Numerical Approximation of Modified Non-linear SIR Model of Computer Viruses

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Abstract: In this paper, the non-linear modified epidemiological model of computer viruses is illustrated. For this aim, two semi-analytical methods, the differential transform method (DTM) and the Laplace-Adomian decomposition method (LADM) are applied. The numerical results are estimated for different values of iterations and compared to the results of the LADM and the homotopy analysis transform method (HATM). Also, graphs of residual errors and phase portraits of approximate solutions for $n = 5, 10, 15$ are demonstrated. The numerical approximations show the performance of the LADM in comparison to the DTM and the HATM.

Keywords: non-linear susceptible-infected-recovered model, differential transform method, Laplace transformations, Adomian decomposition method

1. Introduction

The computer viruses are malware programs that have been able to infect thousands of computers and have hurt billions dollar in computers around the world. The virus should never be considered to be harmless and remain in the system. There are several types of viruses that can be categorized according to their source, technique, file type that infects, where they are hiding, the type of damage they enter, the type of operating system, or the design on which they are attacking. We can introduce some of famous and malicious viruses such as ILOVEYOU, Melissa, My Doom, Code Red, Sasser, Stuxnet and so on. Therefore, it is important that we study the methods to analyze, track, model, and protect against viruses. In recent years, many scientists have been illustrated the epidemiological models of computer viruses [2, 13, 16, 20, 32, 33, 36, 37, 45, 46, 47]. These models have been estimated by many mathematical methods such as collocation method [30], homotopy analysis method [5, 27], variational iteration method [22] and others [33, 41].

One of applicable and important models is the classical Susceptible-Infected-Recovered (SIR) computer virus propagation model [20, 21, 33] which is presented in the following form:

$$\frac{dS(t)}{dt} = f_1 - \lambda S(t)I(t) - dS(t),$$

$$\frac{dI(t)}{dt} = f_2 + \lambda S(t)I(t) - \varepsilon I(t) - dR(t), \tag{1}$$

$$\frac{dR(t)}{dt} = f_3 + \varepsilon I(t) - dR(t),$$

where

$$S(0) = S_0(t), I(0) = I_0(t), R(0) = R_0(t), \tag{2}$$

are the initial conditions of non-linear system of Eqs. (1). Functions and initial values of system (1) are given in Table 1.

Table 1 List of parameters and functions.

Parameters & Functions	Meaning	Values
$S(t)$	Susceptible computers at time t	$S(0) = 20$
$I(t)$	Infected computers at time t	$I(0) = 15$
$R(t)$	Recovered computers at time t	$R(0) = 10$
f_1, f_2, f_3	Rate of external computers connected to the network	$f_1 = f_2 = f_3 = 0$
λ	Rate of infecting for susceptible computer	$\lambda = 0.001$
ε	Rate of recovery for infected computers	$\varepsilon = 0.1$
d	Rate of removing from the network	$d = 0.1$

Recently, several numerical and semi-analytical methods are introduced to solve the mathematical and engineering problems [7, 8, 9, 10, 23, 24, 25, 26, 48] that we can apply them to solve the non-linear model (1). The DTM and the LADM are two important and efficient tools to solve the linear and non-linear problems arising in the mathematics, physics and engineering [29, 34, 35, 40, 43]. Specially, the LADM [11, 14, 31] obtained by combining the Adomian decomposition method [1, 3, 4, 15, 39, 44] and the Laplace transformations [17] similar to the HATM [6, 23, 27, 28], Laplace homotopy perturbation method [12, 38, 42] and so on.

The aim of this paper is to apply the DTM and the LADM to find the approximate solution of non-linear epidemiological system of Eqs. (1). The numerical results are compared with the HATM [6, 23, 27, 28] by plotting the residual erro function for different iterations. Also, the phase portraits of approximate solutions for $n = 10$ and different functions of $S(t)$, $I(t)$ and $R(t)$ are presented. The numerical results show the abilities and capabilities of the LADM in comparison to the DTM and the HATM.

2. Differential transform method

Transformation of the k-th derivative of a function in one variable is as follows

$$F(k) = \frac{1}{k!}\left[\frac{d^k f(t)}{dt^k}\right]_{t=t_0}, \tag{3}$$

and the inverse transformation is defined by

$$f(t) = \sum_{k=0}^{\infty} F(k)(t - t_0)^k \tag{4}$$

where $F(k)$ is the differential transform of $f(t)$. In actual applications, the function $f(t)$ is expressed by a finite series and Eq. (4) can be rewritten as follows:

$$f(t) = \sum_{k=0}^{N} F(k)(t - t_0)^k \tag{5}$$

where N is decided by the convergence of natural frequency. The fundamental operations of DTM have been given in Table 3.

