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Article

Efficient Optimal and Suboptimal Joint Access Point Association and Radio Resource Allocation in WLANs

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Abstract: The centralized joint access point (AP) association and radio resource allocation problem in time-sharing based dense WLANs with heterogeneous station (STA) throughput demands and identical AP transmission power is considered in this paper. The problem is formulated as an NP-hard single non-zero programming (SNZP) optimization problem with three different throughput-based objective functions, including the maximal aggregate (MA) throughput, the max-min fair (MMF) throughput and the proportionally fair (PF) throughput. To solve the NP-hard SNZP problem, based on the optimal branch-and-bound (BnB) searching approach, two novel efficient algorithms, the optimal BnB and the suboptimal depth-first BnB (DF-BnB) algorithms, are proposed, which can be used as convenient performance benchmarks in the study of the problem considered. The complexity of the proposed algorithms is analyzed theoretically. Numerical results are presented to investigate and compare the performance and complexity of the proposed algorithms under the three different design criteria, which can help in choosing an appropriate cost function objective in practical system design.

Keywords: access point association, radio resource allocation, time-sharing, WLAN, branch-and-bound, cross-layer design and optimization

1. Introduction

Wi-Fi wireless local area networks (WLANs) based on IEEE 802.11 standards have been widely deployed in homes, schools, office and apartment buildings, etc. and are the most widely used local area computer networks, providing convenient Internet connections to users with all kinds of portable wireless devices [1]. In a typical infrastructure mode WLAN, an access point (AP) is wirelessly connected and associated with multiple stations (STAs), constituting a basic service set (BSS).

In busy areas with many STAs, WLANs are deployed densely. A large number of APs can be closely placed and STAs can be distributed densely and unevenly. Thus, there may exist overlapping basic service sets (OBSS) in those busy areas. In current IEEE 802.11 standards, STAs use the distributed strongest signal first (SSF) association algorithm to independently search and then associate with the APs which provide them the strongest received signal strength (RSS). In the presence of OBSS, however, the SSF association may result in traffic imbalance among closely deployed APs, and thus degradation in network performance [2,3].

Different technologies have been proposed in literature in order to overcome the difficulties brought by OBSS in dense WLANs, and efficient AP association is one of them. In traditional Wi-Fi standards, WLANs use distributed contention-based medium access control (MAC). In distributed contention-based WLANs, all STAs associated with the same AP are provided with the same throughput. References [4–13] investigated AP association in traditional Wi-Fi WLANs. Nevertheless, heterogeneous STA throughput demands are popular

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requirements for current WLANs supporting different multimedia traffic. On the other hand, it is noted that the Target Wake Time (TWT) scheduled time sharing access in the more recent IEEE 802.11ax standard is a potential implementation method to support heterogeneous throughputs to STAs belonging to the same BSS [14–16]. References [17–23] investigated AP association in time sharing based WLANs, either without considering STAs' throughput demands or considering STAs' special throughput demands only. Besides throughput, other performance metrics, such as fittingness factor, link bandwidth/capacity, AP load, real-time queue related network metrics, resource efficiency, bandwidth satisfaction ratio (BSR), etc. were also used in the investigation of AP association [24–29].

In [30], the downlinks of dense WLANs based on efficient time-sharing MAC was considered, which support general heterogeneous throughput demands to STAs inside the same BSS. Although the iterative single non-zero programming relaxation (iSNZPR) algorithm proposed in [30] has shown the best performance in all kinds of scenarios compared with other algorithms in literature, it is unclear how much of a gap exists between the performance of the iSNZPR algorithm and the optimal one. This paper, therefore, considers the same network scenario as in [30] and tries to answer this question.

As in [30], we jointly consider centralized AP association and radio resource allocation with heterogeneous STA throughput demands and identical AP transmission power in the considered networks, which is formulated as an NP-hard single non-zero programming (SNZP) optimization problem with throughput-based objective functions [30,31]. We consider and compare three different throughput-based designs, including the maximal aggregate (MA) throughput, the max-min fair (MMF) throughput and the proportionally fair (PF) throughput objective functions. Note that in the downlinks of time-sharing WLANs with identical AP transmission power, radio resource is just transmission time of APs. Therefore, the joint AP association and radio resource allocation problem considered in this paper is just a joint AP association and transmission time allocation problem.

We propose novel efficient optimal branch-and-bound (BnB) and suboptimal depth-first BnB (DF-BnB) algorithms based on the optimal BnB searching approach by finding and formulating appropriate branching and bounding methods to the NP-hard SNZP problem so that the general optimal BnB searching method can be effectively applied to and efficiently solve the specific problem studied in this paper. The efficient optimal BnB algorithm provides a performance benchmark that can be used in the study of the problem considered in this paper. The efficient suboptimal DF-BnB algorithm tries to further reduce computational complexity of the optimal BnB algorithm, while offering similar performance as that of the optimal BnB algorithm. The complexity of the proposed algorithms is analyzed theoretically. The performance and complexity of the proposed algorithms under the three design criteria are then investigated and compared with those of some existing algorithms based on numerical results.

In terms of implementation, a centralized controller covering multiple APs and BSS's in a certain area is needed to be added to the system. It is also assumed that the APs and STAs in the system have been upgraded according to IEEE 802.11ax standard, with possible minor modifications. During the negotiation stage, STAs send their throughput demands, along with other necessary information, to the controller. Then the controller determines and sends back to APs and STAs the TWT session parameters like AP association, AP transmission time allocation, etc. based on the results obtained from the centralized algorithms discussed in this paper. It is now the controller that receives STA throughput demands and determines the TWT session parameters, rather than the APs themselves as in previous IEEE 802.11 standards. In addition, the controller also determines AP association, rather than the STAs themselves. Overall, part of the functions of APs and STAs are transferred to the centralized controller, and both are simpler than those in systems based on previous IEEE 802.11 standards.

Below is a summary of the main contributions of this paper.

- We propose two novel algorithms, the efficient optimal BnB and suboptimal DF-BnB algorithms, to solve the NP-hard SNZP optimization problem for centralized joint AP association and radio resource allocation in time-sharing based WLANs with densely deployed APs transmitting at identical power and heterogeneous STA throughput demands. Three different throughput-based objective functions, the MA throughput, the MMF throughput and the PF throughput, are considered.
- We theoretically analyze the complexity of the proposed BnB and DF-BnB algorithms.
- We investigate the fairness and average throughput per-STA performance, as well as the computational complexity of the proposed algorithms under the three different throughput-based design criteria in WLANs with different network scenarios based on numerical results. We also compare them with those of some existing algorithms in literature.

