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Consequences of Holographic Dark Energy Cosmological Model in Gravity

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Abstract: We have investigated the dynamics of spatially homogeneous Bianchi type-I (LRS) space-time filled with two minimally interacting fields-matter and holographic dark energy components with volumetric power laws expansion towards the gravitational field equations of the linear form of gravity. Solving the set of field equation, we obtained the exact solution and observed that the mode of expansion of the model is accelerating throughout the evolution due to destructive assessment of deceleration parameter. Also, it has found that the Gauss-Bonnet invariant and the function of Gauss-Bonnet invariant both are not occur for, the equation of state parameter admits the different values for different values of n which are relevant in the standard range provided by recent theoretical and experimental data along with the model has a Big-Bang type of singularity at singular point.
Keywords: Bianchi typespace-time, holographic dark energy, gravity

1. Introduction

An awesome abundance of observational evidence (SNe-Ia Supernova, CMBR, LSS and WMAP) favor the universe is spatially flat and late-time accelerating expansion which is not fit within the framework of Einstein's General Theory of Relativity. The proposals that have been put forward to explain this observed phenomenon can basically be classified into two categories. An exotic component with negative pressure called mysterious energy or Dark Energy (DE) is introduced into Einstein's General Theory of Relativity (GTR). The dynamical dark energy models are classified into two different categories: (a) the scalar fields, including Quintessence, Phantom, Quintom, K-essence, Tachyon, Dilaton, and so forth. (b) the interacting models of dark energy such as the Chaplygin gas models, Braneworld models, Holographic and Agegraphic models.

For a good review of the dynamics of different dark energy models, Mishra and Biswal [1] have constructed a self-consistent system of Bianchi Type-VI_0 cosmology in five dimensions with a binary mixture of perfect fluid and dark energy where the dark energy is chosen to be either the quintessence or Chaplygin gas using solutions to the corresponding Einstein's field equations as a quadrature and observed that the equation of state parameter for dark energy is found to be consistent with the recent observations (SNe-Ia Supernova with CMBR) and Galaxy Clustering Statistics. Chirde and Shekh [2-4] investigated interacting two-fluid viscous dark energy and magnetized dark energy cosmological models in self creation cosmology and Lyra geometry respectively. The same authors [5, 6] studied plane symmetric dark energy cosmological model in the form of wet, dark fluid in $f(R,T)$ gravity. Dark energy dominated Bianchi type-VI_0 cosmological model with hybrid law of expansion in $f(R,T)$ gravity investigated by Bhoyar *et al.* [7]. Recently, Mishra *et al.* [8] have investigated the anisotropic behavior of the accelerating universe in Bianchi type-V space time in the framework of General Theory of Relativity considering the matter field is of two non-interacting fluids i.e. the usual string fluid and dark energy fluid and the skewness parameters are introduced along three different spatial directions in order to represent the pressure anisotropy. In recent years, an interesting observation is made to determine the nature of dark energy in quantum gravity which is termed as Holographic dark energy [9, 10].Recently, Singh and Kumar [11] study non-viscous and viscous Holographic dark energy models for a homogeneous and isotropic flat Friedmann-Robertson-Walker Universe and find

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that the Hubble horizon as an IR cut-off is suitable for both models to explain the recent accelerated expansion of the Universe. Very recently, Bhoyar *et al.* [12] obtained a spatially homogeneous and anisotropic Bianchi type-I Space-time in $f(G)$ gravity theory using the interaction and non-interaction between Holographic dark energy and dark matter. Chirde and Shekh [13] inspected the dynamics of spatially homogeneous Bianchi type-I space-time filled with two minimally interacting fields matter, Holographic dark energy components with volumetric power, exponential expansion laws towards the gravitational field equations for the linear form of $f(T)$ gravity, They also observed that both models at late times turned out to be flat Universe whereas the power-law model has an initially singular and stable but with expansion, it is unstable while the exponential model is free from any type of singularity and stable throughout the expansion, and predicts that the anisotropy of the Universe will damp out and the Universe will become isotropic.