Table 2 Main operations of DTM.

Original functions	Transformed functions
$f(t) = u(t) \pm v(t)$	$F(k) = U(k) \pm V(k)$
$f(t) = \beta u(t)$	$F(k) = \beta U(k)$
$f(t) = u(t)v(t)$	$F(k) = \sum_{i=0}^{k} U(k)V_{k-s}(k)$
$f(t) = \dfrac{du(t)}{dt}$	$F(k) = (k+1)U(k+1)$
$f(t) = \dfrac{d^m u(t)}{dt^m}$	$F(k) = (k+1)(k+2)\cdots(k+m)U(k+m)$
$f(t) = \int_{t_0}^{t} u(\xi)d\xi$	$F(k) = \dfrac{U(k-1)}{k}, k \geq 1$
$f(t) = t^m$	$F(k) = \delta(k-m)$
$f(t) = \exp(\lambda t)$	$F(k) = \dfrac{\lambda^k}{k!}$
$f(t) = \sin(\omega t + \alpha)$	$F(k) = \dfrac{\omega^k}{k!}\sin(\dfrac{\pi k}{2} + \alpha)$
$f(t) = \cos(\omega t + \alpha)$	$F(k) = \dfrac{\omega^k}{k!}\cos(\dfrac{\pi k}{2} + \alpha)$

By applying the presented method to system of Eqs. (1), we get

$$S_{k+1} = \frac{1}{k+1}\left[f_1 - \lambda \sum_{i=0}^{k} S_i I_{k-i} - dS_k \right],$$

$$I_{k+1} = \frac{1}{k+1}\left[f_2 - \lambda \sum_{i=0}^{k} S_i I_{k-i} - \varepsilon I_k - dR_k \right], \tag{6}$$

$$R_{k+1} = \frac{1}{k+1}\left[f_3 - \varepsilon I_k - dR_k \right].$$

The differential transform method series solution for system (1) can be obtained as

$$S(t) = \sum_{j=0}^{n} S_j t^j,$$

$$I(t) = \sum_{j=0}^{n} I_j t^j, \tag{7}$$

$$R(t) = \sum_{j=0}^{n} R_j t^j,$$

3. Laplace-Adomian decomposition method

We apply the Laplace transformation $\mathcal{L}$ as

$$\mathcal{L}[S(t)] = \frac{S(0)}{z} + \frac{\mathcal{L}[f_1]}{z} - \frac{\lambda}{z}\mathcal{L}[S(t)I(t)] - \frac{d}{z}\mathcal{L}[S(t)],$$

$$\mathcal{L}[I(t)] = \frac{I(0)}{z} + \frac{\mathcal{L}[f_2]}{z} + \frac{\lambda}{z}\mathcal{L}[S(t)I(t)] - \frac{\varepsilon}{z}\mathcal{L}[I(t)] - \frac{d}{z}\mathcal{L}[R(t)], \quad (8)$$

$$\mathcal{L}[R(t)] = \frac{R(0)}{z} + \frac{\mathcal{L}[f_3]}{z} + \frac{\varepsilon}{z}\mathcal{L}[I(t)] - \frac{d}{z}\mathcal{L}[R(t)].$$

By putting the initial conditions we have

$$\mathcal{L}[S(t)] = \frac{S(0)}{z} + \frac{f_1}{z^2} - \frac{\lambda}{z}\mathcal{L}[A] - \frac{d}{z}\mathcal{L}[S(t)],$$

$$\mathcal{L}[I(t)] = \frac{I(0)}{z} + \frac{f_2}{z^2} + \frac{\lambda}{z}\mathcal{L}[A] - \frac{\varepsilon}{z}\mathcal{L}[I(t)] - \frac{d}{z}\mathcal{L}[R(t)], \quad (9)$$

$$\mathcal{L}[R(t)] = \frac{R(0)}{z} + \frac{f_3}{z^2} + \frac{\varepsilon}{z}\mathcal{L}[I(t)] - \frac{d}{z}\mathcal{L}[R(t)],$$

where $A = SI$ and

$$S = \sum_{j=0}^{\infty} S_j, \quad I = \sum_{j=0}^{\infty} I_j \quad R = \sum_{j=0}^{\infty} R_j \quad (10)$$