The organization of the rest of this paper is as follows. The SNZP problem formulation for the centralized joint AP association and radio resource allocation problem is presented in Section 2. In Sections 3 and 4, the proposed efficient optimal BnB and suboptimal DF-BnB algorithms are discussed, respectively. Their complexity is analyzed theoretically in Section 5. In Section 6, numerical results on the performance and complexity of the proposed algorithms are presented, discussed and compared. Conclusions are drawn in Section 7.

2. Problem Formulation

The downlinks of time-sharing based WLANs are considered in this paper. It is assumed that through very careful frequency planning, interfering APs transmit on orthogonal frequency channels and thus co-channel interference is completely overcome, which may be achieved in enterprise networks only [21]. There are M active STAs and N APs in the network with all APs' transmission power being the same. The number of active STAs is no less than the number of APs.

The centralized joint AP association and radio resource allocation problem studied in this paper is formulated as the following single non-zero programming (SNZP) problem [30,32].

$$\arg\max_{\mathrm{Y}} f(\mathrm{Y}) \tag{1a}$$

$$\text{subject to: } 0 \le \sum_{i=1}^{M} y_{i,j} \le 1 \quad j = 1,\dots,N \tag{1b}$$

$$0 \le \sum_{j=1}^{N} y_{i,j} \le 1 \quad i = 1,\dots,M \tag{1c}$$

$$0 \le y_{i,j} \le 1 \quad i = 1,\dots,M; j = 1,\dots,N \tag{1d}$$

$$R_{\min,i} \le Th_i \le R_{\max,i} \quad i = 1,\dots,M \tag{1e}$$

For any $i = 1,\dots,M$:

$$\text{only one } y_{i,j} \ (j = 1,\dots,N) \text{ is nonzero.} \tag{1f}$$

In the above formulation, the matrix $\mathrm{Y} = \{y_{i,j}\} \in \mathrm{R}_{\ge 0}^{M\times N}$ is the variable of this problem, with $\mathrm{R}_{\ge 0}$ denoting the set of non-negative real numbers. A zero value of $y_{i,j}$ represents that STA i is not associated with AP j. A positive value of $y_{i,j}$ represents the transmission time allocated to STA i from the associated AP j. In other words, the value of $y_{i,j}$ represents the situation of both association and radio resource allocation between STA i and AP j.

The cost function $f(\mathrm{Y})$ in (1a) is dependent on the design objective. For MA throughput, we have

$$f_{MA}(\mathrm{Y}) = \sum_{i=1}^{M}\sum_{j=1}^{N} y_{i,j} r_{i,j} \tag{2}$$

For MMF throughput, we have

$$f_{MMF}(\mathrm{Y}) = \min_{i} \sum_{j=1}^{N} y_{i,j} r_{i,j} \tag{3}$$

For PF throughput [33], we have

$$f_{PF}(\mathrm{Y}) = \sum_{i=1}^{M} \log\Big(\sum_{j=1}^{N} y_{i,j} r_{i,j}\Big) \tag{4}$$

In the problem formulation, for convenience, a normalized unit downlink transmission time is adopted, over which the network stays stable. The choice of the exact value of the unit time may be based on other network design considerations, e.g., the channel and network traffic scenarios, etc. and is beyond the scope of this paper. Therefore, the total communication time of any AP or STA is less than or at most equal to one, as shown in (1b) and (1c), respectively. In addition, the value of $y_{i,j}$ should be obviously between 0 and 1, as shown in (1d).

As shown in (1e), STAs' throughput demands fall into ranges from $R_{\min,i}$ to $R_{\max,i}\,(i=1,...,M)$, where Th_i, $R_{\min,i}$ and $R_{\max,i}$ denote the *effective throughput*, the minimum and maximum throughput demands of STA i, respectively, all with unit in bits per normalized unit transmission time. The effective throughput is obtained from the solution of the SNZP problem in (1) and is defined as

$$Th_i = \sum_{j\in A_i} y_{i,j} r_{i,j} \quad i=1,...,M, \tag{5}$$

where

$$A_i = \{j \mid r_{i,j} > 0\} \quad i=1,...,M \tag{6}$$

denotes the set of APs that could potentially be associated with STA i. In the following discussions, the notation $|A_i|$ is used to represent the total number of elements and A_i^k is used to represent the kth element in the set A_i. Note that $|A_i| \geq 1$ since each STA is assumed to be covered by at least one AP. The notation $r_{i,j}$ in (5) and (6) represents the effective transmission bit rate from AP j to its associated STA i, which can be determined by the SINR of the link. For example, according to IEEE 802.11ax standards, Table 1 shows the relationship between SINR and effective data rate for a 20 MHz bandwidth single-input and single-output (SISO) channel [14,34]. By using the received signal strength indicator (RSSI) in WiFi beacon signals [1], we can find the link SINR, thus the effective bit rate quite easily.

Table 1. Relationship between SINR and Effective Data Rate For 20 MHz SISO Channel in IEEE 802.11ax Standard.

SINR (dB)	3.8-7	7-9.4	9.4-13.3	13.3-16	16-20.9	20.9-22	22-23.5	23.5-27.8	27.8-29.5	29.5 -
Data Rate (Mbps)	8	16	24	33	49	65	73	81	98	108

Closely deployed APs may generate overlapping coverage areas, thus the STAs inside can be associated with any of those APs. That is, for STA i located inside an overlapping area, we have $|A_i| > 1$. Considering the convenience of implementation, it is assumed that over the normalized unit time considered, any STA is associated with only one AP, which is indicated in the statement (1f).

3. Efficient Optimal Algorithm

The SNZP problem in (1) is an NP-hard combinatorial problem, whose solution generally can only be found by exhaustive search over all possible combinations of the values of $y_{i,j}$. The computational complexity of the exhaustive search increases exponentially as the size of the problem increases [30]. In order to solve this difficulty, an efficient optimal algorithm based on the generic optimal BnB searching approach, which is termed as the optimal BnB algorithm, for the MA, MMF, and PF throughput-based NP-hard joint AP association and radio resource allocation problem is proposed in this section.

