Computer Networks and Communications
https://ojs.wiserpub.com/index.php/CNC

UNIVERSAL WISER PUBLISHER

Research Article

A New Family of Window Functions Based on Partially Tapered Arrays

Esraa Ammar Mohammed, Jafar Ramadhan Mohammed*

Department of Communication Engineering, College of Electronic Engineering, Ninevah University, Mosul, 41002, IRAQ
E-mail: jafar.mohammed@uoninevah.edu.iq

Received: 26 February 2025; **Revised:** 25 March 2025; **Accepted:** 1 April 2025

Abstract: Conventional standard tapered window functions have been effectively employed in various areas including signal processing and communication applications. Each of these conventional tapered windows has its own advantages and disadvantages regarding efficiency, practical implementation, and far-field radiation performances in terms of directivity, sidelobe level, and half power beam width. This paper first presents a concise overview of several standard window functions, highlighting their main radiation characteristics and design parameters. Then, a new family of window functions called partially tapered windows for antenna array beamforming applications is proposed. The proposed partially tapered windows use only a subset of all the element excitations to control the array radiation pattern characteristics. The central array elements have amplitudes of ones and phases of zeros, while the side element excitations are tapered according to an original used window function. It is found that the proposed tapered arrays are capable to achieve better radiation characteristics than those of the conventional standard windows. Simulation results are presented to validate the proposed tapered arrays.

***Keywords*:** antenna array, window functions, partially tapered arrays

1. Introduction

In antenna array synthesis, generating far-field radiation patterns with improved performance in terms of array gain, directivity, and sidelobe levels is crucial. Window functions with tapered amplitudes are an excellent way to achieve these goals. Generally, window functions are commonly utilized in a number of practical engineering fields such as signal processing, array beamforming, and spectral estimation. Specifically, they have played a significant role in the spectrum prediction [1, 2], digital filter design [2, 3], speech processing [4] and many other fields. There are numerous works in the literature about these conventional window functions and their comparisons. Harris [5] provided a comprehensive overview of much windows functionality such as rectangular, triangular, Hann, Hamming, Dolph, Taylor, and many other windows. Geckinli and Yavuz [6] provided a brief tutorial comparing typical windows. Recent developments on this topic have been presented in [7, 8]. In general, the literature lists 46 common window functions. Some window functions, like triangle and trigonometric functions, are straightforward and easy to implement and design, while some others are not. The most effective approach of designing tapered window functions is by using the Fourier transform method [9] where the far-field radiation patterns can be designed with arbitrary shapes. Some other complex window functions, such as the Dolph-Chebyshev [10, 11], Kaiser-Bessel [12, 13] and other window functions are optimized to get best trade-off between directivity and their peak sidelobe levels [14, 15, 16].

DOI: https://doi.org/10.37256/cnc.3120256636

In this paper, partially tapered arrays are used instead of conventional fully tapered arrays to improve the radiation performances of the array beamforming. First, some original types of window functions have been reviewed and their performances such as directivity, half power beam width and sidelobe levels are analyzed. It is found that they are generally can reduce the sidelobe level with fully tapered windows at the cost of decaying the directivity and the gain [17, 18, 19]. To solve this important problem, we have suggested partially tapered windows to reduce the undesirable sidelobe levels and at the same time to maintain the desirable directivity undistorted. The proposed partially tapered arrays have some mixed properties of the rectangular uniform taper and non-uniform taper window by fixing the magnitudes of some central-top elements at unit amplitudes, while the magnitudes of the two side elements decays according to a specific non-uniform tapered function.

The outline of this paper is as follows; in Section 2 an overview about most commonly used fully tapered windows is illustrated. In Section 3, the proposed partially tapered methods are presented, while Section 4 includes the simulation results. Finally, Section 5 draws the main conclusions of the work.

2. Standard fully tapered windows

The large number of window functions is available in the literature which they are all characterized as fully amplitude tapers. In this section, some commonly used window functions that are using fully tapered elements are illustrated.

2.1 *Rectangular uniform window*

This is the simplest window because it depends on truncating the incoming data results in a rectangular window. It is characterized as providing a maximum directivity and narrower alf power beam width at the cost of highest undesirable side lobes. The window taper function can be determined using Equation (1) as follows:

$$w(n) = 1, \qquad \text{for } \ 0 \le |n| \le \frac{N}{2}. \tag{1}$$

where $n = 1, 2, \ldots, N$ are the array elements and $w(n)$ is the tapered amplitudes. The Rectangular window taper function is the reference one that compared to all other window functions.