Also, the non-linear operator A is called the Adomian polynomials and it is presented as

$$A = \sum_{j=0}^{\infty} A_j, \quad (11)$$

where

$$A_0 = S_0 I_0,$$

$$A_1 = S_0 I_1 + S_1 I_0,$$

$$A_2 = S_0 I_2 + S_1 I_1 + S_2 I_0,$$

$$\quad (12)$$

$$A_3 = S_0 I_3 + S_1 I_2 + S_2 I_1 + S_3 I_0,$$

$$A_4 = S_0 I_4 + S_1 I_3 + S_2 I_2 + S_3 I_1 + S_4 I_0,$$
$$\vdots$$

By substituting series (10) and (11) into (9) we get

$$\mathcal{L}\left[\sum_{j=0}^{\infty} S_j\right] = \frac{S(0)}{z} + \frac{f_1}{z^2} - \frac{\lambda}{z}\mathcal{L}\left[\sum_{j=0}^{\infty} A_j\right] - \frac{d}{z}\mathcal{L}\left[\sum_{j=0}^{\infty} S_j\right],$$

$$\mathcal{L}\left[\sum_{j=0}^{\infty} I_j\right] = \frac{I(0)}{z} + \frac{f_2}{z^2} + \frac{\lambda}{z}\mathcal{L}\left[\sum_{j=0}^{\infty} A_j\right] - \frac{\varepsilon}{z}\mathcal{L}\left[\sum_{j=0}^{\infty} I_j\right] - \frac{d}{z}\mathcal{L}\left[\sum_{j=0}^{\infty} R_j\right], \qquad (13)$$

$$\mathcal{L}\left[\sum_{j=0}^{\infty} R_j\right] = \frac{R(0)}{z} + \frac{f_3}{z^2} + \frac{\varepsilon}{z}\mathcal{L}\left[\sum_{j=0}^{\infty} I_j\right] - \frac{d}{z}\mathcal{L}\left[\sum_{j=0}^{\infty} R_j\right].$$

Now, the following relations can be obtained:

$$\mathcal{L}[S_0] = \frac{S(0)}{z} + \frac{f_1}{z^2},$$

$$\mathcal{L}[I_0] = \frac{I(0)}{z} + \frac{f_2}{z^2},$$

$$\mathcal{L}[R_0] = \frac{R(0)}{z} + \frac{f_3}{z^2},$$

$$\mathcal{L}[S_1] = \frac{\lambda}{z}\mathcal{L}[A_0] - \frac{d}{z}\mathcal{L}[S_0], \qquad (14)$$

$$\mathcal{L}[I_1] = \frac{\lambda}{z}\mathcal{L}[A_0] - \frac{\varepsilon}{z}\mathcal{L}[I_0] - \frac{d}{z}\mathcal{L}[R_0],$$

$$\mathcal{L}[R_1] = \frac{\varepsilon}{z}\mathcal{L}[I_0] - \frac{d}{z}\mathcal{L}[R_0],$$

and for term j we have

$$\mathcal{L}[S_j] = \frac{\lambda}{z}\mathcal{L}\left[A_{j-1}\right] - \frac{d}{z}\mathcal{L}\left[S_{j-1}\right],$$

$$\mathcal{L}[I_j] = \frac{\lambda}{z}\mathcal{L}\left[A_{j-1}\right] - \frac{\varepsilon}{z}\mathcal{L}\left[I_{j-1}\right] - \frac{d}{z}\mathcal{L}\left[R_{j-1}\right], \qquad (15)$$

$$\mathcal{L}[R_j] = \frac{\varepsilon}{z}\mathcal{L}\left[I_{j-1}\right] - \frac{d}{z}\mathcal{L}\left[R_{j-1}\right].$$

Applying the inverse Laplace transformation $\mathcal{L}^{-1}$ for first equations of (14) as follows

$$S_0 = S(0) + f_1 t,$$

$$I_0 = I(0) + f_2 t, \tag{16}$$

$$R_0 = R(0) + f_3 t,$$

By putting S_0, I_0, R_0 in second equations of (14) and using the Laplace transformations we have

$$\mathcal{L}[S_1] = \frac{\lambda}{z}\left(\frac{S(0)I(0)}{z} + \frac{S(0)f_2}{z^2} + \frac{I(0)f_1}{z^2} + \frac{2f_1 f_2}{z^3}\right) - \frac{d}{z}\left(\frac{S(0)}{z} + \frac{f_1}{z^2}\right),$$

$$\mathcal{L}[I_1] = \frac{\lambda}{z}\left(\frac{S(0)I(0)}{z} + \frac{S(0)f_2}{z^2} + \frac{I(0)f_1}{z^2} + \frac{2f_1 f_2}{z^3}\right) - \frac{\varepsilon}{z}\left(\frac{I(0)}{z} + \frac{f_2}{z^2}\right)$$