The BnB searching approach is a well-known tool for solving NP-hard combinatorial optimization problems. It has been successfully used to solve problems such as the traveling salesman problem [35,36], structured prediction in computer vision [37], feature selection in machine learning [38,39], the resource-constrained project duration problem with partially renewable resources in operations management [40], etc. Typically, a BnB searching for a combinatorial problem consists of branching and bounding steps [41]. *Branching* creates multiple subproblems with fewer variables by dividing the feasible region of the original problem into multiple subregions. Then the upper and lower bounds of the solution to each subproblem inside the corresponding feasible subregion are found in the *bounding* step. A searching tree is gradually generated by the subproblems resulting from the branching procedure operated recursively to each feasible subregion, with each subproblem corresponding to a node on the searching tree. A node is *pruned* if for a maximization problem, the upper bound of the solution to the corresponding subproblem is even less than the lower bound or the cost function value of the solution to any other subproblem already checked. Similarly, the pruning rule can be conveniently defined for a minimization problem. A node is solved when the maximum or minimum of the cost function value of the maximization or minimization problem is found within the corresponding subregion. If all nodes on the searching tree are either pruned or solved, then the BnB optimal searching is finished.

Based on the generic BnB searching approach, an optimal BnB algorithm is developed for the MA, PF, and MMF throughput-based NP-hard joint AP association and radio resource allocation problem, which is summarized in Table 2, where the notation " $\cup$ " denotes set sum. The details of the proposed algorithm are explained as follows.

Table 2. Efficient Optimal BnB Joint AP Association and Radio Resource Allocation Algorithm.

Initialization:

$t=0,\ z_0=0, \Omega_0=\phi, \Theta=-\infty, CurrentSolution=\phi, Active=\phi$

Step 1. Set $t=t+1$ and let $z_t=z_{t-1}, \Omega_t=\Omega_{t-1}$.

Step 2. If $z_t \le \Theta$:

go to Step 7.

Otherwise:

set $i'=t, k=1$ and $ActiveTemp=\phi$.

Step 3. Generate the node in depth t such that $a_{i'}=A_{i'}^k$ and update $\Omega_t=\Omega_t \cup \{i'\}$.

Step 4. Solve the subproblem in (10) with $a_i(i\in\Omega_t)$ known.

If the subproblem is infeasible:

go to Step 5.

If the subproblem is feasible:

denote the solution as $y_{t,i,j}^*$,

calculate the cost function value and denote it as z_t;

4a) If $z_t \le \Theta$:

go to Step 5.

4b) If $z_t > \Theta$ and $t<M$:

store this node into $ActiveTemp$ by storing t, z_t, Ω_t and $a_i(i\in\Omega_t)$;

go to Step 5.

4c) If $z_t > \Theta$ and $t=M$:

update $\Theta=z_t$;

update $CurrentSolution=[y_{M,1,a_1}^* \ y_{M,2,a_2}^* \cdots y_{M,M,a_M}^*]^T$;

go to Step 5.

Step 5. Set $k=k+1$.

If $k\le |A_{i'}|$:

go to Step 3.

Otherwise:

go to Step 6.

Step 6. If $ActiveTemp=\phi$:

sort the nodes in $ActiveTemp$ according to the values of z_t in an ascending manner;

append these nodes to the end of $Active$.

Step 7. If $Active \ne \phi$:

pick the node from the end of $Active$;

set t, z_t, Ω_t and $a_i(i\in\Omega_t)$ to the stored values associated with this node and go to Step 1.

Otherwise:

go to Step 8.

Step 8. Using $CurrentSolution$, calculate the effective throughput of STA i,

$Th_i\ (i=1,\ldots,M)$ according to (12);

Output $CurrentSolution$ and $Th_i\ (i=1,\ldots,M)$.

3.1 *Branching*

In the proposed algorithm, nodes are used to represent certain AP association scenarios for STAs in the system. The branching procedure of a node is implemented by trying all possible association schemes to the first unassociated STA in the system represented by the node to create child nodes and corresponding subproblems with fewer variables. The branching procedure is operated recursively to each of the child nodes and subproblems, generating a searching tree whose nodes are saved in the set *Active* in Table 2. The depth of a node on the searching tree is denoted as t, representing that the AP associations of t STAs have been determined.

In theory, a STA can be associated with any AP in the system, i.e., each node on the searching tree will generate N child nodes. Practically, however, considering the effect of SINR on transmission data rate in WiFi networks as shown in Table 1, the number of child nodes generated by a node with STA i being the first unassociated STA is $|A_i|$, which can be much less than N.

3.2 *Bounding*

By removing the only nonlinear constraint in (1f), the SNZP problem in (1) is relaxed to

$$\arg\max_{\mathrm{Y}} f(\mathrm{Y}) \tag{7a}$$

$$\text{subject to: } 0 \le \sum_{i=1}^{M} y_{i,j} \le 1 \quad j = 1,\ldots,N \tag{7b}$$

$$0 \le \sum_{j=1}^{N} y_{i,j} \le 1 \quad i = 1,\ldots,M \tag{7c}$$

$$0 \le y_{i,j} \le 1 \quad i = 1,\ldots,M;\, j = 1,\ldots,N \tag{7d}$$

$$R_{\min,i} \le Th_i \le R_{\max,i} \quad i = 1,\ldots,M \tag{7e}$$

After the relaxation, STAs may be associated with multiple APs simultaneously. On the other hand, for the MA and PF throughput problems, the cost functions in (7a) are (2) and (4), respectively, both of which are strictly concave. For the MMF throughput problem, a property of the cost function $f_{MMF}(\mathrm{Y})$ is [8]

$$f_{MMF}(\mathrm{Y}) \le \frac{f_{MA}(\mathrm{Y})}{M} \tag{8}$$

Thus, $\frac{f_{MA}(\mathrm{Y})}{M}$ can be used as the cost function in finding the upper bounds of the MMF throughput problem. Therefore, the computational complexity of the relaxed optimization problem in (7) becomes polynomial [42–44], and the cost function value corresponding to the solution of the problem provides an upper bound to the original SNZP problem in (1).