2.2 *Triangular window*

To create a $(N+1)$-length triangular window, linearly convolve two rectangular windows of length $\frac{N}{2}$. This window's transform is plainly the square of the Dirichlet kernel. This window's self-convolution makes it the simplest of those having a nonnegative Fourier transform. The functional form is as given in Equation (2):

$$w(n) = 1 - \frac{2n}{N}, \quad \text{for} \quad 0 \le |n| \le \frac{N}{2}. \tag{2}$$

2.3 *The hanning window*

The amplitude tapers of the Hann window is given by

$$W(n) = 0.5 + 0.5\cos\frac{2\pi n}{N}, \quad \text{for} \quad 0 \le |n| \le \frac{N}{2}. \tag{3}$$

2.4 *The hamming window*

The Hamming window is described as a specific member of the family of what called raised cosine functions. Specifically, with the Hamming window, the peak sidelobe level can be greatly reduced at the cost of wider beam width and reduced directivity

$$w(n) = 0.54 + 0.46 \cos\frac{2\pi n}{N}, \quad \text{for} \quad 0 \leq |n| \leq \frac{N}{2} \tag{4}$$

2.5 *Gaussian window*

We are aware that a Gaussian time function has a Gaussian frequency response. To utilize a Gaussian function as a window function, its time span must be trimmed at the ends because it is infinite. The form of this window is as follows:

$$w(n) = e^{-\frac{1}{2}\left[\beta \frac{n}{N/2}\right]^2}, \quad \text{for} \quad 0 \leq |n| \leq \frac{N}{2}. \tag{5}$$

Generally, the β values are 2 , 3, and 3.5. It should be observed that as we increase the value of β, the width of the window decreases, which reduces the severity of the discontinuities at the margins. However, this will raise the main-lobe width, and hence reduce the side-lobe levels, in the transform domain.

2.6 *Kaiser window*

This window approximates between the maximum energy in the main beam and the leakage energy in the sidelobe regions. Kaiser proposes an estimate based on computing the zero-order modified Bessel function of the first kind rather than attempting to calculate the prelate spherical wave function. The equation for Kaiser Window can be written as:

$$w(n) = \frac{Io\left[\pi\beta\sqrt{1-\left(\frac{2n}{N}\right)^2}\right]}{Io[\pi\beta]}, \quad \text{for} \quad 0 \leq |n| \leq \frac{N}{2} \tag{6}$$

2.7 *The Dolph-Chebyshev window*

The Dolph-Chebyshev window aims to obtain a narrowest main beam. It stands out from the other windows in a variety of ways. Unlike more usual window taper functions, this window is defined by the equation below:

$$w(n) = (-1)^n \frac{\cos\left[N\cos^{-1}\left\{\beta\cos\left(\pi\frac{n}{N}\right)\right\}\right]}{\cosh\left[N\cosh^{-1}(\beta)\right]}, \text{for } 0 \leq |n| \leq N-1. \tag{7}$$

where

$$\beta = \cosh[1/N \cosh^{-1}(10^{\alpha})]\alpha = \text{main} - \text{lobe to side} - \text{lobe}$$

This window has the lowest sidelobe level which can be controlled at the expense of increase the half power beam width.

2.8 *The Taylor window*

The Taylor window taper function corresponds to the Dolph-Chebyshev window's constant sidelobe level for an identified number of near-in sidelobes, however then enables a taper beyond this [16]. The Taylor window function is determined as follows:

$$w(n) = \left(\frac{1+\beta}{2}\right) + \left(\frac{1-\beta}{2}\right)\cos\frac{2\pi n}{N}, |n| \leq \frac{N}{2}. \tag{8}$$

As can be seen, in general, all of these fully tapered window functions can be used to reduce the peak sidelobe levels, but, they are come at the cost of lower directivities. This problem is overcome with the proposed partially tapered windows where a better compromise between sidelobe levels and directivity can be obtained as elaborated in the follows section.

