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Channel Precoding for Compute and Forward Relaying in Two Way Relay Network Model

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Abstract: Physical layer Network Coding (PNC) is a promising strategy to improve the spectral efficiency in a relay-based wireless transmission. This paper employs lattice-based channel coding at the source nodes for better performance during Multiple Access Channel (MAC) phase transmission. The achievable transmission rate in various PNC relaying such as Decode and Forward (DF), Amplify and Forward (AF), and Compute and Forward (CF) for the Two Way Relay Network model (TWRN) is derived and simulated through MATLAB software and analyzed. The analysis shows that the rate performance of CF relaying outperforms other relaying techniques in the Additive White Gaussian Noise (AWGN) channel, but it degrades for the Rayleigh fading channel. To mitigate the fading effects, channel aware precoding technique is employed at source nodes, Bit Error Rate (BER) and Symbol Error Rate (SER) performance are analyzed for CF relaying, and Monte Carlo simulations are used to demonstrate the theoretical results. Channel precoding performance of CF relaying is compared with DF relaying.

Keywords: physical layer network coding, channel precoding, compute and forward, decode and forward, amplify and forward

1. Introduction

Relaying the information is needed when the source and destination are beyond a reachable distance, or they cannot exchange information directly due to obstacles or the power limitation of the communicating nodes. In earlier days, relays just forwarded the received message without modifying it. Later in wired transmission, the relays perform network coding operations in which the intermediate nodes forward the functions of the data that it has to forward. This network coding operation increases the network performance and avoids congestion by reducing the amount of data to be forwarded.

Physical layer Network Coding (PNC) is an effective technique in which the packets received from several source nodes are processed and coded at the intermediate relay nodes rather than the nodes at the ends or the destination. PNC improves the network throughput by reducing the data to be forwarded to the end nodes. PNC offers improved security as the individual source node's information is not decoded at the intermediate relay nodes. This technique is more powerful in dense interference-limited networks due to the efficient usage of orthogonal resources. The maximum reliable transmission rate over a channel is called as the achievable rate. The summation of the achievable rate at all the transmitting nodes in a multiuser environment is called as the sum rate.

Muralidharan et al. classified the physical layer network coding scenarios as removable and non-removable singular fade states based on the extent of removal of interference in MAC phase. Adaptively changing the

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network coding map at the relay node based on the channel conditions could reduce the ill effects of multiple access interference during the MAC phase in PNC [1]. Chen et al., in his article [2], proposed the generic lattice network coding in which the constraint on the structure of the nested lattice codes was relaxed and thus enabled various code design techniques. Linear labeling of the nested lattice code points provides compatibility between complex linear arithmetic operations performed by the wireless channel and the linear arithmetic operations performed in the message space for linear network coding. Hao et al. applied Tomlinson-Harashima precoding at the intermediate relay node in PNC TWRC. He proposed modulo-operator-based precoding [3] to reduce the amplitude of the transmitted signal at the relay node, which reduces the effective power consumption at the relay node with less reduction in performance.

Argyriou and Pandharipande implemented PNC for TWRN [4] and extended the implementation approach for practical distributed wireless networks. Long Shi and Soung Chang Liew [5] considered the multiway relay network in which the source nodes were assumed to transmit complex symbols, and the relay computed the linear function of the simultaneously received signals whose optimal coefficients were derived that minimize the symbol error rate. Nazer et al. [6] observed that the modulo-two sum function of the transmitted messages at the relay node could be replaced by a set of functions satisfying the "exclusive law of network coding". The application of lattice codes for Multi-terminal Gaussian channels was discussed by Ling et al. [7].

Aljohani et al. [8] stated that the structured lattice codes perform better in the fast-fading interference channel. Paulo Ricardo et al. [9] proposed the encoding and decoding schemes for multilevel binary low-density parity check codes whose complexity is linearly proportional to the number of coded bits. The clustering and mapping for real-valued Rayleigh fading channels were discussed by Huang et al. [10], which considered the linear combination of two source node's messages for Quadrature Phase Shift Keying (QPSK) and called it the linear PNC. Xu and Fu [11] studied the PNC scheme in which the relay node extracted the sum and the difference of the received signal and then converted the same into a network-coded symbol. Yong Gang et al. analyzed channel estimation in PNC and the effects of improper estimation [3].

The initial idea of network coding operation for multicast networks, which performed arithmetic coding on the digital bit streams, was proposed by Ahlswede et al. [12]. Popovski and Yomo [13] analyzed the transmission rate performance of various PNC relaying schemes such as Decode and Forward, Amplify and Forward, Compress and Forward, and Denoise and Forward. They referred DF Scheme as 3- a step scheme, and all others relaying as a 2-step scheme. In [14], the authors analyzed the performance of PNC, which employs QPSK modulation for (Internet of Things) IoT networks. The proposed model considered multiple antennas at the relay node. This paper deals with the transmit antenna selection scheme to achieve full diversity gain during the broadcast phase.