The same relaxation can be applied to each node on the searching tree to find an upper bound to the corresponding subproblem. In the following discussions, for convenience, AP a_i is denoted as the AP associated with STA i and Ω_t is denoted as the index set of the STAs whose AP associations have been decided after the tth depth calculations on the searching tree. If the joint AP association and transmission time between STA i and AP j after the tth depth calculations on the searching tree is denoted as $y_{t,i,j}$, for any $i \in \Omega_t$ we have

$$y_{t,i,j} \begin{cases} > 0 & \text{for } j = a_i \\ = 0 & \text{for } j \ne a_i \end{cases} \tag{9}$$

Since the AP associations of STA i for $i \in \Omega_t$ are known, the problem in (7) can be reformulated as the following subproblem with less variables

$$\arg\max_{\mathrm{Y}_t} f(\mathrm{Y}_t) \tag{10a}$$

$$\text{subject to: } 0 \le \sum_{i=1}^{M} y_{t,i,j} \le 1 \quad j = 1,\ldots,N \tag{10b}$$

$$0 \le \sum_{j=1}^{N} y_{t,i,j} \le 1 \quad i \notin \Omega_t \tag{10c}$$

$$0 \le y_{t,i,j} \le 1 \quad i \notin \Omega_t \text{ or } i \in \Omega_t,\, j = a_i \tag{10d}$$

$$y_{t,i,j} = 0 \quad i \in \Omega_t,\, j \ne a_i \tag{10e}$$

$$R_{\min,i} \le Th_{t,i} \le R_{\max,i} \quad i = 1,\dots,M \tag{10f}$$

where $\mathrm{Y}_t = \{y_{t,i,j}\} \in \mathrm{R}_{\ge 0}^{M\times N}$ and

$$Th_{t,i} = \begin{cases} y_{t,i,a_i} r_{i,a_i} & \text{for } i \in \Omega_t \\ \sum_{j\in A_i} y_{t,i,j} r_{i,j} & \text{for } i \notin \Omega_t \end{cases} \tag{11}$$

The cost function value of the solution to the relaxed sub-problem in (10) provides an upper bound for the subproblem corresponding to a node at depth t on the searching tree.

3.3 *Pruning*

A variable Θ, which is initialized as negative infinity, is maintained to keep the maximum cost function value corresponding to a feasible solution to the original SNZP problem in (1). The feasible solution, which is denoted as *CurrentSolution* in Table 2, is obtained among all sub-problems solved so far. If we denote $CurrentSolution = y_{i,a_i}^{*} \, (i = 1,\dots,M)$, then the effective throughput of STA i is

$$Th_i^{*} = y_{i,a_i}^{*} r_{i,a_i} \quad i = 1,\dots,M \tag{12}$$

In Table 2, each node is labeled with its depth t and the upper bound z_t for the corresponding subproblem, as well as Ω_t and a_i (for $i \in \Omega_t$).

We will prune any node whose upper bound z_t is less or equal to Θ from the searching tree. There is no need to further generate and check its child nodes, since it is impossible for a global maximum solution to exist in the feasible subregion corresponding to this kind of nodes. The searching finishes when all nodes on the searching tree are solved or pruned.

4. Efficient Suboptimal Algorithm

It is observed that although the average computational complexity of the proposed optimal BnB algorithm is much reduced compared with the exhaustive search algorithm due to the pruning operation, as the size of the problem increases, its complexity still increases rapidly. This shows it is necessary to find even more efficient algorithms. In this section, we propose a more efficient suboptimal algorithm for the MA, MMF, and PF throughput-based NP-hard joint AP association and radio resource allocation problems.

In the proposed suboptimal algorithm, in the hope of reducing computational complexity of the optimal BnB algorithm, instead of checking all nodes on each depth of the searching tree, we only keep and check the node with the maximum upper bound value [8]. The node chosen following this rule is more likely to lead to the global maximum solution. It is seen that the searching route on the searching tree is directly downwards from top to bottom, without lingering on the same depth of the searching tree. Therefore, the proposed suboptimal algorithm is referred to as depth-first BnB (DF-BnB). That is, the last procedure of Step 6 in Table 2 becomes appending the node with the largest value of z_t in *ActiveTemp* to the end of *Active*. Otherwise, all the other steps of the DF-BnB algorithm are the same as those of the BnB algorithm.

5. Complexity Analysis

The product of the number of variables and the number of inequality constraints is an approximate upper bound of the computational complexity in solving a continuous constrained concave maximization problem by using the interior-point algorithm [45].

For the optimal BnB algorithm discussed in Section 3, in the worst case, no node is pruned during the whole process of the algorithm. That is, we need to calculate all the nodes on the M depth searching tree. At the tth $(t = 1,\dots,M)$ depth, up to N^t optimization problems in (10) with up to $(M-t)N + t$ variables need to be solved and thus the worst case complexity of the proposed BnB algorithm at the tth depth is

$$C_{t,BnB} = O(N^{t+2}(M-t)^2) \quad t = 1,\dots,M \tag{13}$$

Based on (13), we have

$$\exists c_t > 0 \; \exists M_0 \; \exists N_0 \; \forall M > M_0, N > N_0 :$$

$$C_{t,BnB} \le c_t N^{t+2}(M-t)^2 \quad t = 1,...,M. \tag{14}$$

Denoting $c_{\max} = \max\{c_t, t = 1,...,M\}$, then we have

$$\exists c_t > 0,\; c_{\max} > 0 \; \exists M_0 \; \exists N_0 \; \forall M > M_0, N > N_0 :$$

$$C_{BnB} = \sum_{t=1}^{M} C_{t,BnB} \le \sum_{t=1}^{M} c_t N^{t+2}(M-t)^2 \le c_{\max} N^{M+2} \cdot \frac{(M-1)M(2M-1)}{6} \tag{15}$$

where C_{BnB} represents the worst case complexity of the proposed BnB algorithm. In the above derivation of (15), it is noted that $c_t \le c_{\max}$ and $N^t \le N^M$ for $t = 1,...,M$. From (15), we have

$$C_{BnB} = O(M^3 N^{M+2}). \tag{16}$$

For the suboptimal DF-BnB algorithm discussed in Section 4, at the tth $(t = 1,...,M)$ depth, up to N optimization problems in (10) with up to $(M-t)N+t$ variables need to be solved and thus the worst case complexity of the proposed suboptimal DF-BnB algorithm at the tth depth is

$$C_{t,DF-BnB} = O((M-t)^2 N^3) \quad t = 1,...,M. \tag{17}$$

Following similar derivations in (15) for the BnB algorithm, it is shown that the worst case complexity of the proposed DF-BnB algorithm is

$$C_{DF-BnB} = O(M^3 N^3). \tag{18}$$

6. Numerical Results

In order to compare performance of different algorithms, two performance metrics, average throughput per-STA and Jain's Fairness Index (FI) [46], are considered. According to [46], the FI is defined as

$$J = \frac{(\sum_{i=1}^{M} Th_i)^2}{M(\sum_{i=1}^{M} Th_i^2)} \in [0,1], \tag{19}$$

whose value represents the fairness level of a certain solution. The higher the value of J, the fairer the corresponding solution. By averaging the simulation results over 100 different random network realizations, we obtain the average performance and complexity presented in this section.

