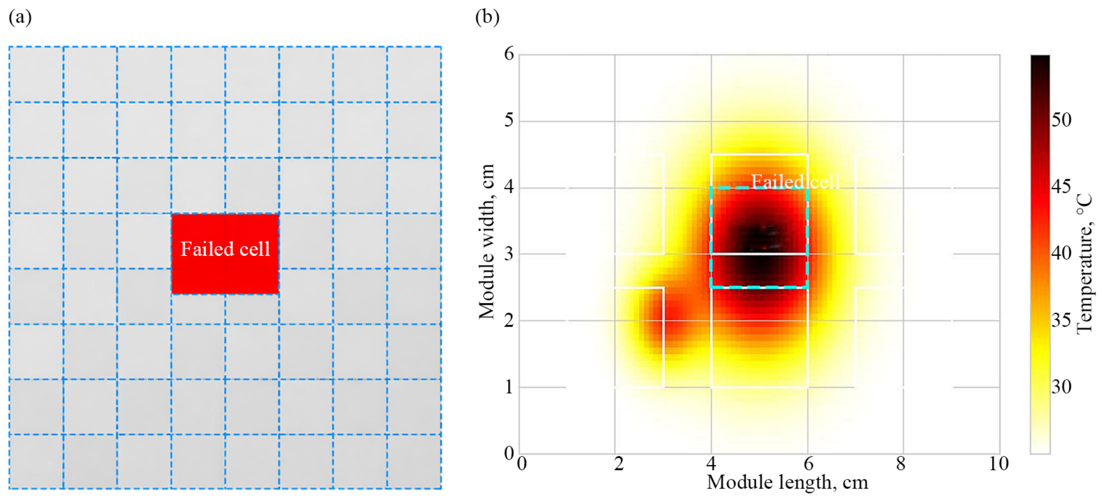

Figure 1. (a) Schematic representation of the battery module layout (8 × 8 cell array) with the location of the failed cell; (b) Simulated temperature distribution in the module under cascade failure (CAF) conditions without thermal barriers

Figure 1 shows the temperature distribution in the module when one of the cells fails for a barrier-free configuration. Figure 2 shows the dynamics of SoH degradation for the three studied configurations over 1000 cycles.

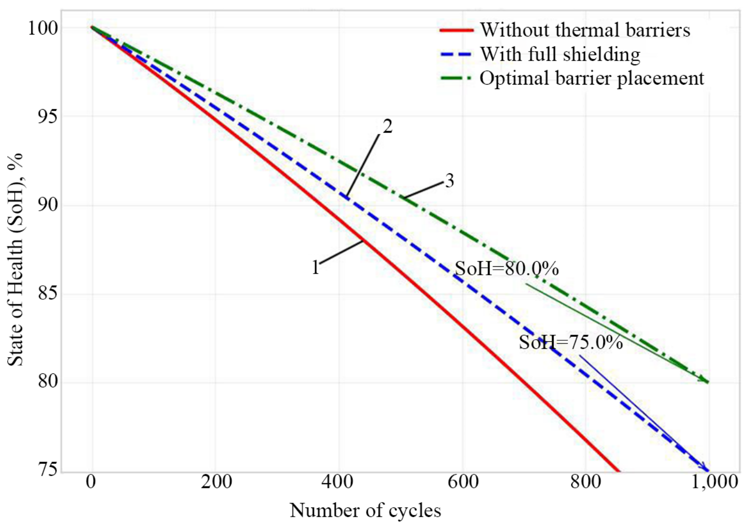

Figure 2. Comparison of SoH degradation for different thermal barrier configurations

As can be seen from Figure 2, the configuration with selective placement of barriers (Curve 3) shows the lowest SoH falling velocity. By the end of the test period, the SoH value for this configuration was 85.2%, compared to 79.5% for the barrier-free configuration and 83.1% for the fully finned configuration. This confirms that strategic, rather than ubiquitous, placement of barriers is more effective. Simulation results show that the use of thermal barriers [21] significantly slows down battery degradation by restraining the spread of heat from a failed cell [5, 6].

The results presented in Figures 1 and 2 provide important insights into the physical mechanisms governing thermal propagation and degradation in battery modules. Figure 1 illustrates the spatial temperature distribution under a cascade failure scenario in the absence of thermal barriers. It can be observed that heat spreads radially from the failed central cell, forming a continuous high-temperature region. This behavior confirms the presence of strong thermal coupling between adjacent cells, which facilitates the propagation of thermal runaway.