$$\qquad - \frac{d}{z}\left(\frac{R(0)}{z} + \frac{f_3}{z^2}\right), \tag{17}$$

$$\mathcal{L}[R_1] = \frac{\varepsilon}{z}\left(\frac{I(0)}{z} + \frac{f_2}{z^2}\right) - \frac{d}{z}\left(\frac{R(0)}{z} + \frac{f_3}{z^2}\right),$$

and by applying the inverse Laplace transform $\mathcal{L}^{-1}$ we can find S_1, I_1 and R_1. By repeating above process, the other terms S_2, $\cdots$, S_j, I_2, $\cdots$, I_j, R_2, $\cdots$, R_j can be obtained. By using the relations

$$S_n = \sum_{j=0}^{n} S_j, \quad I_n = \sum_{j=0}^{n} I_j \quad R_n = \sum_{j=0}^{n} R_j \tag{18}$$

the n-th order approximate solutions can be estimated.

4. Numerical Illustration

In this section, the numerical results of the DTM and the LADM for solving the system of Eqs.(1) are presented. The approximate solutions for $n = 5$ by using the DTM are obtained in the following form

$$S_5(t) = 20 - 2.3t + 0.15425t^2 - 0.00790458t^3 + 0.000309711t^4,$$
$$I_5(t) = 15 - 2.2t + 0.04575t^2 + 0.00573792t^3 - 0.000406169t^4,$$
$$R_5(t) = 10 + 0.5t - 0.135t^2 + 0.006025t^3 - 7.17708 \times 10{-}6t^4,$$

and for $n = 10$ we have

$$S_{10}(t) = 20 - 2.3t + 0.15425t^2 - 0.00790458t^3 + 0.000309711t^4$$
$$\qquad - 7.74864 \times 10{-}6t^5 - 1.35996 \times 10{-}8t^6 + 1.41005 \times 10{-}8t^7$$
$$\qquad - 8.93373 \times 10{-}10t^8 + 3.32927 \times 10{-}11t^9,$$

$$I_{10}(t) = 15 - 2.2t + 0.04575t^2 + 0.00573792t^3 - 0.000406169t^4$$
$$\qquad + 9.82135 \times 10{-}6t^5 + 1.12052 \times 10{-}7t^6 - 1.97453 \times 10{-}8t^7$$
$$\qquad + 9.96904 \times 10{-}10t^8 - 3.2067 \times 10{-}11t^9,$$

$$R_{10}(t) = 10 + 0.5t - 0.135t^2 + 0.006025t^3 - 7.17708 \times 10{-6}t^4$$
$$- 7.97984 \times 10^{-6}t^5 + 2.96686 \times 10^{-7}t^6 - 2.63764 \times 10^{-9}t^7$$
$$- 2.13846 \times 10^{-10}t^8 + 1.34528 \times 10^{-11}t^9,$$

and finally for $n = 15$ the approximate solutions are obtained as

$$S_{15}(t) = 20 - 2.3t + 0.15425t^2 - 0.00790458t^3 + 0.000309711t^4$$
$$- 7.74864 \times 10^{-6}t^5 - 1.35996 \times 10^{-8}t^6 + 1.41005 \times 10^{-8}t^7$$
$$- 8.93373 \times 10^{-10}t^8 + 3.32927 \times 10^{-11}t^9 - 5.68453 \times 10^{-13}t^{10}$$
$$- 2.34166 \times 10^{-14}t^{11} + 2.59234 \times 10^{-15}t^{12} - 1.28786 \times 10^{-16}t^{13} + 3.78868 \times 10^{-18}t^{14},$$

$$I_{15}(t) = 15 - 2.2t + 0.04575t^2 + 0.00573792t^3 - 0.000406169t^4$$
$$+9.82135 \times 10^{-6}t^5 + 1.12052 \times 10^{-7}t^6 - 1.97453 \times 10^{-8}t^7$$
$$+9.96904 \times 10^{-10}t^8 - 3.2067 \times 10^{-11}t^9 + 4.21668 \times 10^{-13}t^{10}$$
$$+2.88892 \times 10^{-14}t^{11} - 2.70438 \times 10^{-15}t^{12} + 1.28307 \times 10^{-16}t^{13} - 3.62709 \times 10^{-18}t^{14},$$

$$R_{15}(t) = 10 + 0.5t - 0.135t^2 + 0.006025t^3 - 7.17708 \times 10^{-6}t^4$$
$$- 7.97984 \times 10^{-6}t^5 + 2.96686 \times 10^{-7}t^6 - 2.63764 \times 10^{-9}t^7$$
$$- 2.13846 \times 10^{-10}t^8 + 1.34528 \times 10^{-11}t^9 - 4.55198 \times 10^{-13}t^{10}$$
$$+ 7.97151 \times 10^{-15}t^{11} + 1.74314 \times 10^{-16}t^{12} - 2.21438 \times 10^{-17}t^{13} + 1.07465 \times 10^{-18}t^{14}.$$