6.1 *Network Setup*

Figure 1 shows the rectangular service areas and layouts of APs of the two WLANs to be considered in this section, the smaller WLAN with 2×2 grid AP layout and the larger WLAN with 4 × 5 grid AP layout. In the figure, APs are represented by the small red triangles. The locations of APs are fixed and the locations of STAs are randomly chosen within the service area in different network realizations following two different random STA layout scenarios. In the uniform scenario, STAs are uniformly distributed over the rectangular service area. In the hotspot scenario, STAs assemble inside a circle centered at the rectangular service area center and with a 100 meter radius.

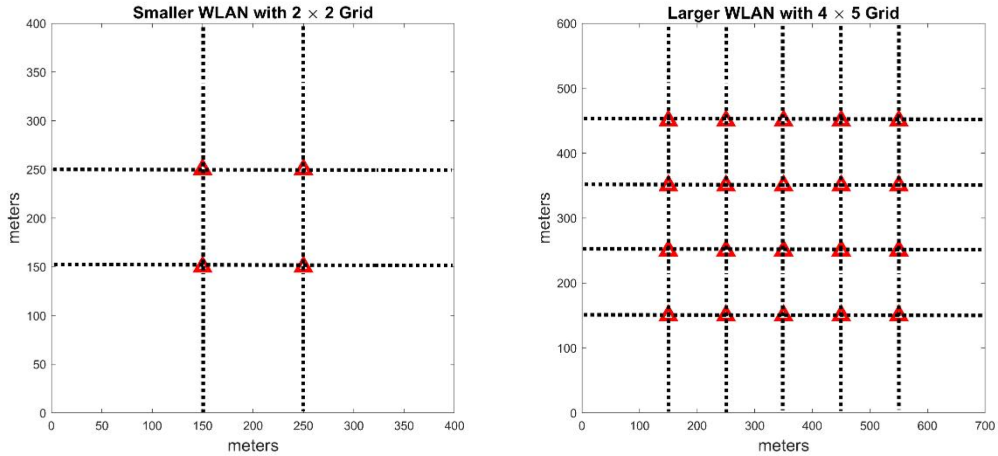

Figure 1. Service area and layout of APs.

We adopt the IEEE 802.11 ax Urban Micro (UMi) outdoor channel model with 2.4 GHz spectrum and 20 MHz bandwidth assigned to each AP. The probability of line-of-sight (LOS) path and the log-normal shadowing of the wireless channels between any AP and STA pair are randomly generated in different network realizations [47]. All APs transmit at the same power. The AP transmission power and the background noise power are 14 dBm and -92 dBm, respectively [48].

In our simulations, there exists three STA categories corresponding to low, medium, and high throughput demands, each with one third of the STAs in the network. The values of $R_{\max}$ are uniformly randomly chosen between 4 and 5 Mbps, 10 and 20 Mbps, and 40 and 50 Mbps, respectively, corresponding to the three categories. The value of $R_{\min}$ is fixed as 1 Mbps for all STAs. For the purpose of comparison, the performance of the legacy SSF algorithm and the iSNZPR algorithm proposed in [30] is also presented. For the iSNZPR algorithm, the parameter β is set as 0.98 which makes sure that in most iterations, only one STA determines its AP association, and the performance is better than that obtained with smaller β [30].

6.2 *Results of Smaller WLAN*

The performance of different algorithms in the smaller WLAN, with different numbers of active STAs in uniform and hotspot layouts, is compared in Figures 2–5. In these figures, the solid, dashed, and dotted curves represent the performance simulation results of the PF, MA, and MMF throughput problems, respectively.

Note that by using the legacy SSF algorithm, even though AP associations are the same under different design objectives, the AP transmission time is allocated differently. This is why the SSF algorithm shows different performance for the MA and the PF throughput problems as shown in Figures 2–5.

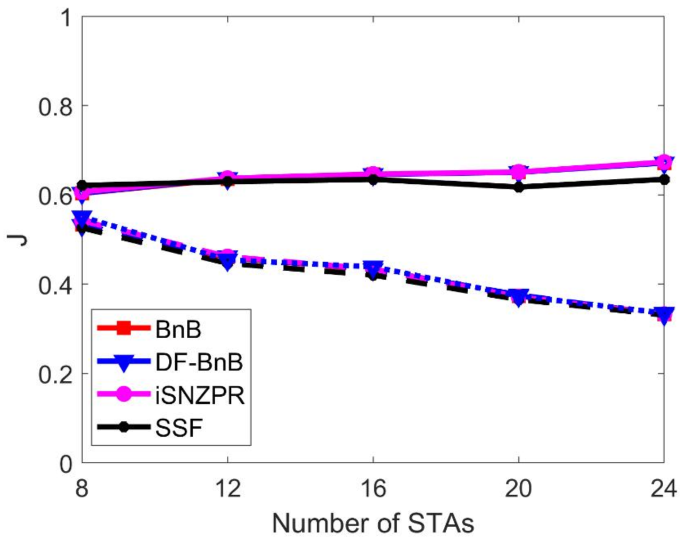

Figure 2. Fairness performance in the smaller WLAN with uniform STA layout.

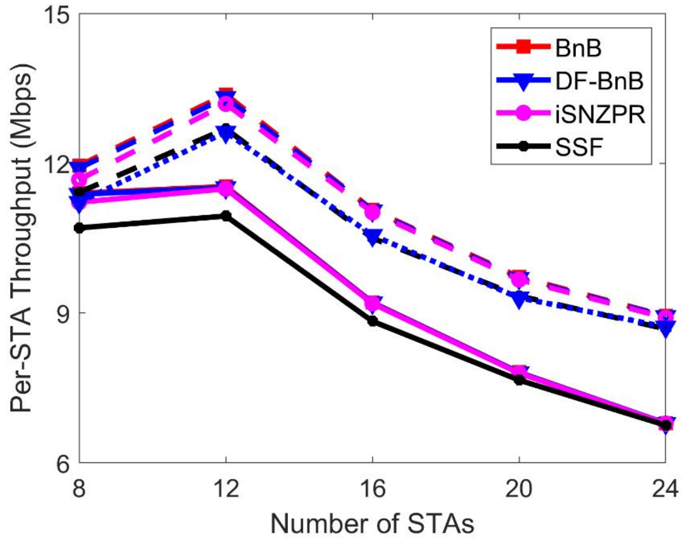

Figure 3. Average throughput per-STA (Mbps) performance in the smaller WLAN with uniform STA layout.