Another category to obtain an accelerating expansion of the Universe is the change in the gravity law through the modification of action in General Theory of Relativity. Various modifications in the action are present, out of which one replaces the Ricci scalar R in the Einstein-Hilbert action by an arbitrary function of R belongs to the well-known $f(R)$ modified gravity. Vacuum solution in cylindrically symmetric space-time in the same gravity studied by Azadi *et al.* [14]. Bianchi type-cosmological models with bulk viscosity in $f(R)$ theory investigated by Katore and Shaikh [15] along with many authors have discussed some features of same gravity in [16-19]. Another generalization is the gravitational action includes an arbitrary function of the Ricci scalar and trace of the stress-energy tensor known as $f(R,T)$ gravity. Several authors have investigated the aspect of cosmological models in this gravity [20-22]. Among the various modifications of Einstein's theory, another one way to look at the theory beyond GR is the Teleparallel Gravity (TG) which uses the Weitzenbock connection in place of the Levi-Civita connection and so it has no curvature but has torsion which is responsible for the acceleration of the Universe. Some relevant works in this gravity are presented in [23-26].

Modified Gauss–Bonnet gravity is another theory that has gained popularity in the last few years [27-28]. It is also known as the $f(G)$ theory of gravity, where $f(G)$ is a generic function of the Gauss-Bonnet invariant G.

Modified Gauss–Bonnet gravity is described by the action

$$S=\frac{1}{2\pi}\int d^4x\sqrt{-g}\left[R+f(G)\right]+S_M(g^{\mu\nu},\psi), \tag{1}$$

where x is the coupling constant, g is the determinant of the metric tensor g_{uv} and $S_M(g^{uv},\psi)$ is the matter action, in which matter is minimally coupled to the metric tensor and ψ denotes the matter fields. This coupling of matter to the metric tensor suggests that $f(G)$ gravity is a purely metric theory of gravity. The $f(G)$ is an arbitrary function of the Gauss–Bonnet invariant G.

$$G\equiv R^2-4R_{\mu\nu}R^{\mu\nu}+R_{\mu\nu\sigma\alpha}R^{\mu\nu\sigma\alpha}, \tag{2}$$

where R is the Ricci scalar and R_{uv} and $R_{uv\sigma a}$ denote the Ricci and Riemann tensors. Gravitational field equations are obtained by varying the action in equation (1) with respect to the metric tensor:

$$\begin{aligned}&R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}+8\left[R_{\mu\alpha\nu\sigma}+R_{\alpha\nu}g_{\sigma\mu}-R_{\mu\nu}g_{\sigma\alpha}-R_{\alpha\sigma}g_{\nu\mu}+R_{\mu\sigma}g_{\nu\alpha}+\frac{1}{2}(R_{\mu\nu}g_{\sigma\alpha}-R_{\sigma\alpha}g_{\nu\alpha})\right]\\&\times\nabla^{\alpha}\nabla^{\alpha}F+(Gf_G-f)g_{\mu\nu}=\kappa T_{\mu\nu}\end{aligned} \tag{3}$$

where ∇_u denotes the covariant derivative and $f(G)$ represents the derivative of f with respect to G.

The Gauss–Bonnet term plays an important role because it may allow avoiding ghost contributions and is helpful in regularizing the gravitational action [29]. It has been suggested that this theory may describe the late-time cosmic acceleration. Moreover, the theory also passes the solar system tests for some specific choices of $f(G)$ gravity models. Some interesting work has been done so far in this theory. Nojiri and Odintsov [30] developed the reconstruction techniques for $f(G)$ gravity and it was demonstrated that how cosmological sequence of matter dominance, deceleration-acceleration transition, and acceleration era could emerge by using a modified theory. Garcia*et al.* [31] explored energy conditions to find the viability of some specific choices of $f(G)$ gravity models. Fayaz *et al.* [32] investigated power-law solutions with an anisotropic background in $f(G)$ gravity and it was concluded that Bianchi type-I power law solutions only existed for some special choices of $f(G)$ gravity models. Abbas *et al.* [33] gave the possibility for the existence of anisotropic compact stars in $f(G)$ gravity. Sharif and Fatima [34] argued the role of Gauss–Bonnet term in the late time accelerated phases of the universe.

