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Geometry Problems in Formalizing the Creation of Machining Processes on Machine Tools

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Abstract: Introduction: The main problems hindering the creation of fully automatic systems of design of parts and technologies of their production are considered. The search for and formalization of regularities and relationships between the design parameters of parts and the technology of their production is an urgent task. At the same time, it has not yet been possible to create a unified general theory of geometric construction and transformation of objects at the macro level. **Purpose**: To show and formulate the key problems of formalizing the design process and identify possible ways to solve them. **Methods**: The study uses a wide range of methods of algebra, geometry, and discrete mathematics, as well as the main provisions of the geometry of non-ideal objects. **Results**: The need for fundamental laws of generation of geometric configurations and their elements is a key formalization problem. The geometry of non-ideal objects allows us to formally solve these problems. A geometric object is represented in six-dimensional space (three linear and three angular dimensions) with binary non-uniform metrics. The considered geometry has a principal possibility to formally generate geometric configurations of objects and their elements. The geometrical structure of a part is unambiguously connected with a set of technological schemes providing the mutual arrangement of surfaces during machining. Rules for the formal generation of a complete set of technological structures that are guaranteed to provide the specified parameters during the machining of parts. **Conclusion**: Sets of surfaces and relations between them define the individual structure of geometric configuration for each specific object. This individuality can be effectively used for the formal recognition of geometric objects. The geometry of non-ideal objects can serve as a fundamental basis for the formal theory of the creation of original technological processes of their processing.

Keywords: formalization of technology design, machining parts on machine tools, geometric configuration, surface shaping, structure of geometric relationships, locating parts

1. Introduction

The most important events in the creation and production of products in mechanical engineering are design and technological preparation, since design answers the question, “what to do?”, and technology answers the question, “how to do it?”. However, despite the development of information technologies and the emergence of automation systems, the processes of design and technological design are performed by humans, since synthesis and decision-making remain at the level of non-deterministic tasks. [1-4].

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There are two generalized approaches in the modern methodology of forming technological processes [2, 3, 5, 6]. The first of these is development using technological processes based on proven analogous processes [2, 7-9]. The most common analog processes in Russia are typical technological processes (TTP), proposed by Sokolovsky [10] in the late 1930s. The essence of the method lies in the selection of a part that is geometrically similar to the one under consideration, for which there is a manufacturing technology and the use of this technology as an analog with a little refinement. For reliable recognition and identification of geometric images of parts or assemblies, Sokolovsky developed a multi-level hierarchical classification. The use of TTP provided a significant reduction in labor intensity and an increase in the efficiency of machine-building industries in the context of the production of frequently changing products. Subsequently, in the United States (US), this method was called group technologies (GT). This is confirmed in a paper published in 1984 by Hyer and Wemmerlöv [11], "Early use of the GT was documented in the Soviet Union in the 1940s. Since then, it has been implemented in many countries in Europe and Asia, mainly in the manufacturing sector. Interest among US manufacturers arose in the mid-1970s, and by now many major corporations (such as John Deere, Caterpillar, Lockheed, General Electric, Black & Decker, and Cincinnati Milacron) have taken advantage of the GT or are planning GT programs."

Further development of the considered approach was the development of technological processes using a pre-compiled set of technological processing schemes corresponding to typical geometric modules of the part design [7, 12]. These works formed the basis of a series of autonomous automated systems for the formation of technological processes (computer-aided design of technological processes, CAD TP) in Russia, for example, T-FLEX Technology, Vertical, TechCard, Technologi CS TRR, Timeline, Sprut-TP and others [13].

A similar approach based on the similarity of repeating parts and, as a result, their manufacturing technologies, was called group technology and was published in the work of Ham in 1976 [14]. The idea of computerization of planning tasks for production processes, expressed by Nibel (1965), was first implemented in 1976 [3, 15]. The use of group technologies became the basis of the first computer-aided process planning (CAPP) systems for preparing plans for the manufacture of parts and products in production [14, 16]. At the same time, planning consists in choosing options for processes, tools, and equipment from among the existing ones and determining the sequence of actions. This approach is usually called variant [3, 5, 6].

The second approach involves the development of a process plan for each component, created "from scratch" with no or minimal human involvement. It envisages the creation of original technological processes for each specific part based on formal models using the tools available for implementation, so it is called generative [2, 7, 8, 9].

Within the generative approach, researchers try to create CAD TP and CAPP systems based on an instrumental arsenal of formal mathematical methods (homogeneous transformation matrices, small displacement torsor, dual quaternions, etc.), artificial intelligence, using neural networks, genetic algorithms, set theory and fuzzy logic, Petri nets, etc. [3, 5, 6, 17-19]. In some cases, local heuristics play the role of connecting elements [19-21].

Nondeterminism in the methods of creating technological processes and their elements do not yet allow the creation of fully automatic CAPP systems [3, 22].

The multiplicity and heterogeneity of approaches and methods used by researchers to solve design problems indicate the absence of a generalized research strategy in this direction.

Analysis of publications in recent years shows that during the development of CAD TP and CAPP, considerable attention is paid to formalizing methods of modeling geometric configurations, considering their deviations from ideality.

Particularly relevant are the issues of recognition and identification of geometric objects, as well as the calculation and optimization of tolerances during the creation of technological processes for manufacturing parts and assemblies [23-28].