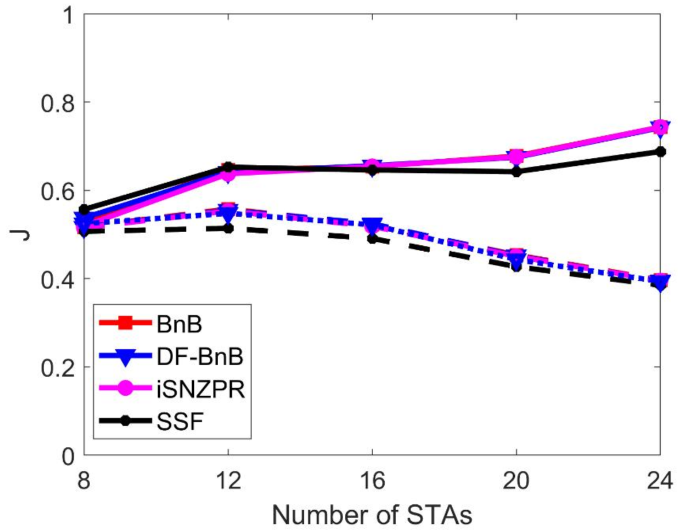

Figure 4. Fairness performance in the smaller WLAN with hotspot STA layout.

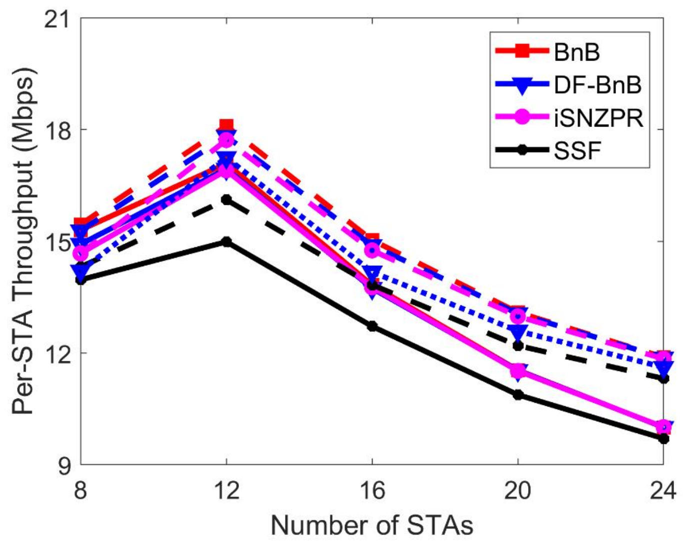

Figure 5. Average throughput per-STA (Mbps) performance in the smaller WLAN with hotspot STA layout.

For the MMF throughput problem, due to the cost function in (3) itself, the problem can only be solved by the proposed BnB and DF-BnB algorithms but cannot be solved by the SSF or the iSNZPR algorithms. In addition, the complexity of the proposed BnB algorithm is too high for it to be used in the performance comparison we conduct. The reason for the extremely high complexity of the BnB algorithm in this case is that the upper bound in (8) is not tight enough. Therefore, for the MMF throughput problem, only the results of the DF-BnB algorithm are presented.

For the fairness performance, it is shown from Figures 2 and 4 that in networks with both uniform and hotspot STA layouts, under both the MA and MMF throughput criteria, all the algorithms considered provide similar fairness among STAs, which is much lower than those under the PF throughput criterion. On the other hand, under the PF throughput criterion, the proposed BnB and DF-BnB algorithms, as well as the iSNZPR algorithm, provide quite similar fairness among STAs, which is slightly better than that provided by the SSF algorithm.

For the average throughput per-STA performance, it can be seen from Figures 3 and 5 that in networks with both uniform and hotspot STA layouts, the proposed BnB and DF-BnB algorithms, as well as the iSNZPR algorithm, have quite close average throughputs per-STA under the MA and PF throughput criteria, respectively. In both STA layout scenarios, when the number of STAs in the network is relatively small, all the other algorithms have higher average throughput per-STA than the SSF algorithm. When the number of STAs is relatively large, all the algorithms have quite similar average throughput per-STA. Among the three throughput criteria, the MA criterion provides the highest and the PF criterion provides the lowest average throughputs per-STA.

In Figures 6 and 7, the computational complexity of different algorithms in the smaller WLAN with different number of active STAs is compared. The measurement used is average CPU time used for solving the joint AP association and radio resource allocation problem for each network realization. From Figures 6 and 7, for both the MA and the PF throughout problems, the complexity of both the BnB and the DF-BnB algorithms is much higher than that of the iSNZPR and the SSF algorithms, and the complexity of the BnB algorithm is higher than that of the DF-BnB algorithm. In addition, the complexity of the algorithms applied in the PF throughout problem is generally higher than that of the corresponding algorithms applied in the MA throughput problem. The complexity of the DF-BnB algorithm applied in the MMF throughput problem is slightly higher than that of this algorithm applied in the MA throughput problem.

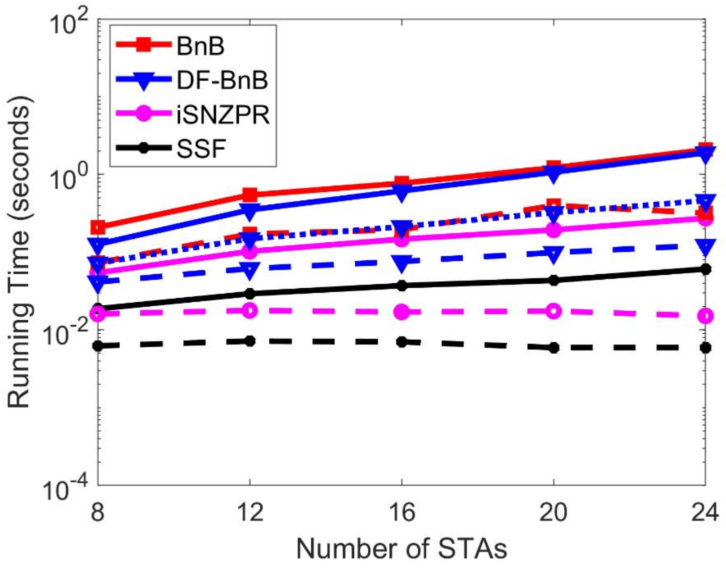

Figure 6. Computational Complexity in the smaller WLAN with uniform STA layout.