In [15], the authors have introduced the source compression using lattice vector quantization in PNC to improve the performance during MAC phase transmission. The error rate performance in Additive White Gaussian Noise (AWGN) and fading wireless atmosphere for Decode and Forward (DF) and Compute and Forward (CF) relaying techniques are derived analytically and verified through MATLAB simulation. A coefficient selection algorithm using an improved Schnorr – Euchner search strategy for decoding linear equations of messages at the intermediate node in PNC for CF relaying is proposed in [16], and the efficiency of the proposed algorithm is compared with other prevailing algorithms of optimal coefficient selection. The BER performance comparison of communication systems for network coding with relay selection and network coding without relay selection is derived and validated through simulation in [17]. In [18], two, three and four timeslot relaying schemes are compared and authors proved that opportunistic relaying could improve both sum-rate and BER performance.

Lattice codes for the two-way two-relay network with the novel concept of “re-distribution transform” for the maximum utilization of intermediate node transmission energy even during unbalanced channel circumstances is proposed in [19]. Lattice decoder with single lattice codebook is required to decode the summation of codewords at the relay nodes. Lattice codes with linear precoding at the base stations is proposed for multiuser multiple input multiple output two-way relay network model with cellular non- cooperative mobile stations [20]. Compress and forward relaying with layered decoding for the Gaussian two-way relay channel is described in [21]. Layered decoding is described as refinement layer is decoded only by the receivers with best channel conditions.

Based on the survey on physical layer network coding for relaying the information and structured lattice codes for improving the capacity of MAC channels, authors proposed the concept of applying the lattice codes in various PNC relaying schemes. The study reveals that CF relaying with lattice coding is superior in AWGN channel, but degrades for fading channel. The performance of CF relaying in fading environment is improved through precoding techniques at the transmitter.

In this work, we considered a two-way relay network model in which all the communicating nodes are equipped with a single antenna transmitting their messages using structured lattice codes. The main contributions of this research work are computing the maximum achievable rates of various PNC relaying techniques such as Decode and Forward (DF), Amplify and Forward (AF), and Compute and Forward (CF) with lattice encoding at the transmitting nodes under AWGN and Rayleigh fading channel. Channel inversion precoding and selective channel aware precoding techniques are applied at the transmitters to compensate for the channel effects in CF relaying. The achievable transmission rate and error rate performance with and without precoding has been demonstrated by Monte Carlo simulation.

This paper is organized as follows, section 2 describes channel coding techniques suitable for the MAC phase transmission, section 3 deals with the network model that we have considered for comparing the performance of various relaying schemes, and Section 4 explains how the performance of CF relaying can be improved by precoding technique, section 5 discusses the simulation conditions and results obtained in this research, and finally, section 6 concludes with the future scope.

2. Channel Coding for Multiple Access Channel

PNC transmission consists of two transmission phases, namely, the MAC phase and the Broadcast phase. The rate during the MAC phase limits the network's performance due to interference from multiple users. The rate during the BC phase is the same as Point to Point (P2P) link, since the link between the relay node and the end nodes is the same as the single-user channel. The channel's capacity in a multiuser wireless environment varies for different encoding techniques, such as independent, random, linear, and structured lattice codes.

Multiple access channel means several senders are transmitting their information simultaneously to a common destination. When the two senders send signals to a common destination through a Gaussian noise channel, the received signal can be written as $Y = X_1 + X_2 + Z$, where Z is the zero-mean Gaussian noise with variance N. If the power constraint of the two transmitting nodes is equal to P and x_{ij} is the codeword corresponding to the message set w_j of senders j = 1,2, and i is the time instances of the channel, then $\frac{1}{n}\sum_{i=1}^{n} x_{ij}^2\left(w_j\right) \leq P;\ w_j \in \left\{1,2,\ldots 2^{nR_j}\right\}$ where R_j is the transmission rate of the senders. The capacity region of Gaussian MAC, when the codewords (X_1, X_2) are independent and identically distributed, is

$$R_1 + R_2 \leq \frac{1}{2}\log\left(1+\frac{2P}{N}\right) \tag{1}$$

Depending on the orthogonal resources shared (time, frequency, code) between the senders, the capacity boundary varies and can be adaptively used to achieve maximum.

When the messages are independent and identically distributed and are the k-dimensional elements of the finite prime field $p\left(w_1, w_2 \in \boldsymbol{F}_p^k\right)$, then the rate at the senders as in [22,23] is

$$R_1 + R_2 \leq \log_2 p\left(w_j\right) - H\left(Z\right) \tag{2}$$

where $p(w_j)$, $H(Z)$ are the probability function of the finite prime field messages and entropy of noise respectively.

However, when the messages are encoded using random linear codes, where encoding is matrix multiplication, $x = Gw$ as given below, G is the generator matrix.

$$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} g_{11} & \cdots & g_{1k} \\ \vdots & \cdots & \vdots \\ g_{n1} & \cdots & g_{nk} \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_k \end{bmatrix} \tag{3}$$

The codewords x are length n-vectors over finite prime field $\boldsymbol{F}_p$, and all the addition and multiplication operations are mod-p operations over the real field. When the codes are linear, the superposition property ensures a linear combination of codewords to be a valid codeword in the code book; the rate region of the Gaussian multiple access channel is max (R_1, R_2) < $\log_2 p(w_j) - H(Z)$. Thus the capacity of the Gaussian MAC channel is increased two-fold for random linear codes.