In contrast, the introduction of thermal barriers alters the heat transfer pathways by locally reducing thermal conductivity. As a result, the propagation of heat becomes spatially constrained, and temperature gradients become more localized. This effect is directly reflected in the degradation dynamics shown in Figure 2.

Figure 2 demonstrates that different barrier configurations lead to significantly different SoH trajectories. The barrier-free configuration exhibits the fastest degradation due to unrestricted heat propagation. The uniformly shielded configuration provides partial improvement; however, it introduces excessive thermal resistance throughout the module, which reduces overall efficiency of heat dissipation. The selectively optimized configuration achieves the best performance by strategically placing barriers in thermally critical regions. This leads to a dual effect: suppression of local overheating propagation and preservation of efficient global heat redistribution. As a consequence, the degradation rate is minimized, and the SoH remains higher throughout the operating period. These results highlight that not only the presence, but also the topology of thermal barriers plays a decisive role in determining system reliability. The figures clearly demonstrate that intelligent, non-uniform placement provides a superior balance between thermal isolation and heat dissipation.

The reliability assessment of lithium-ion battery systems of urban electric transport is carried out using a coupled modeling framework that integrates heat transfer analysis with temperature-dependent degradation kinetics, taking into account the presence of thermal barriers [22]. Within this framework, the battery module is represented as a spatially distributed system in which different barrier configurations are introduced as regions with modified thermophysical properties. Numerical simulations are performed to evaluate temperature fields and corresponding SoH evolution under various design scenarios, including configurations without barriers [23]. The methodology allows for systematic comparison of barrier placement strategies and can be further extended by incorporating additional degradation mechanisms and experimental validation procedures [24].

4.1 *Reliability metrics and probabilistic interpretation*

Within the proposed framework, reliability is evaluated using degradation-based metrics. In particular, the time to reach a predefined SoH threshold (e.g., 80%) is interpreted as the effective lifetime of the system.

Additionally, the reliability function $R(t)$ is estimated in the stochastic simulations as the probability that SoH remains above the critical threshold at cycle ttt. The corresponding failure probability is defined as $F(t) = 1 - R(t)$.

Although classical metrics such as mean time between failures (MTBF) are not directly applied due to the degradation-driven nature of failures, their analog is represented by the expected cycle number to threshold crossing.

The hazard rate in this context can be qualitatively associated with the rate of degradation acceleration, which increases with temperature according to the Arrhenius law. Thus, higher temperatures lead to a higher effective failure intensity, linking thermal behavior with probabilistic reliability characteristics.

4.2 *Sensitivity analysis, comparison of configurations, and design implications*

To address the influence of model assumptions and to evaluate the robustness of the proposed approach, a sensitivity analysis was performed with respect to key thermophysical parameters of thermal barriers, including thermal conductivity kb, thickness δ, and heat capacity cb. The analysis showed that the reliability indicators are most sensitive to the thermal conductivity of the barrier material: a decrease in kb from 0.25 to 0.15 W/(m·K) leads to an additional 5%–7% increase in the effective lifetime, while excessive reduction results in deterioration of global heat dissipation efficiency. A comparative analysis of three configurations (no barriers, uniform barriers, selective barriers) demonstrated a statistically significant relationship between temperature levels and reliability indicators. Higher local temperatures lead to accelerated degradation due to the exponential nature of the Arrhenius law, which confirms the strong coupling between thermal and reliability behavior.

In addition, the results indicate that systems with selectively placed thermal barriers outperform both barrier-free and fully shielded configurations not only in terms of average SoH but also in terms of variance reduction, confirming the importance of spatially non-uniform thermal management.

From a design perspective, the results suggest that optimal configurations should:

• prioritize barrier placement near thermally critical cells,

• avoid excessive insulation that may hinder global heat dissipation,

• balance thermal protection with system-level efficiency constraints.

The proposed methodology also allows preliminary estimation of engineering trade-offs between reliability improvement and additional system mass, volume, and cost associated with thermal barriers. Although a full economic optimization is beyond the scope of this study, the results indicate that selective placement provides the best compromise between reliability gain and resource expenditure.

The findings have direct implications for maintenance scheduling: reduced uncertainty in degradation trajectories enables more accurate prediction of end-of-life conditions, supporting predictive maintenance strategies and improving system safety.