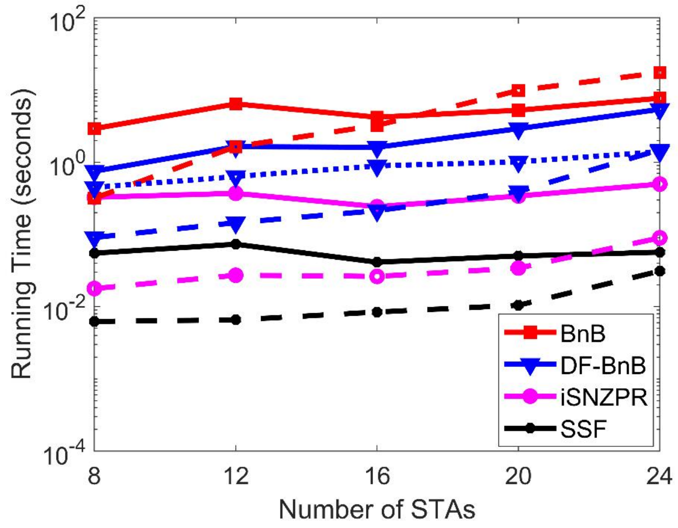

Figure 7. Computational Complexity in the smaller WLAN with hotspot STA layout.

From the above discussion, the proposed BnB algorithm provides the same optimal performance as the exhaustive search algorithm, but with much reduced average computational complexity. The proposed suboptimal DF-BnB algorithm offers similar performance as that of the optimal BnB algorithm in all kinds of scenarios, with further much reduced computational complexity compared with the optimal BnB algorithm. Therefore, the proposed suboptimal DF-BnB algorithm can be used as a convenient performance benchmark in the study of the problem considered in this paper.

6.3 *Results of Larger WLAN*

The performance of different algorithms in the larger WLAN is compared in Figures 8─11. Due to its too high complexity, the performance of the proposed BnB algorithm is not included in these figures.

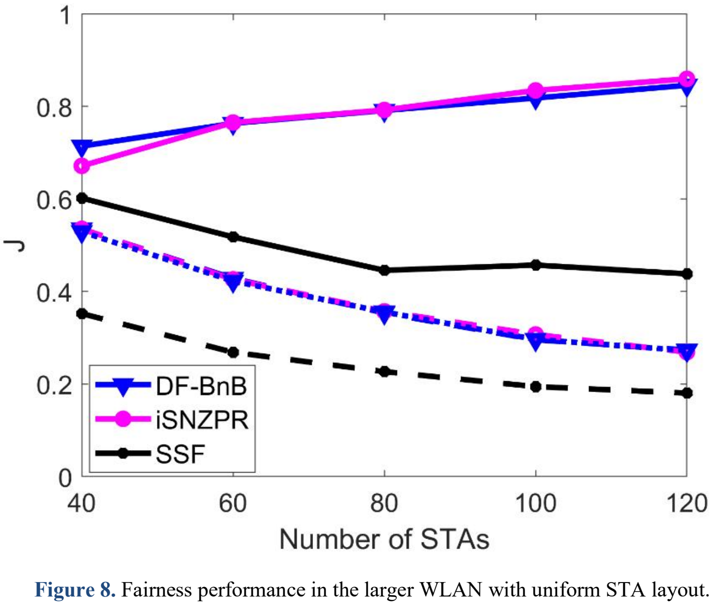

Figure 8. Fairness performance in the larger WLAN with uniform STA layout.

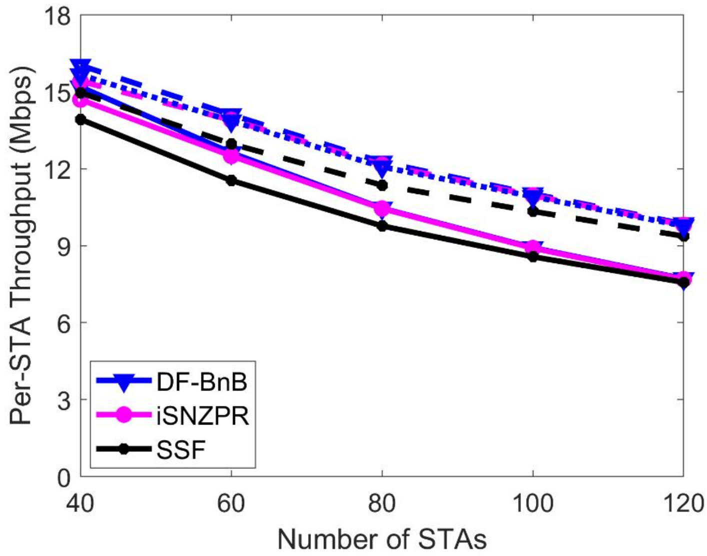

Figure 9. Average throughput per-STA (Mbps) performance in the larger WLAN with uniform STA layout.

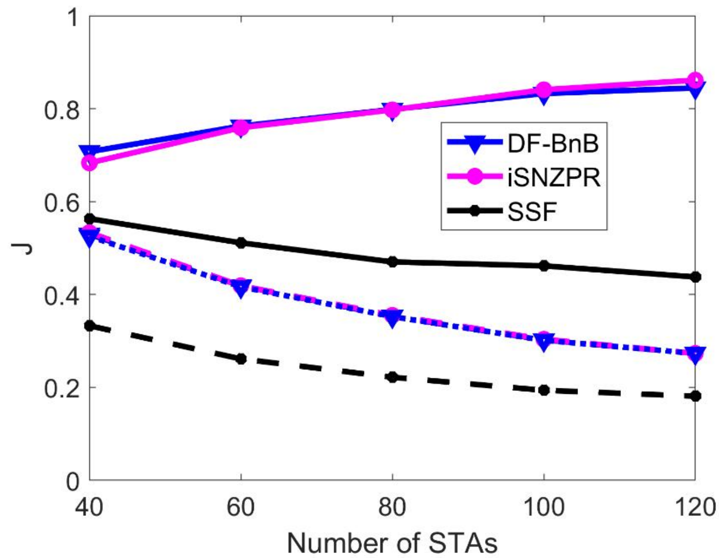

Figure 10. Fairness performance in the larger WLAN with hotspot STA layout.