An urgent task is to determine the regularity and relationship between the geometric design of the part and the technology of its manufacture at the theoretical level.

The authors' research has shown that modern tools of traditional geometry have problems that lead to the impossibility of unambiguous descriptions of geometrical schemes for generating the structure of real machine parts [2, 8, 29, 30].

The paper attempts to present an original approach based on the geometry of non-ideal objects, which provides a solution to two cornerstone problems related to the generation of elementary surfaces and the generation of geometric structures of objects. This will allow the development of CAPP software with a high degree of automation up to the automatic design of technological processes of machining parts on metal-cutting machines. In addition, it can serve as one of the fundamental prerequisites for the development of automatic metalworking machine tools capable of manufacturing parts in a wide range of geometric configurations.

The purpose of this work is to consider the main tasks of technology design, formulate the key problems of formalizing the design process and present a non-trivial approach to solve them.

2. The main tasks of designing the technology of processing parts

A machine part can be represented as a part of a material substance in a solid state, and at the same time, as a closed part of space, limited by a combination of a finite set of surfaces.

The functional characteristics of a part are determined, on the one hand, by a given set of properties (mechanical, chemical, electrical, magnetic, etc.) of the material, and, on the other hand, by the required geometric configuration and the accuracy of its execution. Then the design of technological processes for manufacturing parts is reduced to solving two cornerstone problems [2, 9].

1) Ensuring the specified properties of the material of the part.

2) Ensuring the geometric configuration of the part with regulated accuracy.

In the general case, the manufacturing process of a part can be considered as a finite non-zero set of discrete technological procedures (M^T) on which the sequence order is determined; in this context, under the technological procedure, depending on the type of technology (machining, casting, heat treatment, etc.), transitions, installations, operations are considered. It is believed that during the execution of procedures belonging to the set M^T the initial workpiece is transformed into a finished part by the specified parameters.

Considering the diversity of technologies, tools and equipment used, the set M^T can be divided into two finite non-overlapping subsets [9].

1) A subset of procedures M^{Tm} :$\{Tm_1, Tm_2, ..., Tm_n\}$, that provide the specified properties of the part material (hardness, strength, corrosion resistance, magnetization, etc.), for example, annealing, hardening, normalization, etc.

2) A subset of procedures M^{Tg} :$\{Tg_1, Tg_2, ..., Tg_k\}$ as a result of which the geometric configuration of the part is formed (geometric appearance and specified accuracy), for example, milling a plane, drilling a hole, turning cylindrical surface, threading, etc.

Formally, the conditions for non-intersection of sets are written as $M^T = M^{Tm} \cup M^{Tg} \neq \varnothing$ and $M^{Tm} \cap M^{Tg} \neq \varnothing$.

The problem of ensuring the geometric configuration of a part during machining is extremely difficult to formalize [8, 30, 31]. The problems of formal representation and transformation of geometrical information are given much attention by researchers around the world, as evidenced by the significant number of publications in recent years [3, 4, 19, 20, 23, 24, 32, 33].

The methods, techniques, and tools used in engineering activities to design technical devices are based on the provisions of modern analytical geometry, which formalizes the description and rules of transformation of ideal geometric configurations and their elements [29, 34].

The practice of using mathematical tools of geometry for the correct description of significant geometric characteristics of real technical objects indicates the presence of several problems that cannot be resolved within the framework of existing geometric concepts and definitions [30, 31].

3. Problems with the description of the geometric configuration of parts

The key problem of formalizing the design of technological processes, in our opinion, lies in the fact that at the theoretical level, the fundamental laws of generating geometric configurations and their elements are not formulated [2].

In 1984, Steudel [32] proposed the creation of CAPP based on a generative automated planning strategy, considering the relationship between the configuration of parts and technology. This characterizes the expectation of a "common geometry language" that will link computer-aided design (CAD) and computer-aided manufacturing (CAM) systems. This is consistent with the opinion of Kulikovsky [35], 'in some cases, the application of existing methods and approaches is sufficient to solve the problem, in others, the development of a new mathematical apparatus is required' (p.28).

When solving the formalization problem, it is legitimate to put a hypothetical question – "Are there laws of correspondence between the set of geometric configurations of parts designs $\mathbb{K}: \{K_1, K_2, \ldots, K_n\}$ and the set of technological processes $\mathbb{T}: \{T_1, T_2, \ldots, T_n\}$ for obtaining these configurations?" [2, 32]. Visualization of this question is shown in (Figure 1).

Formally, the question shown in Figure 1 sounds as follows. Can the relationships between a set of part configurations and a set of technologies for obtaining geometric configurations be expressed by the functions of bijection, injection, surjection, or some other?

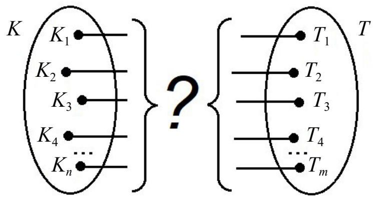

Figure 1. Relationships between sets of part designs and technologies for its processing

A preliminary justification for a positive answer could be the similarity of the part design processes and creating technological processes of their machining.

In general case, the design of the geometric configuration of a part is represented as two independent tasks: 1) defining a set of functionally necessary and suitable in-form surfaces (plane, cylinder, sphere, etc.); 2) defining the necessary mutual arrangement of surfaces, i.e., creating the geometrical structure (architecture) of the part. Similar problems are also solved when designing machining technology on machine tools [2, 8, 36]: 1) determination of generation schemes, methods and tools for obtaining surfaces of desired shapes; 2) forming schemes of the workpiece location about the forming movements of the cutting tool, as well as determining the sequence of their change.