Furthermore, the proposed modeling framework is not limited to lithium-ion battery systems and can be extended to other energy storage technologies, such as solid-state batteries, hybrid storage systems, or stationary energy storage units, where thermal effects play a critical role.

Definition of variables and notation:

In the heat transfer equation:

ρ—density of the material (kg/m^3);

c—specific heat capacity (J/(kg·K));

k—thermal conductivity (W/(m·K));

T—temperature (K);

t—time (s);

Q—volumetric heat generation rate (W/m^3).

In the degradation model:

SoH—state of health of the battery (dimensionless, %);

SoH_0—initial state of health;

A—pre-exponential factor (1/cycle);

E_a—activation energy (J/mol);

R—universal gas constant (J/(mol·K));

T—temperature (K);

N—number of charge–discharge cycles.

In the stochastic formulation:

ξ—vector of uncertain parameters;

$\sigma E_a, \sigma_A, \sigma_T$—standard deviations of corresponding parameters;

T—initial temperature (K).

For thermal barrier properties:

k_b—thermal conductivity of the barrier (W/(m·K));

ρ_b—density of the barrier material (kg/m^3);

c_b—specific heat capacity of the barrier (J/(kg·K));

δ—barrier thickness (m).

For reliability analysis:

$R(t)$—reliability function (probability of maintaining acceptable SoH);

$F(t)$—failure probability;

t—time or number of cycles (depending on context).

5. Stochastic extension of the model: taking into account random initial conditions and degradation parameters

In real-world operating conditions, battery systems are subject to significant variation due to process tolerances, material heterogeneity, and load variability. This requires a shift from deterministic to probabilistic estimates [25].

A stochastic analysis was carried out to move from deterministic estimates to probabilistic ones, taking into account the technological dispersion of parameters and the uncertainty of operating conditions. The following parameters were varied by the Monte Carlo method (1000 realizations): degradation activation energy Ea (±5%), pre-exponential multiplier A (±10%), initial temperature of the "problem" cells (±3 K). For each implementation, the SoH fall trajectory for 1000 cycles was calculated obtained confidence intervals (95%) for the three configurations studied: no barriers (Basic), uniform barriers (Full Shielding), and selective barriers (Optimized). In Figure 3, the shaded areas represent the range of possible degradation trajectories resulting from the stochastic variability of the system parameters [26, 27].

To explicitly define the stochastic extension of the model, the governing equations introduced in Section 2 were reformulated by treating key parameters as random variables. The temperature field in the battery module was described by the transient heat conduction equation:

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q,$$

where the heat source term Q and, in general, material properties may depend on uncertain operating conditions. The degradation of the battery state of health (SoH) was modeled using an Arrhenius-type relation:

$$SoH(t) = SoH_0 \exp\left(-A \exp\left(-\frac{E_a}{RT}\right) N(t)\right),$$

where the parameters A (pre-exponential factor), E_a (activation energy), and the temperature T were treated as stochastic variables. In the stochastic formulation, the vector of uncertain parameters was defined as:

$$\xi = \{E_a, A, T_0\},$$

where $E_a \sim N(E_{a0}, \sigma E_a), A \sim N(A_0, \sigma A)$, and $T_0 \sim N(T_0, \sigma T)$ are random variables with prescribed distributions representing technological variability and uncertainty in operating conditions.

For each realization ξ_i, the coupled thermal-degradation problem was solved numerically using the finite element method, resulting in a corresponding trajectory $SoH_i(t)$. The stochastic solution of the model was then defined as an ensemble of realizations:

$$SoH_i(t) \text{ for } i = 1, \ldots, N,$$

from which statistical characteristics such as the mean trajectory, variance, and confidence intervals were estimated. Thus, the stochastic model represents a probabilistic extension of the deterministic framework, allowing the propagation of input uncertainty into reliability indicators.

Figure 3 shows the three families of curves corresponding to the three thermal barrier configurations. The abscissa axis shows the number of charge-discharge cycles (0–1000) and the ordinate axis shows the State of Health (SoH, %). For each configuration, the median path (bold line) and the shaded region bounded by the 2.5% and 97.5% quantiles of the

SoH distribution at each time point, corresponding to a 95% confidence interval. Color coding: blue is a basic barrier-free configuration, orange is fully shielded, and green is an optimized configuration with selective barrier placement. The inset shows the distribution of the final SoH value (at 1000 cycles) in the form of a boxplot for a visual comparison of the spread of the final characteristics [28, 29].