3. The proposed partially tapered windows

In this part, the rectangular uniform window is combined with other types to create new tapered windows where they have unit magnitudes at the top-center region and decayed magnitudes at the sides. This new configuration provides acceptable tradeoff between sidelobe reduction and directivity degradation. A certain proportion of the central elements can be chosen and set their amplitude excitations to ones, while the side elements keep their original amplitudes according to the used window.

In other words, the proposed partially tapered window uses only a subset of all the element excitations to control the array far-field radiation pattern. The central-top array elements have amplitudes of ones and phases of zeros. Side element excitations are tapered using an appropriate window function or a specific numerical approach. The radiation pattern of the proposed partially tapered array can be written as

$$AF(u) = \sum_{n=1}^{N_{\text{side1}}} w_n e^{jkx_n u} + \sum_{n=N_{\text{side1}}+1}^{N_{\text{side1}}+N_{\text{cen}}} e^{jkx_n u} + \sum_{n=N_{\text{side1}}+N_{\text{cen}}+1}^{N_{\text{cen}}+N_{\text{side1}}+N_{\text{side2}}} w_n e^{jkx_n u} \tag{9}$$

where N_{cen} is the number of unity-amplitude excitation elements in the central-top and N_{side1} and N_{side2} are the number of tapered side elements. The central-top Fourier weights are all equal to one. If the antenna array has a symmetrical even number of equally spaced elements, then $N_{\text{side1}} = N_{\text{side2}} = \frac{N-N_{\text{cen}}}{2}$ and the resultant array radiation pattern is given by

$$AF(u) = 2\sum_{n=N_{\text{cen}}/2+1}^{N} w_n \cos(kx_n u) + \frac{\sin[\frac{N_{\text{cen}}}{2}\frac{\psi}{2}]}{\sin(\frac{\psi}{2})} \tag{10}$$

The amplitude excitations of the side elements N_{side} in the above equation can be computed either according to the original tapered windows or by using least squares approaches. An optimization algorithm can be also use to find these values in an optimum sense.

4. Simulation results and discussions

In all examples, we considered an array consists of 10 elements with inter element spacing equal to $d = \lambda/2$. In the first example, the original standard fully taper windows such as rectangular, triangular, hamming, hanning, Kaiser, Gaussian, Dolph-Chebyshev, and Taylor were considered. The performance measures in terms of peak sidelobe level, half power beam width, leakage power, directivity, dynamic range ratio, and the array complexity of these tapered windows are computed. Table 1 shows these performance measures of the original fully tapered windows.

From Table 1, it can be seen that the conventional rectangular window has highest complexity reduction of 100% since all the element excitation amplitudes are one and zero phases. It is also has the highest peak SLL. The other tapered windows have low peak SLL and relatively low complexity reduction since their amplitude excitations are non-uniform. Thus, to implement these tapered windows, each element needs a separate attenuator and phase shifter. Another advantage of the rectangular window is its unity dynamic range ratio while all other tapered windows have lower values. Figure 1 shows the radiation patterns of the standard fully tapered windows. It can be seen from this figure that the rectangular window has narrower main beamwidth and highest directivity whereas all other windows have wider half power beamwidths and lower directivities. The leakage power of the rectangular window is highest among all other windows.

Table 1. Performance measures of the standard fully tapered arrays

The method	HPBW (degree)	Leakage power (dB)	Peak SLL (dB)	Directivity (dB)	DRR	Complexity reduction (%)
Rectangular	10.2703	−9.7743	−12.9653	9.998	1	100
Hamming	16.036	−15.8719	−35.8053	6.8104	12.1532	0
Hann	15.6757	−15.5189	−34.0208	6.9983	9.7286	0
Kaiser	13.1532	−16.4543	−27.2636	8.3132	4.808	0
Gaussian	14.5946	−16.7813	−34.6614	7.5913	7.2088	0
Dolph-Chebyshev	13.1532	−15.4211	−29.9996	8.4715	3.883	20
Taylor	12.7928	−15.0559	−29.6872	8.5328	3.6936	0
Triangular	14.5946	−16.9128	−28.3125	7.4915	10	20