When well-defined, geometrically structured linear codes such as lattice codes are employed for encoding the messages, then the MAC capacity can still be improved. A linear code C (n, k) over a finite prime field $\boldsymbol{F}_P$ can be converted into a lattice code by extending them over an integer domain by construction A method, $\wedge = x \in \boldsymbol{Z}^n$: x mod $p \in C$ (n, k). The correlation between the signals transmitted through the MAC channel is matched with the channel structure, and hence the computation rate can be improved, which is termed as the structural gain.

A lattice code is a sequence of n-dimensional symbols and a set of M codewords that forms the codebook. Each codeword is encoded using $\log_2 M$ bits. The codeword elements are usually drawn from the finite prime field and represented by $\boldsymbol{F}_p^n$. The codeword thus can be characterized as a point in an n-dimensional space whose coordinates are symbol values. For the structured lattice codes, the rate region of two user Gaussian MAC [22] is

$$R_1 = R_2 \leq \frac{1}{2}\log\left(\frac{1}{2}+\frac{P}{N}\right) \tag{4}$$

Structured linear codes for multiple access channels outperform conventional random coding schemes for decoding linear combinations of messages at the relay nodes in PNC relaying. As structured lattice codes for channel encoding have improved the region of MAC capacity, this method of lattice encoding is employed for analyzing the capacity of various relaying techniques in PNC.

2.1 *Encoding Using Construction A Lattices*

The primary requirement of lattice constellation is the isomorphism between the message elements of finite prime field $\boldsymbol{F}_p$ and lattice points in the complex integer domain. A two-dimensional lattice constellation that satisfies this constraint consists of points in the Gaussian or Eisenstein integer domain.

Gaussian integers are defined as $\boldsymbol{Z}[i] = \{a + bi: a, b \in \boldsymbol{Z}\}$. The prime ideals of Gaussian integers are those whose norms are prime numbers. For example, $1 + 2i$, $2 + 3i$, $4 + 5i$, and $5 + 6i$ are prime ideals of the Gaussian domain that will give the Quadrature Amplitude Modulation (QAM) constellation points with cardinality 5, 13, 41 and 61 respectively. The constellation points in the Eisenstein integer domain are represented by the equation $\boldsymbol{Z}[i] = \{a + b\omega: a, b \in Z, \omega = (-1+i\sqrt{3})/3\}$ [24]. The prime ideals of Eisenstein integer are $1+3\omega$, $2+5\omega$, $3+7\omega$, and $5+9\omega$, etc., whose norms are prime numbers and congruent to 1 mod 3, which give the constellation points size of 7, 19, 37, and 61, respectively.

The constellation mapping table (Table 1) for the lattice modulation in the Gaussian integer domain for 5-QAM using construction A lattices for the TWRN is given below:

Table 1. Constellation mapping table for 5-QAM

Index i	Message elements of source 1 (m_1)	Message elements of source 2(m_2)	$m_1 \oplus m_2$ mod-p addition	Codeword elements of source 1	Codeword elements of source 2	Codeword elements decoded at the relay node
0	0	0	0	0, 0	0, 0	(0, 0)
1	1	0	1	1, 0	0, 0	(1, 0)
2	2	0	2	0, 1	0, 0	(0, 1)
3	3	0	3	0, -1	0, 0	(0, -1)
4	4	0	4	-1, 0	0, 0	(-1, 0)
5	0	1	1	0, 0	1, 0	(1, 0)
6	1	1	2	1, 0	1, 0	(2, 0)
7	2	1	3	0, 1	1, 0	(1, 1)
8	3	1	4	0, -1	1, 0	(1, -1)
9	4	1	0	-1, 0	1, 0	(0, 0)
10	0	2	2	0, 0	0, 1	(0, 1)
11	1	2	3	1, 0	0, 1	(1, 1)
12	2	2	4	0, 1	0, 1	(0, 2)
13	3	2	0	0, -1	0, 1	(0, 0)
14	4	2	1	-1, 0	0, 1	(-1, 1)
15	0	3	3	0, 0	0, -1	(0, -1)
16	1	3	4	1, 0	0, -1	(1, -1)
17	2	3	0	0, 1	0, -1	(0, 0)
18	3	3	1	0, -1	0, -1	(0, -2)
19	4	3	2	-1, 0	0, -1	(-1, -1)
20	0	4	4	0, 0	-1, 0	(-1, 0)
21	1	4	0	1, 0	-1, 0	(0, 0)
22	2	4	1	0, 1	-1, 0	(-1, 1)
23	3	4	2	0, -1	-1, 0	(-1, -1)
24	4	4	3	-1, 0	-1, 0	(-2, 0)

The superimposed signals can be directly mapped to network-coded symbols using the isomorphic property between the message points and code points. Depending on the modulation order, the constellation may be constructed from Gaussian or Eisenstein integer domain. The modulation order in the Gaussian integer domain is $p = 2n\ (n + 1) + 1$, and in the Eisenstein integer domain, $p = 3n\ (n + 1) + 1$, n is any positive integer.