Incited by the above discussion, we focus our attention to investigate $f(G)$ gravity in the anisotropic background with Holographic dark energy. For this purpose, we consider Locally Rotationally Symmetric (LRS) Bianchi type-I space-time. We explore the exact solutions of the LRS Bianchi type-I field equations in modified $f(G)$ gravity. A well-known $f(G)$ gravity model has been used to solve the set of differential equations.

2. Holographic dark energy model in Bianchi type-I space-time

The line element for a spatially homogeneous, anisotropic LRS Bianchi type-I space-time is given by

$$ds^2 = dt^2 - A^2dx^2 - B^2(dy^2 + dz^2), \tag{4}$$

where A and B are the directional scale factors which is a function of cosmic time t.

The corresponding Ricci scalar and Gauss–Bonnet invariant for the space-time (4) are turned out to be

$$R = -2\left[\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}^2}{A^2}\right], \tag{5}$$

$$G = 8\left[\frac{\ddot{A}}{A}\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B}\frac{\ddot{B}}{B}\right], \tag{6}$$

where the dot denotes the differentiation with respect to cosmic time.

Let us consider that the matter content is dark matter and Holographic dark energy such that the energy-momentum tensor T^{v}_{u} are respectively given by

$$T^{\nu}_{\mu} = \rho_m u^{\nu} u_{\mu}, \tag{7}$$

$$\overline{T}^{\nu}_{\mu} = (\rho_h + p_h)u^{\nu} u_{\mu} - p_h g^{\nu}_{\mu}, \tag{8}$$

together with commoving coordinates

$$u^{\nu} = (0,0,0,1) \text{and } u^{\nu} u_{\nu} = 1, \tag{9}$$

where u^{v} is the four-velocity vector of the fluid, p and ρ are the pressure and energy density of the fluid respectively.

From the equation of motion (3), Bianchi type-I space-time (4) for the fluid of stress-energy tensor (7) and (8) can be written as

$$-\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} - 16\frac{\dot{B}}{B}\frac{\ddot{B}}{B}\dot{f}_G + 8\frac{\dot{B}^2}{B^2}\ddot{f}_G - Gf_G + f = k(p_{\wedge}), \tag{10}$$

$$-\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}}{A}\frac{\dot{B}}{B} + 8\left(\frac{\dot{A}}{A}\frac{\ddot{B}}{B} + \frac{\dot{B}}{B}\frac{\ddot{A}}{A}\right)\dot{f}_G + 8\frac{\dot{A}}{A}\frac{\dot{B}}{B}\ddot{f}_G - Gf_G + f = k(p_{\wedge}). \tag{11}$$

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} - 24\frac{\dot{A}}{A}\frac{\dot{B}^2}{B^2}\dot{f}_G + Gf_G - f = k(\rho_m + \rho_{\wedge}), \tag{12}$$

where the dot (.) denotes the derivative with respect to time t.

3. Exact Matter Dominated Solution of the Field Equations

Finally, here we have three differential equations (10) – (12) with six unknowns namely A, B, f, $p_\wedge$, $\rho_\wedge$, ρ_m. Now to solve the system of equations completely, we assume shear scalar is proportional to the expansion scalar (which gives a linear relationship between the directional Hubble's parameters) this assumption gives an anisotropic relation between the scale factors A and B are as follows:

$$A = B^n, \tag{13}$$

where $n > 1$ is an arbitrary constant. If $n = 1$, the matter distribution in the Universe is all over the same, hence the model becomes isotropic otherwise, it turns out to be anisotropic.