Further, an attempt is made to understand the reasons that prevent the formal representation of the laws of correspondence between the design of a part and the technology for obtaining its geometric configuration. The axiomatics and tools of the formal geometry of three-dimensional (3D) linear space cannot describe (display) the schemes for generating geometric configurations of objects and also do not have the means to work with non-ideal geometric shapes and sizes [9, 30, 31].

The following example demonstrates the lack of possibility to formally represent the geometric structure of an object, i.e., to find combinations of surface arrangement relations [2, 8, 9, 31]. Figure 2a shows the location of the planes relative to each other, determined by the three dimensions l_1, l_2 and l_3 represented by real numbers. In the paradigm of classical geometry, such a description is consistent, as is the fact that

$$l_1 = l_3 - l_2; \quad l_2 = l_3 - l_1; \quad l_3 = l_1 + l_2 \tag{1}$$

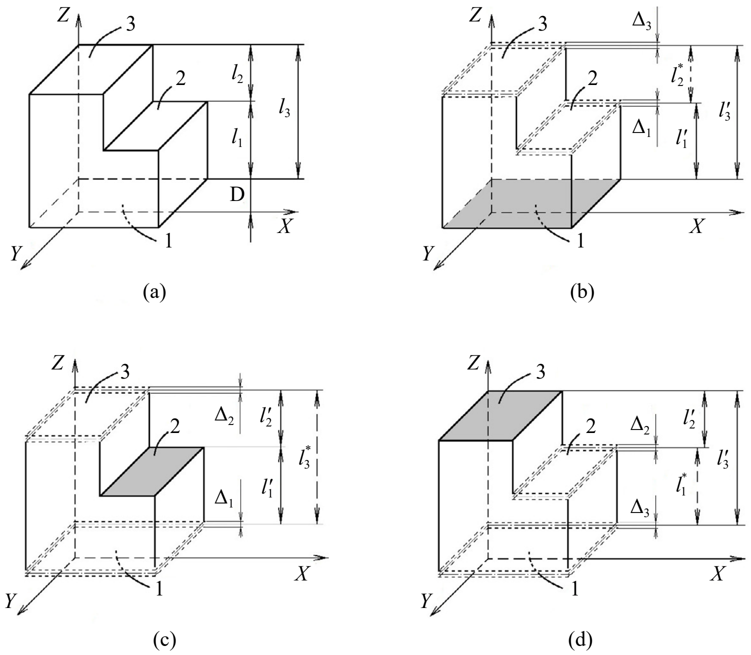

Figure 2. Location of planes 1, 2 and 3: (a) for ideal; (b), (c) and (d) for real geometric configuration

It is known that it is impossible to reproduce or measure any of these dimensions with perfect accuracy. Therefore, in mechanical engineering, it is customary to use a double entry for each size value in the form $l = l' \pm \Delta$, where l' is a deterministic component (it is customary to call the nominal size) and Δ is an indefinite (random) part in the form of a deviation value, which is called the tolerance field. Then, the dimensions (Figure 2a) can be written as $l_1 = l_1' \pm \Delta_1$, $l_2 = l_2' \pm \Delta_2$ and $l_3 = l_3' \pm \Delta_3$.

In real conditions, taking into account the errors, an attempt to reproduce (manufacture) the object shown in Figure 2a results in three different (non-invariant) configurations, as shown in Figures 2b to 2d, which depend on the generation scheme (manufacturing sequence) of surfaces.

Suppose that the tolerance values for the considered dimensions l_1, l_2, l_3 are the same, i.e., $\Delta_1 = \Delta_2 = \Delta_3 = \Delta$, then if we initially produce (machine) plane 1 (Figure 2b) and then planes 2 and 3, at distances $l_1 = l_1' \pm \Delta$ and $l_3 = l_3' \pm \Delta$ respectively, the result between surfaces 2 and 3 would be a certain size $l_2 = l_2' \pm \Delta_2$ instead of size l_2^* with a tolerance field equal to the sum $\pm\Delta_1$ and $\pm\Delta_3$, that is, $l_2^* = l_2' \pm 2\Delta$. Such dimensions l^* are referred to in the literature as the closing link. Similar properties will appear in dimensions $l_3^* = l_3' \pm 2\Delta$ (Figure 2c) and $l_1^* = l_1' \pm 2\Delta$ (Figure 2d) when the first fabricated surface is replaced by 2 and 3, respectively. These examples show that the values of the obtained dimensions l_1^*, l_2^*, l_3^* do not correspond to l_1, l_2, l_3, i.e., contradict expression (1). Thus, for classical geometry the configurations of the objects depicted in Figures 2b to 2d are not simply invariant but identical, i.e., their variations are "transparent" and the differences are indistinguishable [2, 9, 29, 31].

Summarizing the consideration of the example, we can say that the well-known rules in mathematics for adding numbers or vectors in geometry do not allow estimating the accumulated error, which depends on the number of the summand elements.