Now, by applying the LADM we get

$$S_0(t) = 20, \qquad I_0(t) = 15, \qquad R_0(t) = 10,$$

$$S_1(t) = -2.3t, \qquad I_1(t) = -2.2t, \qquad R_1(t) = 0.5t,$$

$$S_2(t) = 0.15425t^2, \qquad I_2(t) = 0.04575t^2, \qquad R_2(t) = -0.135t^2,$$

$$\vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad \vdots$$

$$S_{10}(t) = -5.68453 \times 10^{-13}t^{10}, \quad I_{10}(t) = 4.21668 \times 10^{-13}t^{10}, \quad R_{10}(t) = -4.55198 \times 10^{-13}t^{10},$$

$$\vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad \vdots$$

and finally the approximate solution of the epidemiological model of the computer viruses (1) for $n = 10$ is in the following form

$$S_{10}(t) = \sum_{j=0}^{10} S_j(t) = 20 - 2.3t + 0.15425t^2 - 0.00790458t^3 + 0.000309711t^4$$
$$- 7.74864 \times 10^{-6}t^5 - 1.35996 \times 10^{-8}t^6 + 1.41005 \times 10^{-8}t^7$$
$$- 8.93373 \times 10^{-10}t^8 + 3.32927 \times 10^{-11}t^9 - 5.68453 \times 10^{-13}t^{10},$$

$$I_{10}(t) = \sum_{j=0}^{10} I_j(t) = 15 - 2.2t + 0.04575t^2 + 0.00573792t^3 - 0.000406169t^4$$
$$+9.82135 \times 10^{-6}t^5 + 1.12052 \times 10^{-7}t^6 - 1.97453 \times 10^{-8}t^7$$
$$+9.96904 \times 10^{-10}t^8 - 3.2067 \times 10^{-11}t^9 + 4.21668 \times 10^{-13}t^{10},$$

$$R_{10}(t) = \sum_{j=0}^{10} R_j(t) = 10 + 0.5t - 0.135t^2 + 0.006025t^3 - 7.17708 \times 10^{-6}t^4$$

$$- 7.97984 \times 10^{-6}t^5 + 2.96686 \times 10^{-7}t^6 - 2.63764 \times 10^{-9}t^7$$
$$- 2.13846 \times 10^{-10}t^8 + 1.34528 \times 10^{-11}t^9 - 4.55198 \times 10^{-13}t^{10}.$$

In order to show the accuracy of the presented methods, following residual errors are presented. Also, the numerical results are compared to the obtained results of the HATM for $n = 5, 10$. The results are presented in Tables 3, 4 and 5.

$$E_{n,S}(t) = S'_n(t) - f_1 + \lambda S_n(t)I_n(t) + dS_n(t),$$
$$E_{n,I}(t) = I'_n(t) - f_2 - \lambda S_n(t)I_n(t) + \varepsilon I_n(t) + dR_n(t), \tag{19}$$
$$E_{n,R}(t) = R'_n(t) - f_3 - \varepsilon I_n(t) + dR_n(t).$$

The comparative graphs between the residual errors of the LADM, the DTM and the HATM for $n = 5, 10, 15$ are demonstrated in Figs. 1, 2 and 3. Also, phase portraits of $S - I$, $S - R$, $I - R$ and $S - I - R$ which are obtained by 10-th order approximation of the DTM and the LADM are presented in Figs. 4 and 5. According to the generated results, the LADM has suitable scheme than the DTM and the HATM.