Following the work of Granda and Oliveros [35] and Sarkar [36], the Holographic dark energy density is given by

$$\rho_\wedge = 3(\alpha H^2 + \beta \dot{H}) \text{with } M_p^{-2} = 8\pi G = 1, \tag{14}$$

The equation of state for Holographic dark energy is

$$p_\wedge = \omega_\wedge \rho_\wedge \tag{15}$$

Before finding the solution of these field equations, consider some kinematical quantities of the space-time such as Average scale factor and Spatial volume respectively as

$$a = V^{1/3}, V = AB^2. \tag{16}$$

Another dimensionless kinematical quantity is the mean deceleration parameter which tells whether the Universe exhibits accelerating volumetric expansion or not is

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right), \tag{17}$$

for $-1 \leq q < 0$, $q > 0$, and $q = 0$, the Universe exhibit accelerating volumetric expansion, decelerating volumetric expansion and expansion with constant rate respectively.

The mean Hubble parameter, which expresses the volumetric expansion rate of the Universe, given as

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \tag{18}$$

where H_1, H_2 and H_2 are the directional Hubble's parameter in the direction of x, y and z-axis respectively.

Using equations (16) and (18), we obtain

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{\dot{a}}{a}. \tag{19}$$

To discuss whether the Universe either approach isotropy or not, we define an anisotropy parameter as

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{20}$$

The expansion scalar and shear scalar are defined as follows

$$\theta = u;\ {}^{\mu}_{\mu} = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}, \tag{21}$$

$$\sigma^2 = \frac{3}{2} H^2 A_m. \tag{22}$$

Subtracting equation (11) from equation (10), we obtained

$$\frac{\ddot{A}}{A} - \frac{\dot{B}^2}{B^2} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} = 0, \tag{23}$$

Using equation (13) and (23), we get the directional scale factor as

$$A = (ct-d)^{\frac{1}{1+n-n^2}}, \tag{24}$$

$$B = (ct-d)^{\frac{n}{1+n-n^2}}, \tag{25}$$

where $c = c'\,(1 + n - n^2)$ and $d = d'\,(1 + n - n^2)$.
Hence the model (4) becomes

$$ds^2 = dt^2 - (ct-d)^{\frac{2}{1+n-n^2}} dx^2 - (ct-d)^{\frac{2n}{1+n-n^2}} (dy^2 + dz^2). \tag{26}$$

From the above model (26), it is observed that the metric potentials are different, hence it represents an anisotropic model, but for the constant n if these are identical which represents an isotropic model. Also, the model is constant but at a specific time, $t = t_s = \frac{d}{c}$ the matric potential in the model vanishes hence the model represents the singular. Also, there is no such relation between the constants in the model for which the model shows isotropy.

Some kinematical parameters:

The kinematical parameters such as the Hubble parameter, the anisotropic parameter, the shear scalar, the expansion scalar, and the spatial volume of the model (26), which are of cosmological importance, are respectively given by

$$H = \frac{c(1+2n)}{3(1+n-n^2)} \frac{1}{(ct-d)}. \tag{27}$$

$$A_m = \frac{2n(1-n)}{(1+2n)^2}. \tag{28}$$

$$\sigma^2 = \frac{nc^2(n-1)}{(n^2-n-1)(ct-d)^2}, \tag{29}$$

$$\theta = \frac{(1+2n)c}{(1+n-n^2)} \frac{1}{ct-d}. \tag{30}$$

$$V = (ct-d)^{\frac{1+2n}{1+n-n^2}}. \tag{31}$$

We observe that the spatial volume is constant at t→ 0. Therefore, the model starts evolving with constant volume at t = 0 and expands with cosmic time along with other parameters such as expansion scalar, shear scalar, and Hubble's parameter are constants but at a singular point t_s the spatial volume vanishes and other parameters are diverged. Hence the model (26) has a Big-Bang type of singularity at singular point t_s.