3. Rate Analysis in PNC for TWRN Model

In a two-way relay network, the two source nodes transmit their information via a relay node, depicted in Figure 1. The proposed model assumes the non-existence of direct communication link between sources 1 and 2. The nodes are assumed to have half-duplex transmission due to the practical difficulty of the wireless nodes to avoid the tough interference of their own transmit and receive signal.

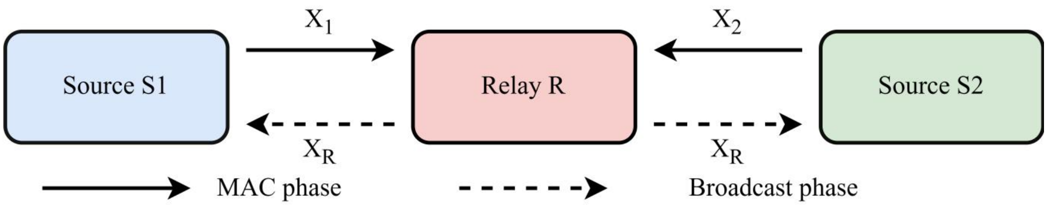

Figure 1. Two-way relay network model

In traditional scheduling, for exchanging single information between two sources, four time slots are required. Using network coding operation at the relay node, manipulating the broadcast nature of the wireless link, it takes three slots to transfer the same amount of information between the user nodes. PNC decreases the time slot into two by exploiting the superposition property of the electromagnetic signal in the air and the network coding operation at the intermediate node.

Source data from the two source nodes $i = 1, 2$, the elements of the finite prime field $m_i^L \in \boldsymbol{F}_P$, are mapped to complex n-dimensional codewords using the encoder such that $\varepsilon : \boldsymbol{F}_P \rightarrow \boldsymbol{C}^n$. The n-dimensional codewords are subjected to the average power constraint $\frac{1}{n}\sum \|X_i\|^2 \leq P$. The sources are concurrently transmitting their messages to the relay node during the MAC phase. Thus the received signal at the relay node is:

$$Y_R = h_{S1R}X_1 + h_{S2R}X_2 + Z \tag{5}$$

h_{S1R}, h_{S2R} are the complex-valued Rayleigh fading channel coefficients, and Z is additive white Gaussian noise. The fading channel coefficients and noise at the MAC stage make it challenging for the relay node to perturb the received overlapped packets. The decoding technique employed at the relay node affects the decision at the end nodes, based on the information transmitted during the BC phase. The received signal at the relay node during MAC and network-coded data to be broadcasted by the relay node for binary data transmission is given in Table 2.

Table 2. PNC mapping table for BPSK transmission

Binary data at Source $S1$	Binary data at Source $S2$	BPSK symbol mapper X_1	BPSK symbol mapper X_2	Received signal at relay node Y_R	Network coded data at relay R	BPSK mapped data X_R
0	0	-1	-1	-2	0	-1
0	1	-1	1	0	1	1
1	0	1	-1	0	1	1
1	1	1	1	+2	0	-1

During the Broadcast phase, the relay forwards the network-coded data to the source nodes. Let X_R represent the network-coded message at the relay node to be broadcast back to source nodes. The received signal at source nodes can be expressed as,

$$Y_{Si} = h_{RSi}X_R + z_{Si};\ i = 1,2 \tag{6}$$

Each source node decodes the message intended for it from the relayed network-coded data and its own information. The channel coefficients between the end nodes and the intermediate node are assumed to be equal during MAC and BC phases ($h_{SiR} = h_{RSi}$). The end-to-end maximum achievable sum rate using PNC relaying is given by, $R_{PNC} = R_{12} + R_{21}$,

R_{12} and R_{21} are the transmission rates at the source nodes [25] and are given by

$$R_{12}, R_{21} = \min\{R_{MAC}, R_{BC}\} \tag{7}$$

$$R_{12} + R_{21} = \frac{1}{2}\left(\min\left(R_{1R}, R_{R2}\right) + \min\left(R_{2R}, R_{R1}\right)\right) \tag{8}$$

R_{MAC} is the rate at which each of source nodes are transmitting to the relay node which depends on the PNC relaying scheme, and R_{BC} (R_{R1} and R_{R2}) is the rate at which the intermediate node forwards to the source nodes which is equal to the Shannon's information capacity for the P2P channel.

Depending on the processing performed at the relay node, PNC relaying is classified as Decode and Forward (DF), Amplify and Forward (AF), and Compute and Forward (CF). This research compares the achievable rates in the above relaying techniques of PNC and channel inversion techniques are applied to improve the transmission rate under Rayleigh fading channel.

3.1 *Decode and Forward Relaying*

In Decode and Forward (DF) PNC relaying, the relay decodes the individual source node's message and reencodes using a network coding operation. $S1$ and $S2$ communicate concurrently to relay node during the MAC phase, and relay broadcasts the XOR of the decoded message to source nodes, during BC phase. The relay node uses the Successive Interference Cancellation (SIC) procedure for extracting each source node's information.