Table 3 Numerical comparison of residual error $E_{n,S}(t)$ between LADM, DTM and HATM for $n = 5, 10$.

t	$E_{5,S}(t)$-LADM	$E_{5,S}(t)$-DTM	$E_{5,S}(t)$-HATM	$E_{10,S}(t)$-LADM	$E_{10,S}(t)$-DTM	$E_{10,S}(t)$-HATM
0.0	0	0	0	0	0	0
0.2	1.98295×10^{-11}	6.22316×10^{-8}	0.0000116327	4.44089×10^{-16}	4.44089×10^{-16}	4.22713×10^{-11}
0.4	4.38413×10^{-10}	9.99398×10^{-7}	0.0000116327	0	1.77636×10^{-15}	8.71449×10^{-10}
0.6	1.87637×10^{-9}	5.07724×10^{-6}	0.0000214386	1.77636×10^{-15}	5.9508×10^{-14}	4.95175×10^{-9}
0.8	1.93464×10^{-9}	0.0000161	0.00015492	2.57572×10^{-14}	7.94032×10^{-13}	1.57635×10^{-8}
1.0	1.18760×10^{-8}	0.0000394305	0.000429509	2.30038×10^{-13}	5.97122×10^{-12}	3.43742×10^{-8}

Table 4 Numerical comparison of residual error $E_{n,I}(t)$ between LADM, DTM and HATM for $n = 5, 10$.

t	$E_{5,I}(t)$-LADM	$E_{5,I}(t)$-DTM	$E_{5,I}(t)$-HATM	$E_{10,I}(t)$-LADM	$E_{10,I}(t)$-DTM	$E_{10,I}(t)$-HATM
0.0	0	0	7.10543×10^{-15}	0	0	0
0.2	2.08857×10^{-10}	7.88132×10^{-8}	0.000013859	4.44089×10^{-16}	4.44089×10^{-16}	4.24451×10^{-11}
0.4	6.48732×10^{-9}	1.26470×10^{-6}	0.000036861	4.44089×10^{-16}	1.77636×10^{-15}	8.8009×10^{-10}
0.6	4.78103×10^{-8}	6.42035×10^{-6}	0.0000397647	1.77636×10^{-15}	4.44089×10^{-14}	5.06861×10^{-9}
0.8	1.95500×10^{-7}	0.0000203449	8.51494×10^{-6}	3.15303×10^{-14}	5.96412×10^{-13}	1.65033×10^{-8}
1.0	5.78838×10^{-7}	0.000049794	0.000140914	2.89546×10^{-13}	4.50395×10^{-12}	3.7476×10^{-8}

Table 5 Numerical comparison of residual error $E_{n,R}(t)$ between LADM, DTM and HATM for $n = 5, 10$.

| t	$E_{5,R}(t)$-LADM	$E_{5,R}(t)$-DTM	$E_{5,R}(t)$-HATM	$E_{10,R}(t)$-LADM	$E_{10,R}(t)$-DTM	$E_{10,R}(t)$-HATM
0.0	0	0	0	0	0	0
0.2	5.69638×10^{-10}	6.38387×10^{-8}	1.95392×10^{-6}	1.11022×10^{-16}	3.33067×10^{-16}	1.04154×10^{-12}
0.4	1.82284×10^{-8}	6.38387×10^{-6}	0.0000285358	3.33067×10^{-16}	1.22125×10^{-15}	8.17403×10^{-12}
0.6	1.38422×10^{-7}	5.17094×10^{-6}	0.000112891	3.33067×10^{-16}	4.56857×10^{-14}	1.57476×10^{-10}
0.8	5.83309×10^{-7}	0.0000163427	0.000290976	9.43690×10^{-15}	6.10734×10^{-13}	1.69486×10^{-9}
1.0	1.78012×10^{-6}	0.0000398992	0.000601621	8.77631×10^{-14}	4.55164×10^{-12}	8.65934×10^{-9}
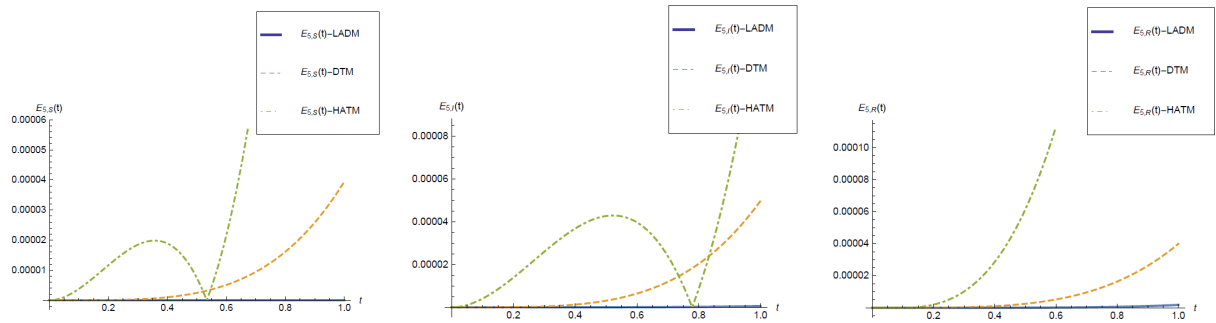
Figure 1 Comparison between error functions of LADM, DTM and HATM for $S_5(t)$, $I_5(t)$, $R_5(t)$.
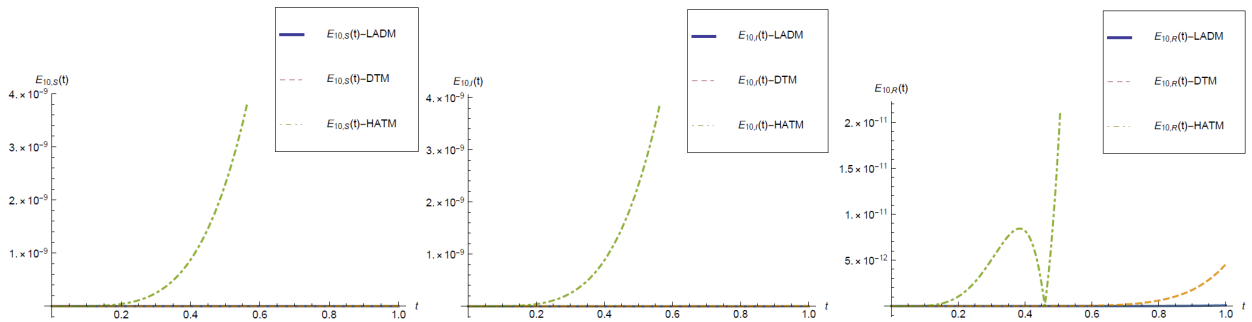