Deceleration parameter is

$$q=-\frac{(3n^2-n-3)}{(1+2n)}. \tag{32}$$

From equation (32), it is observed that the deceleration parameter is constant with a negative sign and not associated with expansion. Hence, the mode of expansion of the model is accelerating and throughout the evolution, the deceleration parameter is constant. For $-1\leq q<0$, $q>0$, and $q=0$, the Universe exhibit accelerating volumetric expansion, decelerating volumetric expansion, and expansion with constant rate respectively. For, the model is isotropic and accelerating, and for, the model is anisotropic which is also an accelerating.

Gauss–Bonnet invariant is defined as

$$f(G)=G=\frac{8nc^4(n^3+n^2-2)}{(1+n-n^2)^4}\frac{1}{(ct-d)^4}. \tag{33}$$

Equation (33) represents the Gauss–Bonnet and function of Gauss–Bonnet invariants which is a function of cosmic time and depends on n. Our derived model gives two types of expansion isotropic and anisotropic for n = 1 and n > 1 respectively. But it is seen that in an isotropic expansion the Gauss–Bonnet invariant and function both are zero hence it seems that in an isotropic expansion the Gauss–Bonnet invariant and function both do not exist.

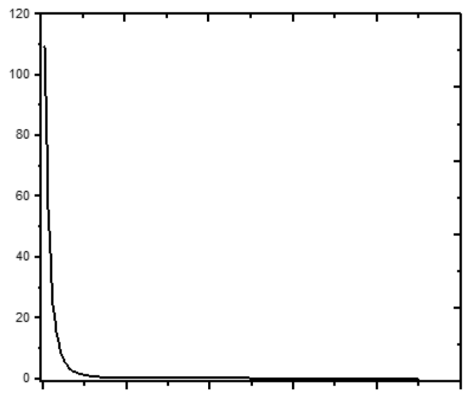
Pressure is defined as

$$p_\wedge=\frac{1}{k}\left\{\frac{c^2(1+n)}{(1+n-n^2)}\frac{1}{(ct-d)^2}-\frac{c^2(1+2n)}{(1+n-n^2)^2}\frac{1}{(ct-d)^2}\right\}. \tag{34}$$

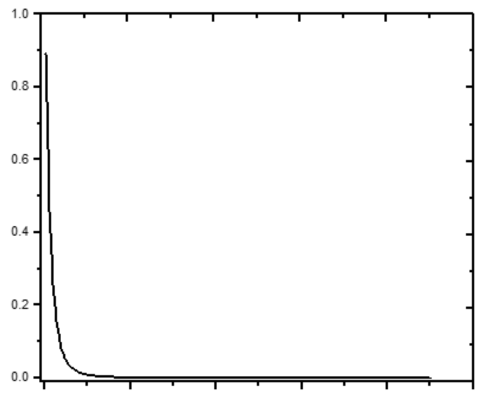
Energy density of Holographic dark energy is

$$p_\wedge=\left\{\frac{3c^2(2+n)\left[\alpha(n+2)-\beta(1+n-n^2)\right]}{(1+n-n^2)^2}\frac{1}{(ct-d)^2}\right\}. \tag{35}$$

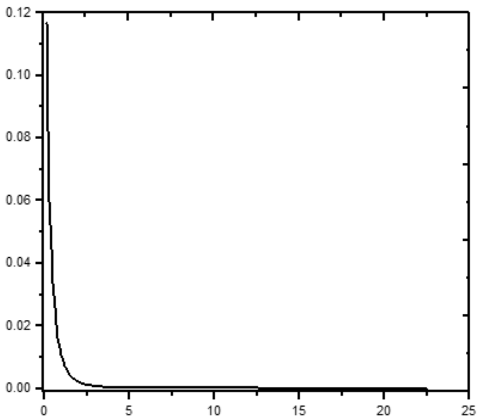
It is observed that the energy density is a function of time t, for, it is zero and for it always decreases positively with the expansion. At the initial stage t→ 0 the universe has infinitely large energy density $\rho\rightarrow\infty$ but with the expansion of the universe it declines and at large $t\rightarrow\infty$ it is null $\rho\rightarrow 0$. The behavior is clearly depicted in the following figures.