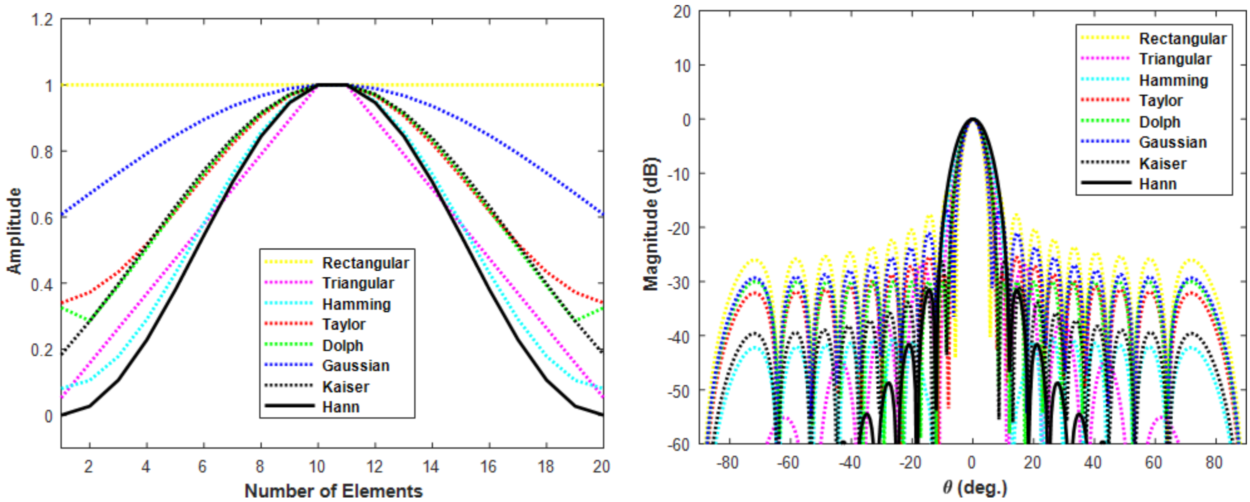

Figure 1. The radiation patterns (left) and the amplitude excitations (right) of the standard fully tapered arrays

In the second example, the partially tapered arrays that have been presented in Section 3 are considered. The proposed partially tapered windows use only a subset of all the element excitations to control the array far-field radiation pattern. The central array elements have amplitudes of ones and phases of zeros, while the side element excitations are tapered according to the original window function. As previous example, we used the same performance measure and compared them. Table 2 shows the results of performance measures of the proposed partially tapered arrays, while Figure 2 shows their far-field radiation patterns. It can be seen that by designing a new taper window with fixed values of one at the central elements similar to the rectangular taper window and edge elements similar to that of the original taper window good results can be obtained.

After applying the proposed strategy, it was noted that when making 60% of the central elements unity, the directivity can be improved while the peak SLL and the leakage power were slightly changed with compared to those of the original fully tapered arrays. From Table 2, it can be seen that the HPBW value of the partially Hamming window is 14.955 degrees, while the original Hamming window was 16.036 degrees, indicating that the HBPW has been decreased, which is a good indicator because the HPBW is directly proportional to the directivity and it leads to increase it by a value at least 0.2 dB. More important the complexity reduction of the partially hamming window is increased from 0 to about 60%. Similarly, the directivity of the Hanning window increases by 0.16 dB when 60% of the central elements are selected to be uniform like the rectangular window and the edge elements like the original Hanning window. Nevertheless, any other percentage of the central elements can be used. In general, it is noticed that the radiation performance of the partially tapered arrays become closer to that of the standard rectangular array when increasing the number of central elements. Also, when reducing the number of central elements, the radiation performances of the partially tapered arrays become closer to those of the fully tapered arrays. Thus, this design parameter controls the radiation characteristics and gives a best compromise between peak sidelobe level reduction and directivity loss.

The Kaiser window increases peak side lobe level and leakage power by 9.5 dB and 4.5 dB, respectively. However, directivity decreases by 0.12, and there is no noticeable improvement in the taper window's specifications. The modification will have no major effect on the Gaussian window because the increase in directivity is only 0.04 dB. However, it will have a considerable effect on the peak side lobe level and leakage power, boosting their values while decreasing complexity by 60%.

The value of directivity will reduce by approximately 0.12 dB for the Dolph-Chebyshev window, affecting the value of the peak side lobe level, which will become -18.4604 from its previous value of -29.999 dB, and the value of the leakage power, which will become -13.0216 dB

Finally for the triangular window, the HPBW value will reduce to 14.2324 degrees, resulting in an increase in directivity to 7.53 dB with an increase in the complexity reduction as well.