The position and movement of objects in the environment are inextricably linked with the turns and rotations of objects. In the engineering practice of creating, manufacturing and operating aircraft, spacecraft, ships and other mechanical systems, the concept of six degrees of freedom (6DoF) is used [37-40]. It determines the number of independent parameters that characterize the configuration, as well as the possibility of changing the relative orientation of objects in the environment. In this case, the object under consideration can change orientation in three linear (along the X, Y and Z axes) dimensions, as well as rotate in three angular directions relative to the specified axes (heading, roll, pitch).

In the last decade, research has intensified in the field of solving practical problems of geometric transformation, including linear movements and angular rotations. Considerable attention is paid to the formalization of the representation of geometric deviations that occur in the processing and optimization of manufacturing tolerances. Most studies are based on the Small Displacement Torsor (SDT) method [18, 21, 41]. Other methods use homogeneous transformation matrices (HTM) [42, 43], vector loops [44], variational geometric constraints [45], deviation flow chains [42], etc. up to the use of quaternions [19, 40].

In formal geometry, the dimensional parameters of objects are specified (modeled) by single numerical values. For example, linear dimensions are given as L 125.8 mm or R 12.5 mm; corner -∠42°5; location relations - ⊥ (perpendicularity) or ∥ (parallelism) [46, 47]. In contrast to this, in the world practice of mechanical engineering, dimensions are represented by two independent parameters: 1) the nominal value of the size and; 2) the normalized tolerance, i.e., the value of permissible deviations, for example, L 125.8 ± 0.15 mm or R 12.5 ± 0.05 mm; angular - ∠42°45' ± 10'; location ratio - ⊥ 0.05 mm (non-perpendicularity) or // 0.1 mm (non-parallelism) [48, 49].

The considered examples demonstrate only a part of the problems associated with the completeness and correctness of the representation and transformation of geometric objects. To solve the problems considered in the article, it is proposed to use the geometry of non-ideal objects, which can serve as a theoretical basis for creating mathematical kernels of geometric modeling in six-dimensional space [2, 29, 31].

4. Main provisions of the geometry of non-ideal objects

One of the fundamental differences of the geometry under consideration is the fundamental possibility and availability of tools for the formal representation of the patterns of generation and interaction of objects.

Geometric space is interpreted as a space (set) of conditions allowing the existence of a complete variety of real or virtual geometric objects of various shapes and positions. At the same time, the dimension of space is determined by six independent parameters (three linear movements along orthogonal axes and three angular rotations around them), which are necessary and sufficient to describe states (configurations and positions), as well as schemes for generating geometric objects [29].

The basis of the six-dimensional space is a set of six orthonormal vectors, three of which are linear $(\vec{e}_x, \vec{e}_y, \vec{e}_z)$, and three are angular $(\vec{\propto}_x, \vec{\propto}_y, \vec{\propto}_z)$. These vectors are oriented concerning the Cartesian system of axes as shown in Figure 3 [2, 9, 29, 31].

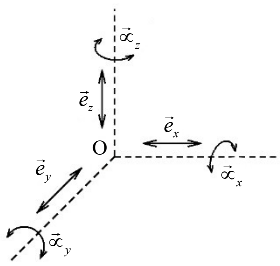

Figure 3. Basis of six-dimensional space

To determine the angular positions of objects in the geometry under consideration, the concept of an angular vector $(\vec{\propto})$ is introduced, which is an angular value (angle) measured in the plane between two elements of a geometric object or objects (Figure 4). The trajectory of the angular vector lies in the plane of rotation and is an arc or a circle. The direction of the angle vector is the same as the direction of rotation or rotation.

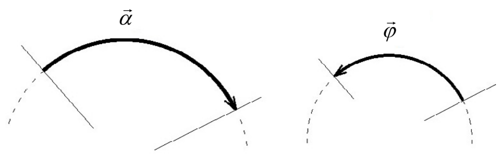

Figure 4. Examples of visualization of the angular vector

To measure linear and angular quantities in the geometry of non-ideal objects, fuzzy arithmetic tools are used [50]. In this context, a fuzzy number of the form $a \pm \Delta$ is represented by two parameters, the first of which (a) determines the deterministic component, and the second (Δ) - the non-deterministic (stochastic) value of its possible deviation.

Linear and angular size values are written as $l = l^M \pm 3\sigma_l$ and $\alpha = \alpha^M \pm 3\sigma_\alpha$, respectively, where l^M and α^M are the mathematical expectation of linear and angular values, and σ_1 and σ_α is the root mean square (standard) deviation of these random variables.

Geometric object is represented as a fuzzy closed subspace with a nonzero volume, bounded by a finite set k ($k \geq 1$) of intersecting or mating surfaces uniquely located relative to each other. The specific mutual position and orientation of the surfaces are represented by a connected six-dimensional structure (scheme) of relations between the surfaces forming a geometric configuration. In this case, surfaces, lines and points are not geometric objects but are considered as elements of a geometric object. The main elements for creating and defining the geometric configuration of an object are surfaces [2, 9, 29].

Surface is considered a boundary separating two different environments. In the general case, the set of surfaces S includes a complete set of fuzzy smooth infinite manifolds. A surface is a basic element, i.e., a "basic brick" for constructing the configuration of an object. Three kinds of special surfaces are distinguished from the whole manifold, which belong to the elementary ones: a plane (S^p) a circular cylinder (S^c) and a sphere (S^s) Elementary surfaces include surfaces that can be formed kinematically using two independent elementary plane production lines (a straight line and a circle) at their mutually orthogonal orientation [2, 9, 29].