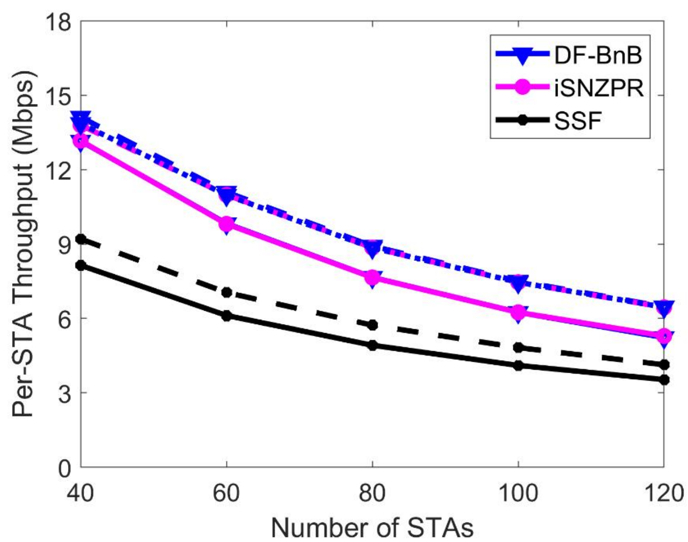

Figure 11. Average throughput per-STA (Mbps) performance in the larger WLAN with hotspot STA layout.

It is shown from Figures 8–11 that mostly similar performance is obtained in the larger network as that in the smaller network, except that as the number of STAs increases, the fairness of the SSF algorithm tends to decrease in the larger network, instead of increasing in the smaller network. The proposed DF-BnB algorithm and the iSNZPR algorithm in [30] show even greater performance improvement, and thus even greater benefit over the legacy SSF algorithm in this larger network case than in the smaller network case.

The computational complexity of different algorithms in the larger WLAN with different number of active STAs is compared in Figures 12 and 13.

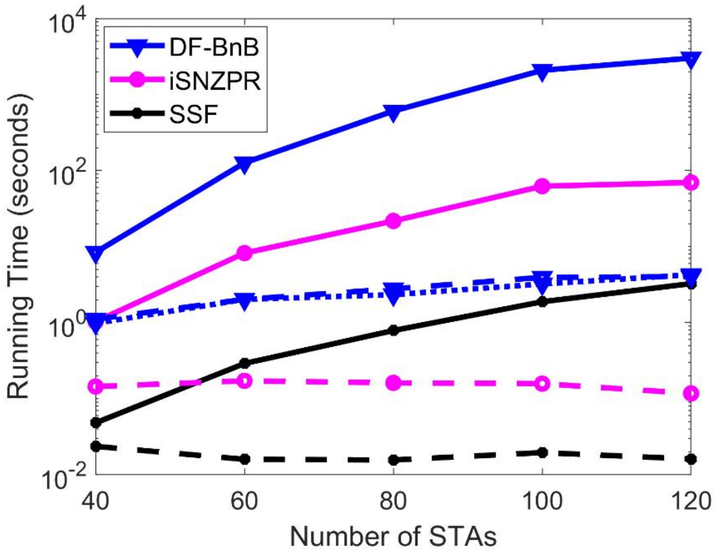

Figure 12. Computational Complexity in the larger WLAN with uniform STA layout.

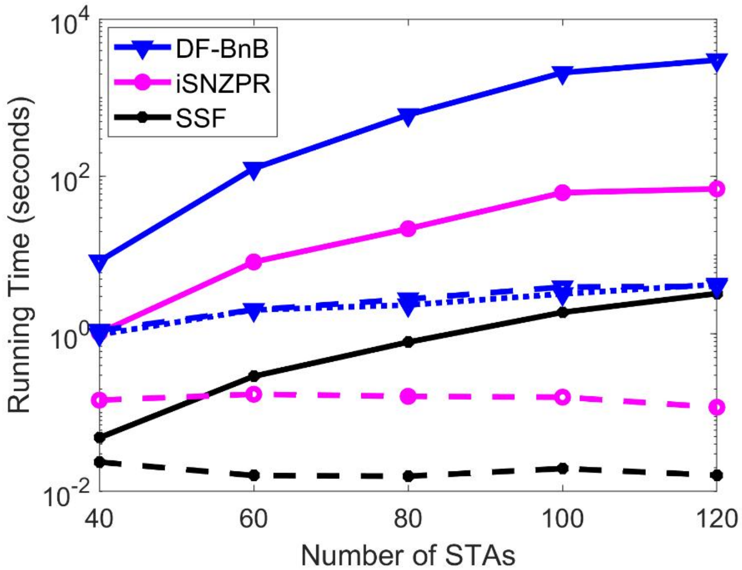

Figure 13. Computational Complexity in the larger WLAN with hotspot STA layout.

From the discussions in this section, it becomes clear that the iSNZPR algorithm offers quite close to optimal performance with reasonably low computational complexity.

7. Conclusion

In this paper, we have investigated the NP-hard SNZP optimization problem for centralized joint AP association and radio resource allocation with heterogeneous STA throughput demands. Three different throughput-based objective functions, the MA throughput, the MMF throughput, and the PF throughput, are considered in our study. In order to solve the NP-hard SNZP problem efficiently, the novel optimal BnB and suboptimal DF-BnB algorithms have been proposed, whose complexity has also been analyzed. Based on numerical results, the fairness and average throughput per-STA performance and complexity of the proposed algorithms have been investigated and compared with those of some existing algorithms in different WLAN scenarios.

The proposed suboptimal DF-BnB algorithm provides near-optimal performance, with much reduced computational complexity compared with the proposed optimal BnB algorithm. Therefore, the proposed DF-BnB algorithm provides an effective and efficient performance benchmark which can be used practically in the studies of the problem considered in this paper.

The three different throughput-based designs have also been investigated and compared, which can help in choosing an appropriate cost function objective in a practical system design. It has been shown by numerical results that in terms of fairness performance, the PF throughput criterion is much better in both STA layout scenarios. In terms of average throughput per-STA performance, the MA and MMF throughput criteria provide similar performance, which is better than that of the PF throughput criterion in both STA layout scenarios.

Conflict of Interest

There is no conflict of interest for this study.

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