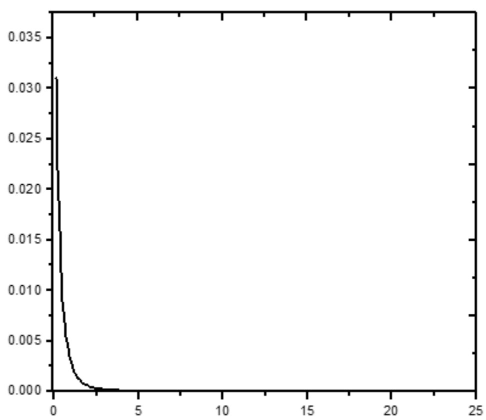
Energy density versus cosmic time for *n* = 2



Energy density versus cosmic time for *n* = 3



Energy density versus cosmic time for *n* = 4



Energy density versus cosmic time for *n* = 5

Energy density of matter is

$$\rho_m = \left\{ \frac{c^2(2+n)\left[n-3\alpha(n+2)+3\beta\right]}{(1+n-n^2)^2} \frac{1}{(ct-d)^2} \right\}. \tag{36}$$

Energy density of matter shows the same deeds as that of the energy density of Holographic dark energy.

Equation of state parameter for Holographic dark energy is

$$\omega_{\wedge} = \left\{ \frac{n^3}{(n+2)(n-3n\alpha-6\alpha+3\beta)} \right\}. \tag{37}$$

From equation (37), it is observed that the Equation of state parameter of Holographic dark energy is independent of cosmic time (constant) hence which is not deflected throughout the expansion of the model. The results from SNe-Ia data collaborated with CMBR anisotropy and galaxy clustering statistics yield $\omega_{\wedge}$ as $\omega_{\wedge}$ = -0.97 (WMAP, SNe-Ia results) at a 68% confidence level for dark energy. These results are consistent with the time-variable equation of state parameter and also for time free $\omega_{\wedge}$. The quintessence models, ($\omega_{\wedge} > -1$) (explanation of observations of accelerating universe) involving scalar field and phantom model, ($\omega_{\wedge} < -1$) (expansion of universe increases to infinite degree infinite time) give rise to the time-dependent parameter $\omega_{\wedge}$. Some other limits of the equation of state parameter are obtained from observational results that came from SNe-Ia data and a combination of SNe-Ia data with CMB anisotropy and Galaxy clustering statistics are 1.67 $< \omega_{\wedge} <$ 0.62 and -1.33 $< \omega_{\wedge} <$ -0.79 respectively. The latest result in 2009, obtained after a combination of cosmological data sets coming from CMB anisotropy, luminosity distances of high redshift SNe-Ia, and galaxy clustering constrain shows the range of the dark energy equation of state is -1.44 $< \omega_{\wedge} <$ -0.92. In the isotropic model, the equation of state parameter admits the value -0.1666 corresponding to n = 1, also it has found that for the value of constant n in the range [2, 5] the equation of state parameter admits a value -0.5, -0.9, -1.333, -1.777 which exists in the standard range given above.

$$\overline{r} = \left\{ \frac{3\left[\alpha(n+2)-\beta(1+n-n^2\right]}{(n-3n\alpha-6\alpha+3\beta)} \right\}. \tag{38}$$

It is observed that the coincidence parameter is independent of cosmic time, at very early to large stage of evolution, it is constant and remains constant throughout the evolution, by this means avoiding the coincidence problem (unlike ΛCDM).