Lines are formed as a result of the intersection or smooth conjugation of fuzzy surfaces of arbitrary shape, as shown in Figure 5. At the same time, lines are considered elements of the kinematic generation of surfaces. Among the set of fuzzy lines ($\mathbb{L}$) one can single out elementary lines: a straight line (L^s) and a circle (L^c) Elementary lines, in turn, are identified in two types of vectors: linear $(\vec{e})$ and angular $(\vec{\propto})$.

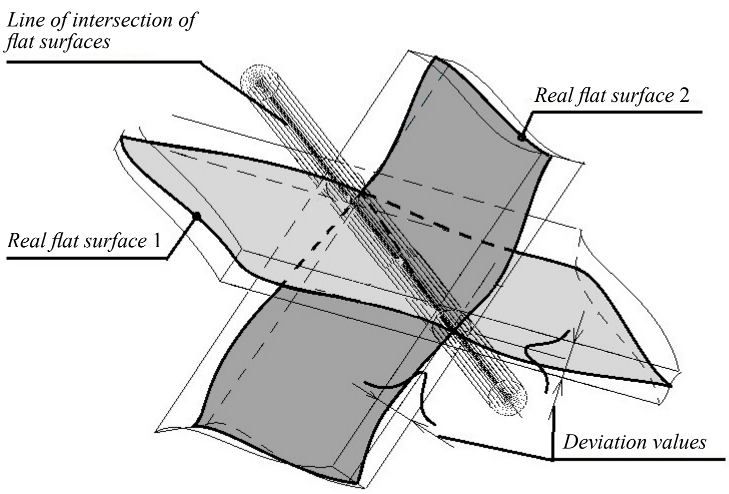

Figure 5. Line generation

Point is the result of the intersection of fuzzy lines and is an auxiliary element in the form of a fuzzy spherical space at the intersection of these lines. The position in the said space of the mathematical expectation of the *i*-th point is determined by three linear coordinates $(\vec{e}_x, \vec{e}_y, \vec{e}_z)$, and the sphere diameter is determined by the value of the standard deviation [2, 9, 29, 31].

Geometric configuration of an object is determined by the outlines of the boundaries separating the object and the environment. Boundaries are represented by a set of mutually oriented surfaces that intersect or conjugate in a certain way. Thus, the geometric configuration of an object is determined by the composition, i.e., a set of surfaces, as well as a structure, i.e., a set of values of the parameters of their mutual arrangement.

Formally, the geometric configuration of an object is described by a pair of sets *G* (*S*, *D*): 1) a non-empty finite set of surfaces $S:\{s_1, s_2, \ldots, s_k\}$, i.e., $\forall S(S \notin \varnothing)$, which determines the composition of elements; 2) a finite set of necessary and sufficient relations (dimensional relations) between the surfaces *D*, which determines the structure of the geometric configuration. In the six-dimensional space, it is convenient to represent the pair of sets *G* (*S*, *D*) as six planar graphs, in which vertices correspond to surfaces (composition) and edges to the set of dimensional relations between surfaces (structure).

The set *D* can be empty $(D = \varnothing)$ only in the case when the configuration of the object is determined by one surface (*S* = 1) from among the closed ones, for example, sphere, torus, etc. [2, 9, 13]. In the general case, the set $D:\{D^{\vec{e}_x}, D^{\vec{e}_y}, D^{\vec{e}_z}, D^{\vec{\propto}_x}, D^{\vec{\propto}_y}, D^{\vec{\propto}_z}\}$ is obtained by the union of non-overlapping subsets of dimensional relationships in each dimension, i.e., $D = D^{\vec{e}_x} \cup D^{\vec{e}_y} \cup D^{\vec{e}_z} \cup D^{\vec{\propto}_x} \cup D^{\vec{\propto}_y} \cup D^{\vec{\propto}_z}$ while $D^{\vec{e}_x} \cap D^{\vec{e}_y} \cap D^{\vec{e}_z} \cap D^{\vec{\propto}_x} \cap D^{\vec{\propto}_y} \cap D^{\vec{\propto}_z} = \varnothing$. Then, the structure of the geometric object is represented as six independent sets of relations in the dimension space $\vec{e}_x, \vec{e}_y, \vec{e}_z, \vec{\propto}_x, \vec{\propto}_y, \vec{\propto}_z$. For example, for dimension (coordinates) $\vec{e}_x$, the relationship between surfaces will be written as $D^{\vec{e}_x}:\{d_1^{\vec{e}_x}, d_2^{\vec{e}_x}, \ldots, d_q^{\vec{e}_x}, \ldots, d_n^{\vec{e}_x}\}$, where $d_q^{\vec{e}_x}$ is a binary relationship between the mapping of surfaces $s_i^{\vec{e}_x}$ and $s_j^{\vec{e}_x}$ to the dimension $\vec{e}_x$, i.e., $d_q^{\vec{e}_x} = (s_i^{\vec{e}_x}, s_j^{\vec{e}_x}), i \neq j$. The contents of the subsets $D^{\vec{e}_y}, D^{\vec{e}_z}, D^{\vec{\propto}_x}, D^{\vec{\propto}_y}, D^{\vec{\propto}_z}$ are written similarly [2, 29].

