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Research Article

A Fresh Perspective on the Concatenation Model in Optical Fibers with Kerr Law of Self-Phase Modulation

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Abstract: The current work revisits the concatenation model having Kerr law of self-phase modulation and takes a fresher look with three different forms of integration technologies. The extended simple equation approach, the tanh-coth method, and the improved modified extended tanh-function approach yielded a spectrum of soliton solutions to the model. These reveal a spectrum of 1-soliton solutions to the model and they are all classified as well. The surface plots are also presented.

***Keywords*:** traveling waves, extended simplest equation method, tanh-coth approach, improved modified extended tanh-function

1. Introduction

One of the most unique models proposed to study the propagation of solitons through optical fibers is the concatenation model. This model was conceived in 2014 [1, 2]. It is the conjunction of three well-known equations that is studied in nonlinear optics. They are the nonlinear Schrodinger's equation (NLSE), the Lakshmanan-Porsezian-Daniel (LPD) model, and the Sasa-Satsuma equation (SSE). These three equations are concatenated and a new model was established during 2024, hence the name.

There have been several works that were reported from this model. These include the Painleve analysis, retrieval of solitons, and conservation laws with the usage of the undetermined coefficients and the multiplier approach respectively. Later, the trial equation approach was also implemented to recover the soliton solutions to the model [3-7]. Further down the road, several more studies were conducted. These include the evolution of quiescent optical solitons for nonlinear chromatic dispersion (CD) and the model was further analyzed with the absence of self-phase modulation (SPM). In this context, quiescent optical solitons were also studied with polarization-mode dispersion and the revealed

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interesting results were also reported. The model was also studied numerically with the application of the Laplace-Adomian decomposition principle where the surface plots for bright and dark soliton solutions are reported along with the error analysis. The inclusion if the spatio-temporal dispersion gave way to an extended version of the concatenation model that was analyzed, and the mitigation of the Internet bottleneck effect was also proposed for the model.

The current paper takes a fresher look at the model to recover the soliton solutions. These are the applications of a new set of integration technologies. They include the extended simple equation approach, the tanh-coth method, and the improved modified extended tanh-function approach. Thus, from such a fresh visitation to the model with these newly proposed integration structures a spectrum of soliton solutions is revealed. The results are indeed very promising as well as encouraging. These solutions are enlisted and they are exhibited in the rest of the paper along with the corresponding integration algorithms.

2. Governing model

The concatenation model is formulated as [1-6]:

$$i\psi_t + a\psi_{xx} + b|\psi|^2\psi + c_1\left[\sigma_1\psi_{xxxx} + \sigma_2(\psi_x)^2\psi^* + \sigma_3|\psi_x|^2\psi + \sigma_4|\psi|^2\psi_{xx} + \sigma_5\psi^2\psi^*_{xx} + \sigma_6|\psi|^4\psi\right]$$

$$+ic_2\left[\sigma_7\psi_{xxx} + \sigma_8\left[\psi\right]^2\psi_x + \sigma_9\psi^2\psi^*_x\right] = 0. \tag{1}$$

The wave profile, including its spatial and temporal derivatives, can be described by the complex function $\psi(x, t)$. The linear temporal evolution of solitons is given by the first term, while a is the coefficient of CD and b represents SPM. The concatenation model is the conjoined version of three familiar and frequently visible models. For $c_1 = c_2 = 0$, the model collapses to NLSE, while $c_1 = 0$ and $c_2 = 0$ give the familiar SSE and LPD equations, respectively.

3. Traveling wave solution

The solutions of Eq. (1) are supposed as [8-15]:

$$\psi(x,t) = u(\xi)e^{i\theta(x,t)}, \tag{2}$$

where $\xi = x - \gamma t$ and $\theta(x, t) = -kx + \omega t + \theta_0$ is the phase component of the wave. Also, $u(\xi)$ is the amplitude component of the wave. Here γ is the soliton speed, k is the soliton frequency, ω is the wavenumber and θ_0 is the phase constant. Using Eq. (2) and their derivatives, Eq. (1) is transformed to

$$[-i\gamma u' - \omega u] + a[u'' - 2iku' - k^2u] + bu^3$$

$$+c_1\sigma_1\left(u'''' - 4iku''' - 6k^2u'' + 4ik^3u' + k^4u\right) + c_1(\sigma_2 + \sigma_3)\left(uu'^2 - 2u'kiu^2 - k^2u^3\right)$$

$$+c_1(\sigma_4 + \sigma_5)\left(u^2u'' - 2iku^2u' - k^2u^3\right) + c_1\sigma_6u^5 + c_2\sigma_7\left(iu''' + 3ku'' - 3ik^2u' - k^3u\right)$$

$$+c_2(\sigma_8 + \sigma_9)u^2\left(iu' + ku\right) = 0. \tag{3}$$

Eq. (3) can be decomposed into real and imaginary parts, which yield a pair of relations.