The density parameter of dark matter and Holographic dark energy are as follows:

$$\Omega_m = \frac{(n+2)(n-3\alpha(n+2)+3\beta)}{(1+2n)^2}, \tag{39}$$

$$\Omega_{\wedge} = \frac{(n+2)(\alpha(n+2) - \beta(1+n+n^2))}{(1+2n)^2}. \tag{40}$$

The overall density parameter as

$$\Omega = \frac{(n+2)(n - 2n\alpha - 4\alpha + 2\beta - n\beta + n^2\beta)}{(1+2n)^2}. \tag{41}$$

From the above equation, one can observe that the sum of the energy density parameter approaches a constant value.

So, from initial to late time the Universe becomes flat. Therefore, for a sufficiently large time, this model predicts that the anisotropy of the Universe will dam pout and the Universe will become isotropic. This result also shows that in the early Universe, i.e. during the radiation and matter-dominated era the Universe was anisotropic and the Universe approaches isotropy as dark energy starts to dominate the energy density of the Universe.

4. Conclusions

In the investigation of the spatially homogeneous Bianchi type-I space-time with two minimally interacting fields matter and holographic dark energy components with volumetric power laws expansion towards the gravitational field equations for the linear form of $f(G)$ gravity, it is observed that the metric potentials are altered, hence it represents the anisotropic model but for the constant n if these are identical and represents an isotropic model. Also, both are constant but at a specific time, $t = t_s = \frac{d}{c}$ the metric potential in the model vanishes hence the model represents the singular model. Also, except for there is no such relation between the constants in the model for which it shows isotropy. The spatial volume is constant, at t→ 0. Therefore, the model starts evolving with constant volume and expands with cosmic time along with other parameters such as expansion scalar, shear scalar, and Hubble's parameter are constants, but at a singular point t_s the spatial volume vanishes and other parameters are diverged. Hence the derived model has a Big-Bang type of singularity at singular point t_s.

The deceleration parameter is constant with a negative sign and not associated with expansion. Hence, the mode of expansion of the model is accelerating throughout the evolution, the deceleration parameter is constant. For $-1 \le q < 0$, $q > 0$, and $q = 0$, the Universe exhibit accelerating volumetric expansion, decelerating volumetric expansion, and expansion with constant rate respectively. For, the model is isotropic and accelerating, and for, the model is anisotropic which is also an accelerating.

The Gauss–Bonnet invariant and function of Gauss–Bonnet invariants both are functions of cosmic time and depend on n. Our derived model gives two types of expansion isotropic and anisotropic for $n = 1$ and $n > 1$ respectively. But it is seen that in an isotropic expansion the Gauss–Bonnet invariant and function both are zero, hence it looks that in an isotropic expansion the Gauss–Bonnet invariant and function both do not exist. The energy density is a function of time t and always decrease positively with the expansion. At the initial stage t→ 0 the universe has infinitaly large energy density $\rho \to \infty$ but with the expansion of the universe it declines and at large $t \to \infty$ it is null $\rho \to 0$.

It is observed that the Equation of state parameter of Holographic dark energy is independent of cosmic time (constant) hence which is not deflected throughout the expansion of the model. In an isotropic model, the equation of state parameter admits the value -0.1666 corresponding to $n = 1$, also, it has been found that for the value of constant n in the range [2, 5] the equation of state parameter admits a value -0.5, -0.9, -1.333, -1.777 which exists in the standard range given by recent theoretical experiments along with the coincidence parameter is also independent of cosmic time, at very early to large stage of evolution, it is constant and remains constant throughout the evolution, by this means avoiding the coincidence problem (unlike ΛCDM). The energy density parameter approaches a constant value. So, from initial to late time the Universe becomes flat. Therefore, for a sufficiently large time, this model predicts that the anisotropy of the Universe will dampen out and the Universe will become isotropic. This result also shows that in the early Universe, i.e. during the radiation and matter-dominated era the Universe was anisotropic and the Universe approaches isotropy as dark energy starts to dominate the energy density of the Universe.

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