Table 2. Performance measures of the proposed partially tapered arrays

The method	HPBW (degree)	Leakage power (dB)	Peak SLL (dB)	Directivity (dB)	DRR	Complexity reduction (%)
Rectangular	10.2703	−9.7743	−12.9653	9.998	1	100
Hamming	14.955	−9.9696	−15.2808	7.0202	12.5	60
Hann	14.955	−10.1147	−15.7624	7.1581	10	60
Kaiser	13.1532	−11.9636	−17.7016	8.1919	4.8808	60
Gaussian	14.2342	−10.8539	−16.8117	7.5875	7.3891	60
Dolph-Chebyshev	13.1532	−11.8297	−18.4604	8.3636	3.883	60
Taylor	11.7117	−13.0216	−16.9524	9.2156	2.4083	60
Triangular	14.2342	−11.1281	−16.1887	7.5294	10	60

In the third example, the effect of changing the number of central elements has been investigated. Table 3 shows the results of 0% which is corresponds to the conventional fully tapered window, 20%, 40%, 60%, 80% (which are 4 different cases of the proposed partially tapered arrays), and 100% which is corresponds to the standard fully rectangular tapered window where all of its element amplitudes were unity. Generally, we can see that when the number of central elements is chosen to be small, the partially tapered window acts like the original fully tapered window. On the other hand, it becomes most likely like a standard rectangular window with all amplitudes at one value when increasing the number of central elements. Thus, this approach provides enough flexibility to achieve the required performance.

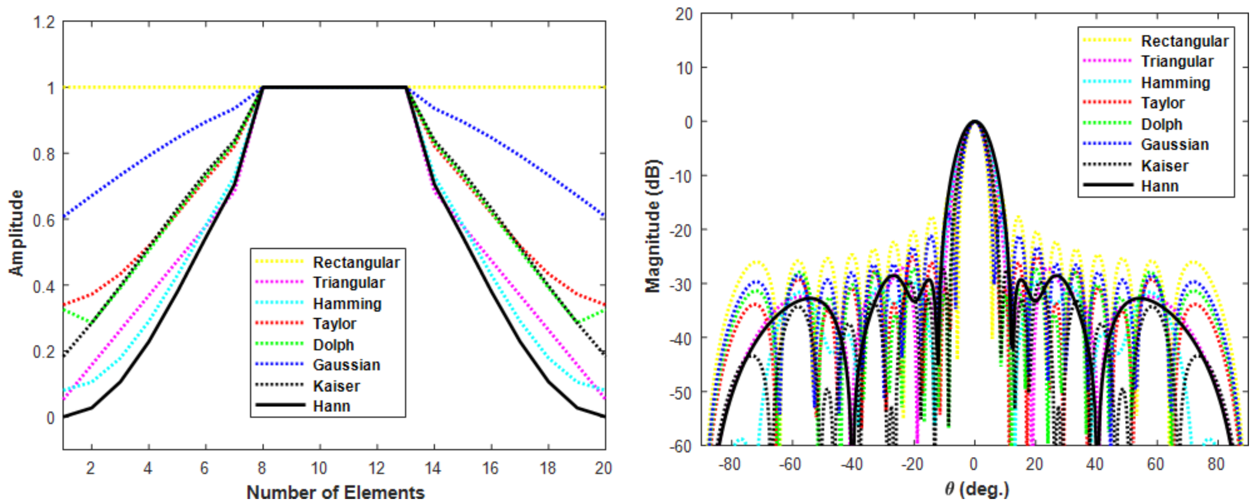

Figure 2. The radiation patterns (left) and the amplitude excitations (right) of the proposed partially tapered arrays

Table 3. Performance measures of the proposed partially tapered arrays versus number of central elements

Percentage of central elements	HPBW (degree)	Leakage power (dB)	Peak SLL (dB)	Directivity (dB)	DRR	Complexity reduction (%)
0%	12.7928	−15.0559	−29.6872	8.5328	3.6936	0
20%	12.0721	−11.9337	−17.166	8.8511	3.2503	20
40%	11.7117	−12.8084	−16.6214	9.1957	2.4843	40
60%	11.7117	−13.0216	−16.9524	9.2156	2.4083	60
80%	11.3514	−12.0999	−14.0327	9.3428	2.4083	80
100%	10.2703	−9.7743	−12.9653	9.998	1	100