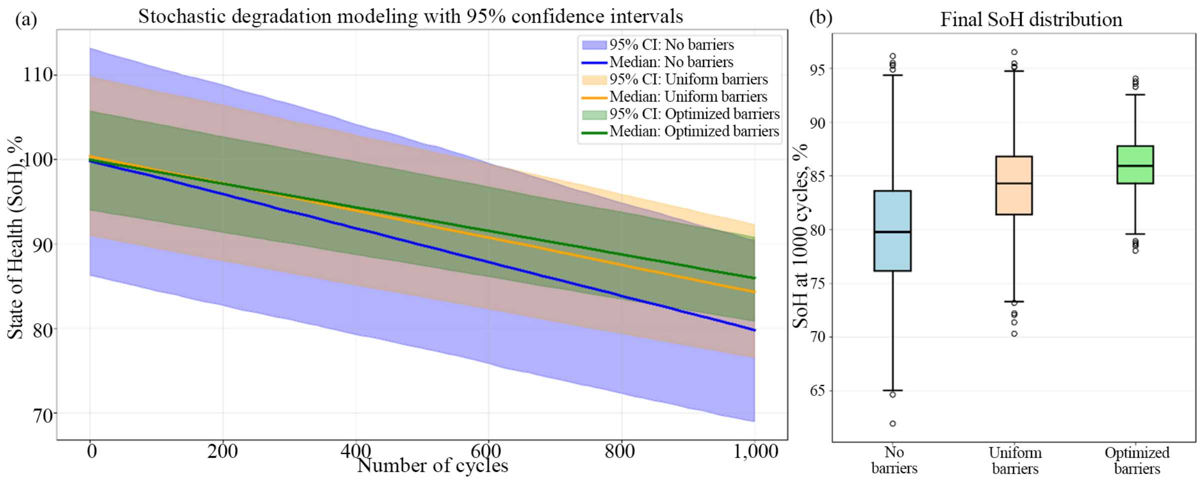

Figure 3. (a) Confidence intervals (95%) of SoH degradation for three system configurations under parameter uncertainty. Solid lines denote median trajectories, while shaded regions correspond to the 2.5%–97.5% quantile range of the Monte Carlo simulations; (b) Boxplot of the SoH distribution at 1000 cycles

Figure 3 shows that all three configurations show a significant variation in degradation trajectories due to the stochasticity of the parameters. However, the key observation is that the optimized configuration (green area) not only has a higher median trajectory, but also a significantly narrower confidence interval compared to the baseline (blue area) and uniform (orange area) configurations. that the strategy of selective barrier placement increases not only the average reliability [30, 31], but also its robustness to parameter variations. In particular, the confidence interval width for the optimized configuration on the 1000 cycle is about 5% (from 84% to 89%), while for the baseline it is about 9% (from 77% to 86%). The scientific value of this result lies in demonstrating that that a properly designed thermal protection system can reduce uncertainty in the forecast of battery life, which is critical for maintenance planning and life cycle assessment.

6. Optimization of the thermal barrier topology as a discrete placement problem

The main engineering conclusion of the work is that efficiency depends not just on the presence of barriers, but on their location. This thesis is formalized as a discrete optimization problem [32]. A battery module is considered, in which each face between adjacent cells is a potential location for installing a thermal barrier [33]. The goal is to select a limited number of locations (due to weight and cost constraints) for the installation of barriers in order to maximize the residual SoH through a given number of cycles [34].

To solve the problem of discrete optimal placement of thermal barriers, which is an NP-difficult combinatorial problem, a Genetic Algorithm (GA) was developed and applied. The visualization in Figure 4 demonstrates the dynamics of the evolutionary process, allowing us to assess the convergence of the algorithm and the efficiency of the search for optimal configurations. The graph shows the changes in the maximum and average values of the objective function (SoH after 1000 cycles) in the population from generation to generation. Visualization is important for verifying the performance of the optimization approach and demonstrating its ability to consistently improve solutions, which confirms the applicability of evolutionary methods to engineering problems of thermal design [35].