In DF relaying, though the intermediate relay node is not interested in the individual source node's message, it detects it before forwarding. Hence, the scheme is also called Detect and Forward. In DF relaying, the achievable sum rate is limited by the MAC interference and the type of detection technique at the relay node since decoding at the user nodes $S1$ and $S2$ has the upper bound capacity. The received signal at the relay node during the MAC phase is,

$$Y_R = h_{S1R} X_1 + h_{S2R} X_2 + Z \tag{9}$$

When the channel from $S1$ to R is better, then the transmission rate is given as in [25],

$$R_{1R} = \frac{1}{2}\log\left(1 + \frac{P\left|h_{S1R}\right|^2}{P\left|h_{S2R}\right|^2 + N}\right) \tag{10}$$

$$R_{2R} = \frac{1}{2}\log\left(1 + \frac{P\left|h_{S2R}\right|^2}{N}\right) \tag{11}$$

However, when the channel from $S2$ to R is better, then the transmission rate is given as,

$$R_{2R} = \frac{1}{2}\log\left(1 + \frac{P\left|h_{S2R}\right|^2}{P\left|h_{S1R}\right|^2 + N}\right) \tag{12}$$

$$R_{1R} = \frac{1}{2}\log\left(1 + \frac{P\left|h_{S1R}\right|^2}{N}\right) \tag{13}$$

Upon receiving the signals from the source nodes during the MAC phase, the transmission rate at the relay node during the broadcast phase is,

$$Y_{Si} = h_{RSi} X_R + Z_{Si};\ i = 1, 2 \tag{14}$$

$$R_{Ri} = \frac{1}{2}\log\left(1 + \min\left\{\left|h_{RS1}\right|^2, \left|h_{RS2}\right|^2\right\}\frac{P}{N}\right) \tag{15}$$

Then achievable sum rate in DF relaying for the Rayleigh fading channel can be calculated using equation (7) as

$$R_{DF\,Rayl} = \frac{1}{2}\left(\min\left\{R_{1R}, R_{R2}\right\} + \min\left\{R_{2R}, R_{R1}\right\}\right) \tag{16}$$

Similarly, for the AWGN channel, the sum rate in DF relaying assuming all the fading coefficients equated to 1 is given as

$$R_{DF\ AWGN} = \frac{1}{2}\log\left(1+\frac{2P}{N}\right) \tag{17}$$

Though the intermediate relay is not interested in individual source nodes' messages, the DF relay decodes individual users' messages and then reencodes using network coding. Hence the rate is limited to achievable MAC capacity region.

3.2 *Amplify and Forward Relaying*

In Amplify and Forward (AF) PNC relaying, the relay scales the received signal by an optimal scaling factor and broadcasts the amplified version of it to the source nodes. The relay node acts as a repeater and passes the received signal even if it cannot decode. However, the noise also gets amplified along with the desired signal. This relaying, where the relay node amplifies the received noisy signal, is suitable only for an error-free environment. All the decoding and signal processing operations are performed at the end nodes. The power of the received signal at the relay node during the MAC phase is given by, $P(|h_{S1R}|^2+|h_{S2R}|^2) + Z$. Since all the nodes are assumed to have equal power constraint P, the received SNR has to be scaled by a factor α given as,

$$\alpha = \sqrt{\frac{P}{P\left(|h_{S1R}|^2+|h_{S2R}|^2\right)+N}} \tag{18}$$

Hence the received signal at source nodes during the BC phase is,

$$Y_{Si} = \alpha h_{RSi}\left(h_{S1R}X_1 + h_{S2R}X_2 + Z\right);\ i = 1,2 \tag{19}$$

The SNR at source nodes $S1$ and $S2$ for the received signal from Equation 19 is given as,

$$SNR_1 = \frac{\alpha^2 P|h_{RS1}h_{S2R}|^2}{\alpha^2 N|h_{S1R}|^2 + N} \tag{20}$$

$$SNR_2 = \frac{\alpha^2 P|h_{RS2}h_{S1R}|^2}{\alpha^2 N|h_{S2R}|^2 + N} \tag{21}$$

The achievable sum rate by AF relaying in Rayleigh fading channel environment using equation 7 is given as,

$$R_{AF\ Ray} = \frac{1}{2}\left(\log\left(1+SNR_1\right)+\log\left(1+SNR_2\right)\right) \tag{22}$$

Similarly, the sum rate for the AWGN channel assuming fading coefficients as one is

$$R_{AFAWGN} = 2\left(\log\left(\frac{P}{N}\cdot\frac{P}{3P+N}\right)\right) \tag{23}$$