In the general case, the creation of a geometric object starts with an arbitrary arrangement of the first (any) base surface in six-dimensional space. It is this base surface that determines (sets) the direction of those coordinate axes that correspond to its geometric image (spatial portrait). This image is defined by a set of necessary and sufficient measurements for unambiguous determination of the position of the surface in question. For example, a plane is always defined by one linear and two angular measurements, and any cylindrical surface by two linear and two angular measurements. Only three linear measurements are used to determine the position of a sphere.

Any subsequently added surface must be related to the original surface in those dimensions which coincide with the spatial portrait of the first surface. The coordinate dimensions that do not match are added to the structure of the geometrical configuration generation, and so on. In this case, the surface is also treated as a base surface. The number of base surfaces cannot exceed three. All other added surfaces are joined to the already existing set of surfaces.

Thus, the structure (scheme) of the geometric configuration is formed. This scheme is unique (one and only) for each particular geometric object. It can be effectively used for formal recognition of geometric objects. Also, the geometric configuration formation structure is the starting point for formally defining the many possible variants of the sequential machining scheme for all surfaces of the part.

More detailed examples will be presented below.

5. Principles of object generation

In the general case, the process of generating (creating) the geometric configuration of an object consists in forming the boundaries of a closed subspace by the relative arrangement of all surfaces included in its composition (*S*) by the required structure (*D*). Thus, any change in either composition or structure leads to the formation of a new configuration. The conditions for the correctness and uniqueness of the existence of a geometric object are the closedness of the internal space, as well as the connectedness and acyclicity of its structure [29, 30, 36].

As a result of the research, it was found that for any elementary surface, no more than four bonds in the coordinate space $(\vec{e}_x, \vec{e}_y, \vec{e}_z, \vec{\propto}_x, \vec{\propto}_y, \vec{\propto}_z)$ are necessary and sufficient. Moreover, for planes, this is one linear and two angular bonds, for a cylindrical surface - two linear and two angular ones relative to the same axes, and for a spherical one - three linear ones [2, 29, 30].

Figure 6 shows examples of describing the position of elementary surfaces in space $\vec{e}_x, \vec{e}_y, \vec{e}_z, \vec{\propto}_x, \vec{\propto}_y, \vec{\propto}_z$. Formally, the position of each of them can be represented by a unit $\vec{S}\{\vec{e}_x, \vec{e}_y, \vec{e}_z, \vec{\propto}_x, \vec{\propto}_y, \vec{\propto}_z\}$. Then, the plane (Figure 6a) is described as $\overrightarrow{S^p}\{0,0,1,1,1,0\}$, the cylinder (Figure 6b) is described as $\overrightarrow{S^c}\{0,1,1,0,1,1\}$, and the sphere (Figure 6c) is described as $\overrightarrow{S^s}\{1,1,1,0,0,0\}$, where the unit indicates the presence of a connection in the corresponding dimension, and zero is its absence. Visually, these vectors are conveniently depicted in the form of six-cell tables, as shown in Figure 6.

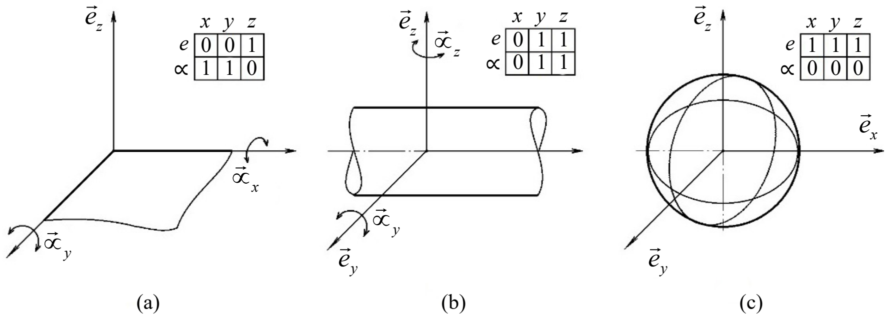

Figure 6. Representation of positions of elementary surfaces in orthonormal space: (a) plane; (b) cylinder; (c) sphere

As indicated above, the structure of the geometric configuration is determined by the sets of links $D^{\vec{e}_x}, D^{\vec{e}_y}, D^{\vec{e}_z}$, $D^{\vec{\propto}_x}, D^{\vec{\propto}_y}, D^{\vec{\propto}_z}$, which are conveniently represented as six graphs in the dimension space $\vec{e}_x, \vec{e}_y, \vec{e}_z, \vec{\propto}_x, \vec{\propto}_y, \vec{\propto}_z$. For any of the six directions, the set of connections must correspond to a connected acyclic graph, i.e., a tree graph. The set of vertices of each graph is determined by the presence of a unit in the corresponding coordinate direction in the surface vectors [2, 8, 9, 29, 30].

In Figure 6, one can also see all admissible (possible) connections between elementary surfaces. From the six-cell tables (Figure 6), it can be seen that the graph G^{e_x} with the coordinate e_x contains the only vertex S^S the graph G^{e_y} can contain two vertices S^c and S^s therefore, the cylinder and the sphere can be related by the size along the coordinate e_y and three vertices (S^p, S^c and S^s) fall into the graph $G^{\propto_z}$ and, therefore, connections between them are allowed. The graph $G^{\propto_x}$ contains one vertex S^p the graph $G^{\propto_y}$ contains two vertices S^p and S^c and the graph $G^{\propto_z}$ is empty because it does not contain any vertex.