5. Conclusions

It has been found that the partially tapered arrays have many advantages with compared to their original counterparts where the proposed partially tapered windows use only a subset of all the element excitations to control the array radiation pattern. The central array elements had amplitudes of ones and phases of zeros, while the side element excitations were tapered according to the original window function. Thus, the proposed tapered arrays are capable to achieve better radiation characteristics such as low peak side lobe level, better directivity, lower leakage power, narrower beam width and better complexity reduction than those of the standard windows. Moreover, the designer may control the far-field radiation characteristics of the partially tapered arrays by simply changing the number of central elements to get a best trade-off between peak sidelobe reduction and directivity loss.

Conflicts of interest

The authors declare no conflict of interest.

References

[1] R. B. Blackman and J. W. Tukey, *The Measurement of Power Spectra*. New York, NY, USA: Dover, 1958.

[2] A. V. Oppenheim and R. W. Schafer, *Digital Signal Processing*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1975.

[3] E. A. Robinson and M. T. Silvia, *Digital Signal Processing and Time Series Analysis*. San Francisco, CA, USA: Holden-Day, 1978.

[4] L. R. Rabiner and R. W. Schafer, *Digital Processing of Speech Signals*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1978.
[5] F. J. Harris, "On the use of windows for harmonic analysis with the discrete Fourier transform," *Proceedings of the IEEE*, vol. 66, pp. 51–83, Jan. 1978.
[6] N. C. Geckinli and D. Yavuz, "Some novel windows and a concise tutorial comparison of window families," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 26, pp. 501–507, Dec. 1978.
[7] Y. H. Ha and J. A. Pearce, "A new window and comparison to standard windows," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 37, pp. 298–301, Feb. 1989.
[8] S. Choi, T. Sarkar, and S. S. Lee, "Design of two-dimensional Tseng window and its application to antenna array for the detection of AM signal in the presence of strong jammers in mobile communication," *Signal Processing*, vol. 34, pp. 297–310, Dec. 1993.
[9] A. H. Nuttall, "Some windows with very good side lobe behavior," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 29, pp. 84–91, Feb. 1981.
[10] J. C. Burgess, "Optimum approximation to Dolph-Chebyshev windows," *IEEE Transactions on Signal Processing*, vol. 40, pp. 2592–2594, Oct. 1992.
[11] J. K. Gautam, A. Kumar, and R. Saxena, "Dolph-Chebyshev window and its coefficient computing methods," In Proc. National Seminar on Electronics Systems and Applications, New Delhi, India, Mar. 25-27, 1994, pp. 134–137.
[12] A. R. Reddy and S. K. Lahiri, "On window functions," *International Journal of Electronics*, vol. 56, pp. 809–813, June 1984.
[13] J. F. Kaiser, "Non recursive digital filter design using I0-Sinh window function," In Proc. IEEE International Symposium on Circuits and Systems, San Francisco, CA, USA, Apr. 1974, pp. 20–23.
[14] L. R. Rabiner, "Tutorial on isolated and connected word recognition in signal processing II: Theories and applications," in *Signal Processing II: Theories and Applications*, H. W. Schüssler, Ed. Amsterdam, Netherlands: Elsevier, 1983.
[15] K. M. M. Prabhu, *Window Functions and Their Applications in Signal Processing*. Boca Raton, FL, USA: Taylor & Francis, 2014.
[16] A. W. Doerry, *Catalog of Window Taper Functions for Sidelobe Control* [online]. Available: https://www.osti.gov/servlets/purl/1368638. [Accessed Feb. 15, 2025].
[17] J. R. Mohammed, "Phased array antenna with ultra-low sidelobes," *Electronics Letters*, vol. 49, no. 17, pp. 1055–1056, Feb. 2013.
[18] J. R. Mohammed, "Array pattern synthesis using a new adaptive trapezoid window function for sidelobe suppression and nulls control," *Progress in Electromagnetics Research M*, vol. 129, pp. 83–90, Oct. 2024.
[19] J. R. Mohammed, "An optimized phase-only trapezoid taper window for array pattern reshaping," *Progress in Electromagnetics Research C*, vol. 152, pp. 245–251, Feb. 2025.