3.3 *Compute and Forward Relaying*

Compute and Forward relaying harnesses the interference by letting several intermediate relay nodes to compute the independent linear combination of source messages from the overlapped signal received from several source nodes. The relay node in compute-and-forward relaying exploits the natural linear computation of the wireless channel. It decodes the integer linear equation of messages instead of decoding individual messages. For the TWRN, two source nodes transmit its lattice-coded message to a central relay node. The source nodes can extract the other node's message after receiving enough combined signals. The received signal Y_{R1} at the relay node is given by,

$$Y_{R1} = h_{S1R1}X_1 + h_{S2R1}X_2 + Z \tag{24}$$

h_{S1R1} and h_{S2R1} are the complex channel coefficients, and Z is the additive white Gaussian noise. The relay computes:

$$X_{R1} = a_1X_1 + a_2X_2 \tag{25}$$

a_1, a_2 are the elements from $\boldsymbol{Z}[i]$. The choice of integers a_1 and a_2 depend on the fading factors h_{S1R1} and h_{S2R1}, respectively. The lattice property of the codes guarantees that any integer linear combination of codewords is a valid codeword. But the wireless medium produces a linear combination with complex valued coefficients.

The relay $R1$ scales the received channel output by an optimal scaling factor α_{MMSE} so that the scaled version of the received signal is closer to the integer linear combination of codewords. By choosing α to be a minimum mean square error coefficient, the computation rate can be maximized, and the α_{MMSE} is given as,

$$\alpha_{MMSE} = \frac{a h_1{}^H P}{1 + P\|h_1\|^2} \tag{26}$$

$h_1 = \begin{pmatrix} h_{S1R1} \\ h_{S2R1} \end{pmatrix}$ and P is the transmission power and α is chosen to minimize the error due to the approximation of complex-valued channel coefficients as integers.

During the second time slot, the relay transmits X_{R1} with a_1 and a_2 inserted in the preambles of the packets. Since the source nodes know their own data, it gets excluded to retrieve the other user's information. The received signal at the source node $S1$ can be written as,

$$Y_{S1} = h_{R1S1} X_{R1} + h_{R2S1} X_{R2} + Z \tag{27}$$

The achievable sum rate in CF relaying under Rayleigh fading channel environment with estimated optimal equation coefficients at the relay node,

$$R_{CFRay} = \frac{1}{2}\log\left(\frac{P}{|\alpha|^2 + p\|\alpha h_1 - a\|^2}\right) \tag{28}$$

The achievable sum rate in C-F relaying for the AWGN channel is

$$R_{CFAWGN} = 2\log\left(\frac{1}{2} + \frac{P}{N}\right) \tag{29}$$

4. Channel Precoding in PNC for Fading Channel

4.1 *Channel Inversion Precoding*

The performance degradation due to fading in TWRN in CF relaying at the intermediate relay node can be compensated completely by employing the channel inversion precoding at the source nodes.

Fading reduces the minimum distance between the network-coded symbols, and fading correction can be introduced at the source nodes. The water-filling algorithm at the source nodes in which transmission power is altered for transmission rate adaptation is efficient for correlated fading. However, the water-filling technique only significantly improves independent and identically distributed (iid) fading capacity. For iid channels, non-power-constrained channel inversion precoding at the source nodes can turn the fading channel to the AWGN channel. Encoding and decoding circuitry are simpler, and processing speed is greater, as the nodes are always transmitting at constant rates independent of fading states. Channel inversion cannot be applied at the relay node for the BC phase in PNC relaying, as the relay node cannot precode for the two different channel instances to the two source nodes.

A traditional training-based channel estimation technique, along with exploring the redundant information of network coding transmissions during the broadcast phase, is used to apply the precoding. The sources send the training packets, and the relay node estimates the channel and sends it back to the user nodes. Knowing the CSI, the source nodes precode the symbols in their next transmission, assuming a static, slow fading channel in MAC and BC phases. Source nodes can have their instant transmit power very high to avoid saturation. Source $S1$ transmits $\frac{x_1}{h_{S1R}}$, and source $S2$ transmits $\frac{x_2}{h_{S2R}}$. The transmission power of the nodes has been changed from P to $\frac{P}{|h_{S1R}|^2}$ for $S1$ and $\frac{P}{|h_{S2R}|^2}$ for $S2$. Thus, the fading channel has been turned into an AWGN channel at the cost of high saturation power as a limitation at the source nodes. There will be an additional increase in transmission power at the two source nodes, and let the average increase in transmission power μ be represented as

$$\mu = \frac{P/|h_{S1R}|^2 + P/|h_{S2R}|^2}{2P} = \frac{|h_{S1R}|^2 + |h_{S2R}|^2}{2|h_{S1R}|^2|h_{S2R}|^2} \tag{30}$$

Even when the channels are symmetric from sources to relay, the relay cannot precode during the broadcast phase, and hence the average achievable sum rate is restricted by the BC phase transmission. When CSI information is used and precoding is perfect, the MAC phase transmission rate for fading channel is equal to the AWGN rate. The increase in average transmission power from P to μP makes the received power at the relay P.

The increase in rate performance for CF relaying after precoding is due to the elimination of error due to the Diophantine approximation trade-off between the MMSE scaling factor and channel noise amplification. However, the transmitting nodes have power limitations and cannot be arbitrarily high. Therefore when the channel is in a deep fade, precoding cannot be used efficiently, and we proposed a selective channel precoding technique.