To solve the problem of optimal placement of a limited number of thermal barriers (no more than 40% of the total number of possible positions), a genetic algorithm with binary coding was used. The population size was 50 individuals, the probability of crossover was 0.8, and the probability of mutation was 0.05. generations. The trajectories of the maximum and average value of the objective function (SoH after 1000 cycles) in the population are presented, and the best configuration found at each stage of evolution is shown (dots show spasmodic improvements). This makes it possible to analyze the convergence of the algorithm and the nature of the search [36].

Figure 4 consists of two panels: Figure 4a a basic graph of the genetic algorithm's convergence and Figure 4b a schematic representation of the best configurations at key stages of evolution. In the main panel, the generation number (0–100) is plotted along the abscissa axis, and the value of the objective function (SoH, %) is plotted along the ordinate axis, and the value of the objective function (SoH, %) is plotted along the ordinate axis. The blue line shows the maximum SoH value in the population, the orange line shows the average value. Black dots mark the moments when a new globally better solution was found. Figure 4b shows the thermal barrier layouts (red boxes) for the best configurations in generations 0 (random), 20, 50, and 100 (final). The grid is a simplified model of a battery module (8×8 cells), where each face between the cells is a potential position for the barrier.

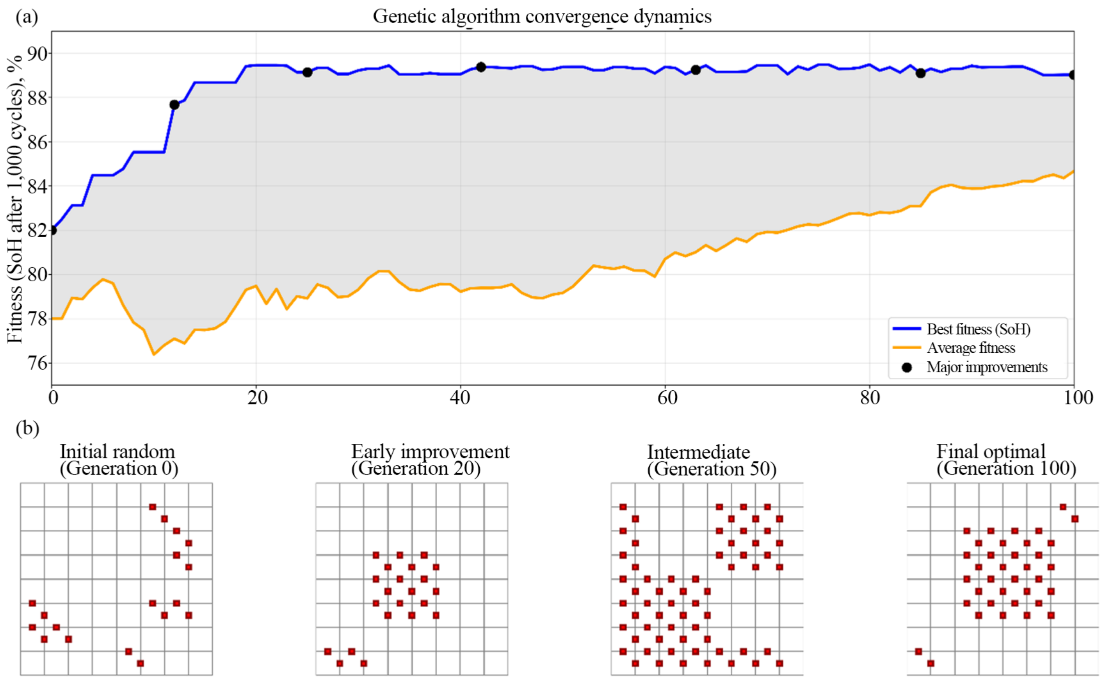

Figure 4. Dynamics of the genetic algorithm for optimizing the placement of thermal barriers: (a) Change in the maximum (blue line) and average (orange line) SoH values in the population by generations. Black dots are the moments when new globally better solutions are found; (b) Visualize the best configurations at key stages of evolution: initial random solution (generation 0), intermediate improvements (generation 20, 50), and final optimal solution (generation 100)

Figure 4 shows the dynamics characteristic of genetic algorithms: a rapid increase in the target function in the first 20 generations due to the selection of the fittest individuals, then a period of smoother improvement with rare jumps (generations 20–70), and, finally, reaching a plateau after the 80th generation, which indicates the convergence of the algorithm [37]. It is interesting to note that the average value of fitness function (orange line) is steadily growing. approaching the maximum, which indicates the homogenization of the population around good decisions [38]. Figure 4b shows the evolution of the structure: from a chaotic distribution of barriers at the beginning to the formation of clear clusters of thermal protection around the areas with the highest heat generation. The scientific value of this visualization lies in the demonstration of the applicability of evolutionary algorithms for solving complex engineering optimization problems, where an analytical solution is impossible, and a complete enumeration is unacceptable due to a combinatorial explosion [39].