5.1 *Generation of elementary surfaces*

The basis for the generation of surfaces in the geometry of non-ideal objects is the kinematic method, when the formation of a surface occurs as a result of the interaction of two generating lines, i.e., movement of one line (generator) relative to another (guide) [2, 30].

For any elementary surface, there are at least two free coordinates that do not affect its orientation in space. In six-cell tables (Figure 6), these coordinates are marked with zeros. By the established patterns in the space under consideration, any paired combination of free coordinates uniquely corresponds to a pair of independent generating lines forming this surface. Moreover, free linear coordinates correspond to a straight line, and angular coordinates correspond to circles [2, 9, 29-31, 49, 51].

For example, in Figure 6a, the six-cell table of the S^p plane contains zeros in the coordinates $\vec{e}_x, \vec{e}_y$ and $\vec{\propto}_z$, i.e., these coordinates do not determine the position of the plane. Then, the set of pair combinations is three: 1) $\vec{e}_x$ and $\vec{e}_y$, 2) $\vec{e}_x$ and $\vec{\propto}_z$, 3) $\vec{e}_y$, and $\vec{\propto}_z$. Here, the first pair of generating lines consists of two straight mutually orthogonal lines, and the other two include one straight line located along the X (or Y) axis and one circle relative to the Z axis. Taking into account the reversibility of elementary surfaces, i.e., the replacement of a generator by a guide and vice versa, each pair of generators will form two schemes for generating the considered surface [2, 52-54].

The considered example shows that for each type of elementary surface, there is an unambiguous rule for determining the complete set of shaping schemes represented by pairs of generating lines. Each pair of generating lines strictly corresponds to four processing methods [9]. Then the power of the set of processing methods for a plane is defined as $|s^p| = 24$, for a cylindrical surface $|s^c| = 8$, and a sphere $|s^s| = 8$.

Thus, each element of the set of surfaces of the structure of any part $S:\{s_1, s_2, \ldots, s_k\}$ (k is the number of surfaces), exactly corresponds to one element of the set of technological processing procedures $T^S:\{T_1^S, T_2^S, \ldots, T_k^S\}$. In other words, there is a bijective relation between the forms of the surfaces of the part and the technologies for their processing, i.e., $f: S \leftrightarrow T^S$ [2].

A surface of any complexity can be approximated with a sufficient degree of accuracy by a finite set of elementary lines using algorithms for the formation of parametric Bezier surfaces [55, 56]. Then the representation of any geometric configuration is formalized by setting the structure of dimensional relationships that determine the relative position of the surfaces of the part in each of the six dimensions.

5.2 *Generation of geometric structure*

A more complex relationship was established between the structure of the geometric configuration of the structure: $\{D_1, D_2, \ldots, D_v\}$ (v is the number of dimensional relationships) and a set of technological schemes to ensure the relative orientation of surfaces during processing $T^D:\{T_1^D, T_2^D, \ldots, T_w^D\}$ (w is the number of allowed technological schemes).

The results of the research show that for any part design with a correctly specified structure of dimensional relationships (D_i^K), there is a unique set of options for the structures of technological processes that are guaranteed to provide a given relative position of the surfaces $T_i^D:\{t_1^i, t_2^i, \ldots, t_q^i, \ldots, t_w^i\}$, $(T_i^D \in T^D)$. The implementation of each option (t_q^i) from this set is guaranteed to lead to the result specified by the design of the part, taking into account the tolerances of the relative position of the surfaces [2, 8, 35, 38, 53].

Consider an example of forming machining process structures. Figure 7a shows an example of displaying dimensional relationships between the surfaces of a fragment of the geometric structure of an object, which includes three planes (P1, P2, P3) and one cylindrical surface (C1). Linear dimensional relationships l_1 and l_2 are expressed by fuzzy numbers 75 ± 0.023 mm and 60 ± 0.023 mm, respectively. Given a perpendicularity tolerance (⊥ 0.02 mm), the values of the angular dimensions will be written as 90° ± 0.005° or 90° ± 0.3'.

The position of surfaces in space is given by single six-dimensional vectors and presented in the form of six-cell tables (Figure 7b). The given linear and angular dimensional relationships between surfaces (Figure 7a) are displayed on the adjacency graphs (Figure 7c) for each of the six dimensions $(\vec{e}_x, \vec{e}_y, \vec{e}_z, \vec{\propto}_x, \vec{\propto}_y, \vec{\propto}_z)$. The set of vertices of each adjacency graph is determined by the unit in the corresponding dimension of the six-cell table. For example, surfaces P1 and C1 have units in dimension $\vec{e}_x$ (Figure 7b).

Upon careful consideration of the dimensional relationships between the surfaces given in Figure 7a, it can be seen that no connection determines the angular location of the P2 surface relative to the X-axis (measurement $\vec{\propto}_x$), which indicates a violation of the connectivity of the adjacency graph $\vec{\propto}_x$ (Figure 7c) and, as a consequence, the integrity of the geometric configuration of the object. To unambiguously determine the position of P2, it is necessary to set either perpendicularity concerning P3, or parallelism concerning C1. Let us set the angular deviation of the surfaces P2 and C1 to 0° ± 0.005°, which corresponds to a parallelism tolerance of 0.02 mm for the considered linear dimensions.