Figure 2 Comparison between error functions of LADM, DTM and HATM for $S_{10}(t)$, $I_{10}(t)$, $R_{10}(t)$.
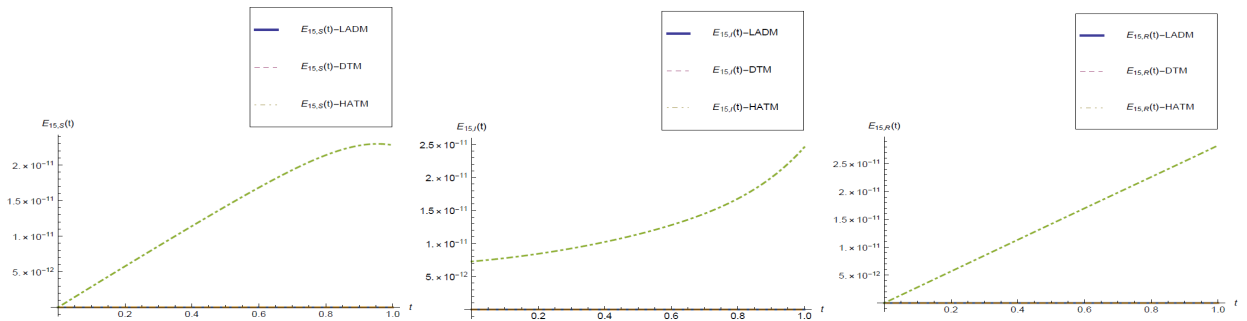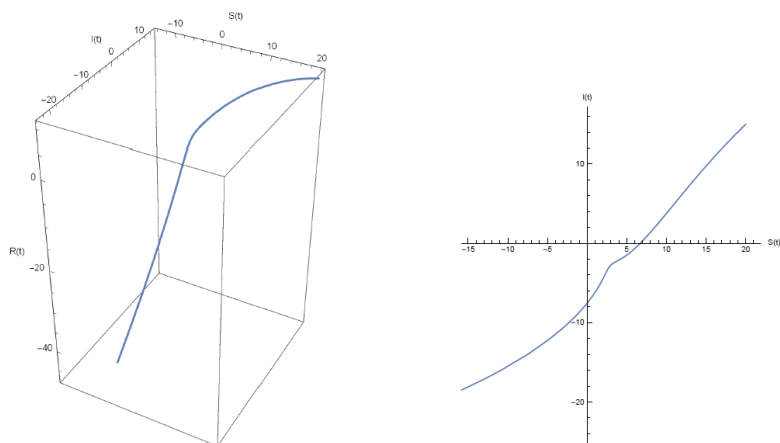