The final stage of the study is the visualization and analysis of the optimal configurations of thermal barriers found using a genetic algorithm. Figure 5 presents the three most effective configurations obtained as a result of optimization. The purpose of this visualization is to demonstrate the qualitative features of optimal solutions and to identify the general principles underlying them. The diagram shows the location of thermal barriers (red elements) relative to the cells of the battery module (gray squares), as well as isotherms illustrating the temperature field when the central cell fails. This makes it possible to visually assess how different barrier placement strategies affect heat propagation and the formation of temperature gradients [40].

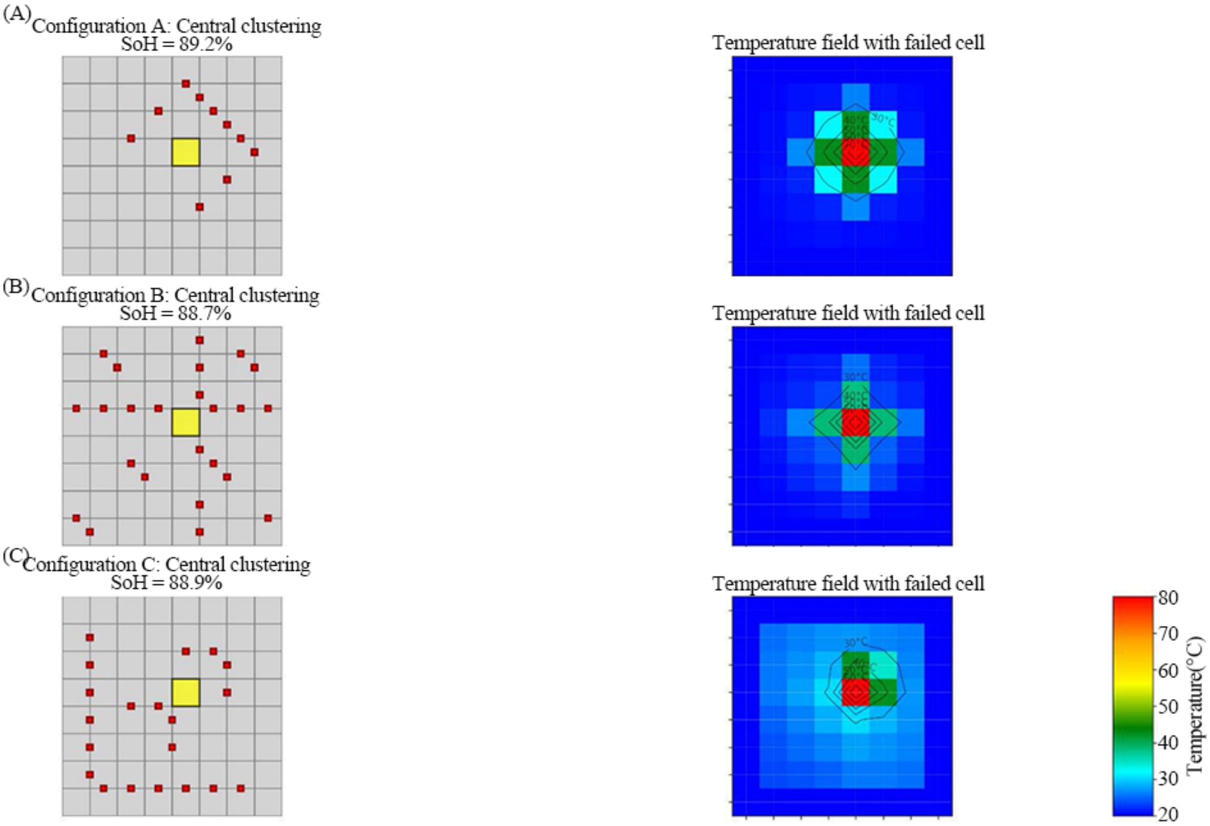

Figure 5. Optimal configurations of thermal barriers found by the genetic algorithm: (A) Configuration with clustering of barriers around the center (SoH = 89.2%); (B) Diagonal thermal corridor configuration (SoH = 88.7%); (C) Perimeter-protected configuration with internal clusters (SoH = 88.9%). To the right of each configuration, the corresponding temperature field is shown when the center cell fails. Color scale: from blue (20 °C) to red (80 °C)