4.2 *Selective Channel Precoding*

In selective channel precoding, channel inversion is performed for MAC phase transmission in TWRN between source nodes to relay node only if the precoded powers $\frac{P}{|h_{S1R}|^2}$ and $\frac{P}{|h_{S2R}|^2}$ are less than a certain fixed threshold power P_{th}. When the channel between any sources to the relay node is in the deep fade, then that source node stops transmission. The other node transmits after precoding, considering the link as a single-user link, and PNC is not employed in transmission. If both the channels are in a deep fade, both source nodes stop transmissions, thus saving the transmission power.

5. Simulation Results and Discussion

In the simulation, an ideal frame and symbol synchronization has been assumed, and the channel is believed to be slowly fading and symmetric channel. Source nodes in the network employ lattice encoding to encode their messages. The average achievable sum-rate performance has been simulated for all the relaying schemes discussed above under the AWGN and Rayleigh fading channel environment for 10,000 instances. All the nodes in the network are transmitting at the same power level ($P_1 = P_2 = P_R = P$), and the noise is circularly symmetric with the Gaussian distribution whose variance is N.

The simulation parameters considered in the proposed network model to analyze the performance of PNC relaying schemes are shown in Table 3.

Table 3. Simulation parameters

Parameter	Value
Number of channel instances	10000
Transmit power	10dBm
Noise power	{-100, -70} dBm
Noise power spectral density	-174dBm/Hz
Channel coding technique	Lattice encoding of 5-QAM

Figure 2 shows the transmission rate performance for AWGN channel for various PNC relaying techniques. The noise power is varied for simulated channel instances and the average SNR is calculated for plotting the achievable rate for different PNC relaying techniques. The performance graph shows that for the AWGN channel, CF relaying scheme outperforms both DF and AF relaying. From the simulation results, we inferred that in the lower SNR region, CF relaying performs almost similar as DF relaying technique. But, in the high SNR region, Compute and forward relaying considerably achieves the upper bound. Beyond the SNR of 2dB, the sum rate starts to improve in CF, and CF relaying gives almost seven bits/s/Hz, whereas it is only 3.85 bits/s/Hz in DF relaying at 20dB SNR. In DF relaying, though the intermediate relay node is not interested in individual source node's data, it decodes before forwarding the network-coded data. In AF relaying, the relay node amplifies the received superimposed signal along with noise; hence, this scheme cannot be applied to a noisy environment. As the relay node is not interested in individual source nodes messages, in CF relaying the linear combination of messages are decoded from the received overlapped signal, thus eliminating the drawbacks of DF and AF relaying.

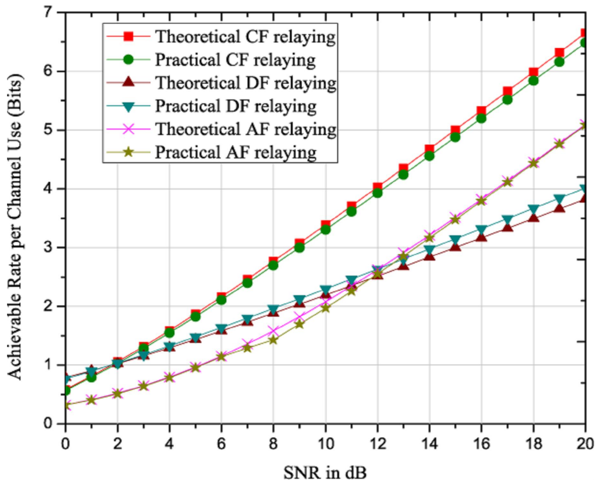

Figure 2. Achievable rate in AWGN channel

Figure 3 shows the comparison of the transmission rate for various PNC relaying techniques for the Rayleigh fading environment. AF relaying performs well only at a high SNR regime because the noise amplitude is greater near a low SNR regime, which gets amplified by the scaling factor. The relay node in AF relaying doesn't decode and thus retransmits the enhanced version of the signal that it has received during the BC phase. The transmission rate is greater in AF in comparison with CF relaying at a high SNR regime due to less noise effect in this region.

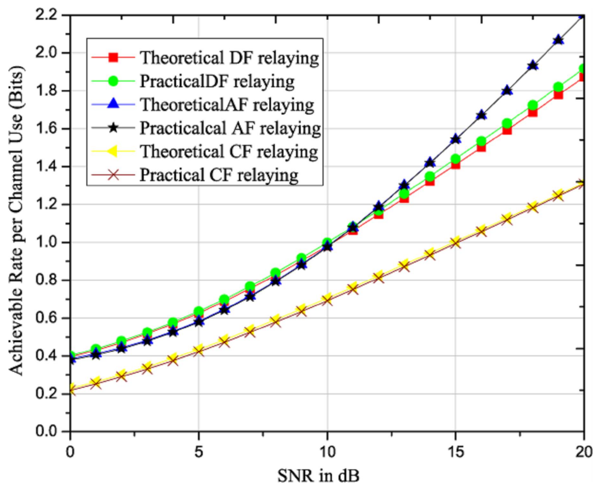

Figure 3. Achievable rate in Rayleigh fading channel

The reason for decrease in the performance for CF relaying is due to channel estimation errors and equation coefficient approximation errors. In CF relaying, additional noise is introduced due to the integer approximation of complex valued channel coefficients while decoding the independent linear equation of messages. The approximation is suitable when using the one-dimensional constellations such as Pulse Amplitude Modulation (PAM) or BPSK. However, the sum rate can be improved by adapting channel precoding techniques at the transmitting nodes.