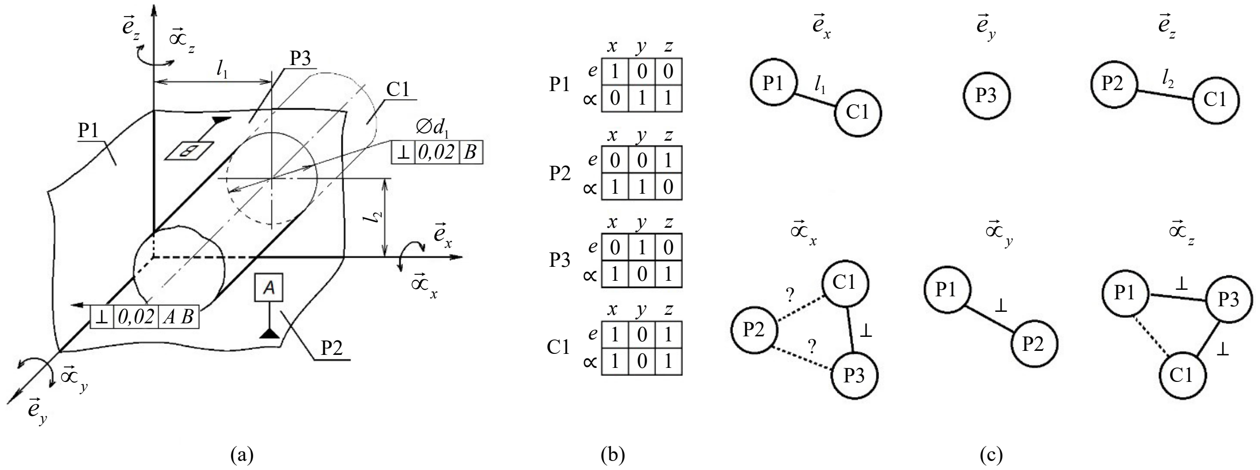

Figure 7. Dimensional relationships between the surfaces object: (a) a fragment geometric structure of the object; (b) surface position vectors; (c) graphs of dimensional relationships

The formation of the structure of the technological process is modelled as "cutting out" a part of the space by a sequential arrangement of surfaces by the specified parameters (Figure 7a). In this case, the problem of ensuring the relative position of the surfaces of the part is solved, without relative reference to the surrounding space. Then, the first arbitrarily located surface sets a part of the coordinates corresponding to its characteristics (Figure 7b), relative to which the geometric structure of the object is "built". Taking into account that modelling can be started with the processing of any surface of the object, the number of basic options for the structure of the technological process is equal to the number of surfaces from the total composition.

For example, if the plane P2 is chosen as the first surface, which specifies the direction of three coordinates $\vec{e}_z, \vec{\propto}_x, \vec{\propto}_y,$ then the second step leading to the solvability of the problem will be the location of the surface P1, which is connected to P2 by two-dimensional bonds in the coordinates $\vec{e}_z$ and $\vec{\propto}_x$ (Figure 7c). As a result, the surface P2 determined two more coordinate directions $\vec{e}_x$ and $\vec{\propto}_z$. The next step makes it possible to uniquely orient the surface P3 connected in two directions $\vec{\propto}_x$ and $\vec{\propto}_z$ with the surface C1 by the given perpendicularity condition (Figure 7c). Then the last surface P1 is oriented along three already-defined coordinates $\vec{e}_x, \vec{\propto}_y, \vec{\propto}_z$ in accordance with the given parameters (Figure 7a and Figure 7c). The variant of the structure of the technological process obtained from the initial node P2 will be called the generating branch. The remaining three generating branches are modelled in a similar way, in which the initial surfaces are P1, C1 and P3. Note that in the example under consideration, all variants lead to the solvability of the problem, i.e., to the processing of a complete set of surfaces with guaranteed achievement of a given relative position of the surfaces subject to dimensional tolerances. The considered example confirms the existence of a correspondence between the geometric structure of the object D_i^K and a finite set of technological processes that provide a given relative position of the surfaces T_i^D.

Based on the foregoing, it is possible to answer in the affirmative the question (Figure 1) about the existence of a correspondence between the design of the part and the technology of its processing, provided that its geometric configuration is correctly and consistently represented in six-dimensional space.

6. Conclusions

The machining design models reviewed demonstrate the key problems in representing the geometric configuration of real objects. As a formal basis for solving these problems, the geometry of non-ideal objects with a six-dimensional space having three linear and three angular coordinates is proposed. The considered geometry has a principal possibility to formally generate geometric configurations of objects and their elements. The given examples demonstrate the existence of rules for the formal generation of both elementary surfaces and a complete set of technological structures that are guaranteed to provide the specified parameters of the mutual arrangement of surfaces.

Sets of surfaces and relations between them define the individual structure of geometric configuration for each specific object. This individuality can be effectively used for the formal recognition of geometric objects.

The presented materials allow us to assert the existence of strict correspondences between the design of the part and the technology of its processing, provided that its geometric configuration is correctly and consistently represented in six-dimensional space.

Thus, the geometry of non-ideal objects can serve as a fundamental basis for the formal theories of designing mechanisms and their parts, as well as the creation of original technological processes for their processing. In turn, it becomes possible to create geometric modeling kernels in a six-dimensional space, which will allow the development of CAD and CAD systems with a high degree of automation up to the automatic design of technological processes for processing parts on metal-cutting machines.

Conflict of interest

There is no conflict of interest for this study.

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