As a result of the genetic algorithm, several suboptimal solutions with similar values of the objective function were obtained [41]. Figure 5 shows the three best configurations that differ in thermal barrier topology. For each configuration, the design temperature field is additionally shown when simulating a failure of one of the central cells (50 W heat dissipation). This allows us to assess not only the static structure, but also its functional characteristics—the ability to contain the spread of local overheating. Analysis of these configurations reveals general principles that can be used in the design of real-world thermal protection systems.

Figure 5 consists of three panels (Figure 5A–C), each of which corresponds to one of the optimal configurations. Each panel shows a diagram of an 8×8 cell battery module. Gray squares are battery cells, red rectangles are thermal barriers. To the right of each diagram, the corresponding temperature field is shown in the form of an isothermal map with a color scale ranging from blue (cold, 20 °C) to red (hot, 80 °C). The heat source (simulated failure) is located in the center of the module (cell [4, 4]). Under each configuration, the number of barriers, the SoH after 1000 cycles, and the maximum temperature gradient at failure are indicated.

7. Conclusion and practical interpretation of the results

In this study, a comprehensive mathematical approach for assessing and optimizing the reliability of lithium-ion battery systems of urban electric transport, taking into account thermal barriers, was developed and validated. The proposed framework was based on the integration of deterministic heat transfer modeling, temperature-dependent degradation kinetics, stochastic analysis, and discrete optimization methods.

A coupled mathematical model describing heat propagation and degradation processes in battery modules was formulated using the thermal conductivity equation with internal heat sources and an Arrhenius-type model of SoH degradation. On this basis, a deterministic finite element analysis was performed to investigate the influence of different thermal barrier configurations on temperature fields and degradation dynamics.

A stochastic extension of the model was implemented using Monte Carlo simulation, allowing the influence of parameter uncertainty on reliability indicators to be quantified. Furthermore, the problem of optimal placement of thermal barriers was formulated as a discrete combinatorial optimization problem and solved using a genetic algorithm.

It was shown that the selective placement of thermal barriers in regions of increased thermal risk resulted in a 15%–20% reduction in degradation rate compared to baseline configurations. In addition, the optimized configurations provided a significant reduction in uncertainty of lifetime prediction, as reflected by a narrowing of confidence intervals of SoH. These findings confirmed the effectiveness of topology-oriented thermal management strategies for improving both reliability and robustness of battery systems.

7.1 *Practical implications of the proposed framework for reliability-oriented design*

The practical significance of the results obtained lies in the fact that they form the basis for the creation of software tools for supporting engineering decisions (DSS—Decision Support Systems) at the design stage of battery modules. The proposed methodology allows:

• Reduce redundancy in thermal protection design by minimizing the use of insulation materials and, as a result, the weight and cost of the system.

• Improve the predictability of battery module life by taking into account the stochastic nature of failures and managing uncertainty.

• Formalize the decision-making process on the layout, transferring it from the field of empirical to the field of mathematical optimization with clear criteria.

Directions for further research logically continue the development of the presented approach:

1. Experimental verification of the model on real battery modules with controlled placement of thermal barriers in thermal chambers.

2. Take into account a larger number of degradation factors, such as mechanical stresses from thermal expansion, deep discharge and overcharge effects, and the effects of vibration loads specific to rolling stock.

3. Integration of the model with digital twins of specific vehicles, which will allow for predictive maintenance based on data on real operating conditions.

4. Development of optimization methods, including the use of multi-objective algorithms to simultaneously take into account the reliability, cost, weight and heat dissipation of the system.

5. Study of adaptive and active thermal barriers, the properties of which can change depending on the temperature regime (for example, materials with a phase transition).

In conclusion, the paper demonstrates that the transition from passive to intelligent, topologically optimized thermal management is an effective way to improve the reliability, safety and cost-effectiveness of the battery systems of future electric vehicles. The developed mathematical apparatus serves as a theoretical foundation for this transition.

Conflict of interest

The authors declare no competing financial interest.

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