CF relaying performance approaches near to upper bound for AWGN channel. Still, for fading environment, the rate of this relaying technique is lowered, which can be improved by channel inversion precoding at the source nodes. The effect of the fading channel can be nullified, and the channel can be transformed to behave as an AWGN channel but at the cost of unrestricted power constraint. Channel-aware precoding is a smarter technology, and it performs precoding at the source nodes only when the fading level is within a certain threshold. When the channel is in a deep fade, both sources stop their transmission, and when one of the channels is good, that source node alone transmits after implementing the channel inversion precoding.

The source nodes in channel aware precoding fixed the threshold power as P_{th}, and the nodes perform channel inversion only if the power is above the threshold power. Thus, precoding techniques for fading channels in CF relaying achieve a higher rate, as shown in Figure 4.

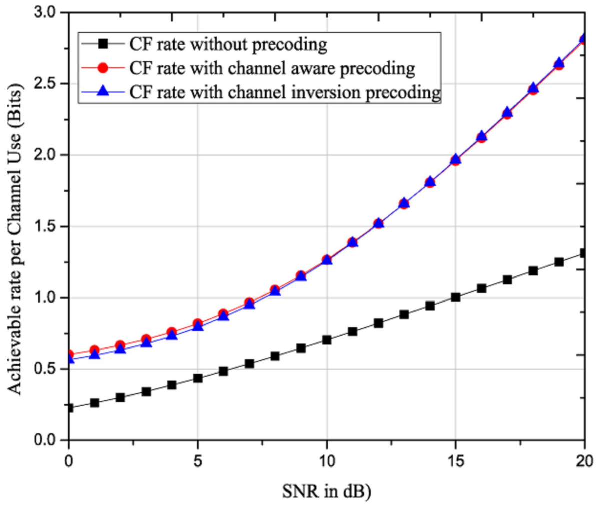

Figure 4. Achievable rate of CF relaying with precoding

Lattice encoding using construction A(Z^2) lattice of 5-QAM, as shown in Table 1, is employed at source nodes, and lattice decoding is used at the relay node to decode the integer linear combination of messages from the two source nodes. The symbol error rate performance at the relay node to decode the linear equation of messages is plotted in Figure 5.

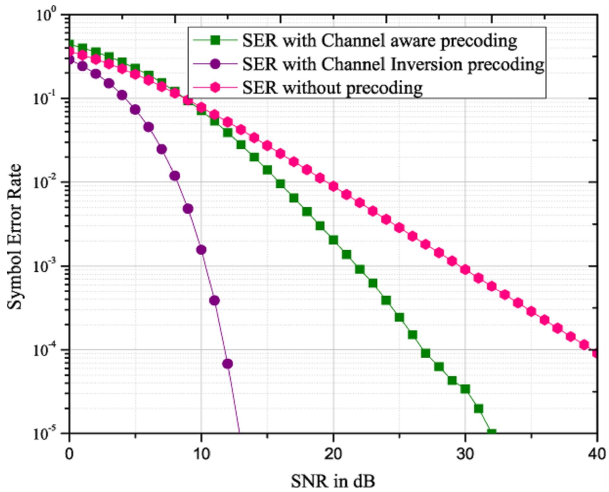

Figure 5. Error rate in CF relaying with and without precoding

6. Conclusion and Future Work

In this paper, the authors used lattice encoding for transmitting the messages in PNC transmission. The transmission rate performance of DF, AF and CF relaying for AWGN and Rayleigh fading channel for the TWRN model is reviewed. CF relaying is found to perform better under AWGN environment, but degradation in sum-rate performance occurs for fading environment. Hence for CF relaying, to mitigate the effects of fading, channel inversion techniques at source nodes are introduced, and its performance analyzed. With precoding, the transmission rate in CF relaying is improved to 2.8 bits per channel use at the SNR of 20 dB, whereas the rate remains only at 1.26 bits without precoding. To confirm the improved performance by precoding, the symbol error rate performance is also simulated and compared for CF relaying with and without precoding.

The MAC phase transmission limits the transmission rate in PNC relaying schemes due to multiuser interference. The authors applied structured lattice codes for encoding the messages. For improving the sum rate performance in various PNC relaying techniques, the transmission rate from source nodes can be reduced by adopting specific source compression techniques, such as optimum vector quantization based on the distribution of source samples. Also, for optimizing the performance of the network, optimal power allocation can be considered to achieve the desired BER.

Conflict of Interest

There is no conflict of interest for this study.

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