Figure 3 Comparison between error functions of LADM, DTM and HATM for $S_{15}(t)$, $I_{15}(t)$, $R_{15}(t)$.
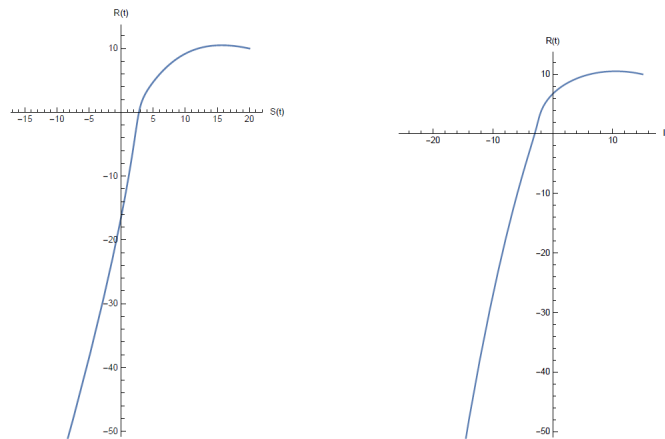
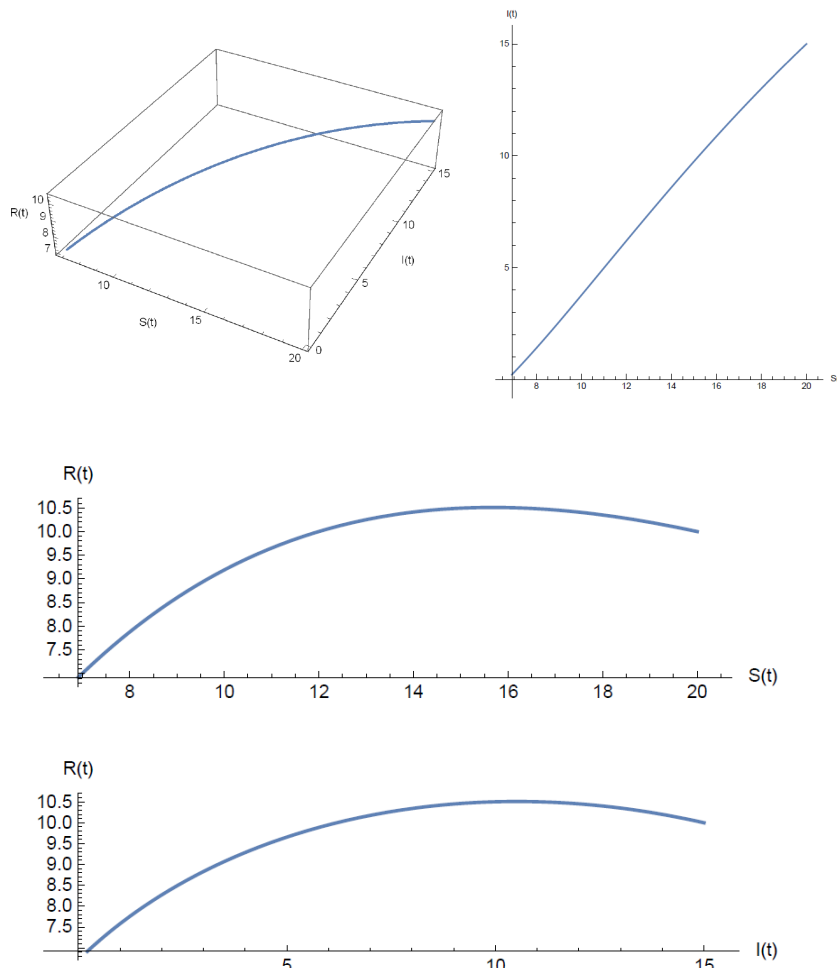**Figure 4 Phase portraits of $S_{10}(t)$, $I_{10}(t)$, $R_{10}(t)$ by using the LADM.**

Figure 5 Phase portraits of $S_{10}(t)$, $I_{10}(t)$, $R_{10}(t)$ by using the DTM.

5. Conclusion

In this study, two robust and applicable methods, the DTM and the LADM were applied to solve the non-linear epidemiological model of computer viruses. These models are among of applicable models in computer engineering that we can apply to track, analyze and predict the computer viruses in a network. In order to show the efficiency and accuracy

of presented method, the residual errors for different iterations n = 5, 10 were presented based on the LADM, DTM and HATM. Also, the graphs of residual error were demonstrated to show the abilities of the presented methods. According to these results the LADM has more applicable and more accurate than DTM and the HATM. We can improve the mentioned model by adding some other parameters into the model. Also, this model can be applied to provide the other models in many other sciences. We will work on fractional and fuzzy mathematical model of computer viruses for future researches.

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