The real part from Eq. (3) is:

$$c_1\sigma_1 u^{(4)} + \left[a + 3c_2\sigma_7 k - 6k^2 c_1\sigma_1 \right] u'' + c_1(\sigma_4 + \sigma_5)u^2 u''$$

$$+ c_1(\sigma_2 + \sigma_3)uu'^2 + \left[c_1\sigma_1 k^4 - ak^2 - c_2\sigma_7 k^3 - \omega \right] u$$

$$+ \left[b + kc_2(\sigma_8 + \sigma_9) - c_1(\sigma_2 + \sigma_3 + \sigma_4 + \sigma_5)k^2 \right] u^3$$

$$+ c_1\sigma_6 u^5 = 0, \tag{4}$$

while the imaginary part reads as:

$$(c_2\sigma_7 - 4kc_1\sigma_1)u''' + \left[4k^3 c_1\sigma_1 - 3k^2 c_2\sigma_7 - \gamma - 2ak \right] u'$$

$$+ \left[c_2(\sigma_8 + \sigma_9) - 2kc_1(\sigma_2 + \sigma_3 + \sigma_4 + \sigma_5) \right] u'u^2 = 0. \tag{5}$$

From Eq. (5), the soliton speed is:

$$\gamma = -2k\left(4k^2 c_1\sigma_1 + a\right), \tag{6}$$

whenever

$$c_2(\sigma_8 + \sigma_9) = 2kc_1(\sigma_2 + \sigma_3 + \sigma_4 + \sigma_5), \tag{7}$$

and

$$c_2\sigma_7 = 4kc_1\sigma_1. \tag{8}$$

Eq. (4) can be written as

$$c_1\sigma_1 u^{(4)} + \beta_2 u'' + c_1(\sigma_4 + \sigma_5)u^2 u'' + c_1(\sigma_2 + \sigma_3)uu'^2 + \beta_5 u + \beta_6 u^3 + c_1\sigma_6 u^5 = 0, \tag{9}$$

Where,

$$\beta_2 = \left[a + 6k^2 c_1\sigma_1 \right], \tag{10}$$

$$\beta_5 = -\left[k^2(a + 3k^2 c_1\sigma_1) + \omega \right], \tag{11}$$

and

$$\beta_6 = \left[b + c_1(\sigma_2 + \sigma_3 + \sigma_4 + \sigma_5)k^2 \right]. \tag{12}$$

For $\beta_2 = \beta_5 = \beta_6 = 0$, then Eq. (9) can be written as:

$$\sigma_1 u^{(4)} + (\sigma_4 + \sigma_5)u^2 u'' + (\sigma_2 + \sigma_3)uu'^2 + \sigma_6 u^5 = 0. \tag{13}$$

Therefore, we get:

$$a = -6k^2 c_1\sigma_1,\ \omega = 3k^4 c_1\sigma_1,\ b = -c_1(\sigma_2 + \sigma_3 + \sigma_4 + \sigma_5)k^2. \tag{14}$$

4. Methodologies

In this section we will apply three different methods to solve Eq. (13). These methods are the extended simple equation method (ESEM), the Tanh-Coth method, and the improved modified extended Tanh function method (IMETF).

4.1 *Extended simple equation method (ESEM)*

In this section, the extended form of the simple equation method (ESEM) is introduced to obtain the soliton solutions [8, 9].

Step 1: Consider the form of the solution for Eq. (13) as [16-25]:

$$u(\xi) = \sum_{j=-1}^{j=1} B_j f^j(\xi). \tag{15}$$

Here, B_j is a real constant.

Step 2: Find the positive integer N that appeared in Eq. (13) by employing the balance rule between non-linear terms and the highest-order derivative.

Step 3: Suppose that $f(\xi)$ satisfies the following differential equation:

$$f'(\xi) = b_0 + b_1 f(\xi) + b_2[f(\xi)]^2, \tag{16}$$

where b_0, b_1, and b_2 are arbitrary constants.

Step 4: For different values of b_i, the solutions of Eq. (1) are given below:

When $b_0 = 0$,

$$f(\xi) = \frac{b_1 e^{b_1(\xi+\xi_0)}}{1 - b_2 e^{b_1(\xi+\xi_0)}},\quad b_1 > 0, \tag{17}$$

and

$$f(\xi) = -\frac{b_1 e^{b_1(\xi+\xi_0)}}{1 + b_2 e^{b_1(\xi+\xi_0)}},\quad b_1 < 0. \tag{18}$$

When $b_1 = 0$,

$$f(\xi) = \frac{\sqrt{-b_0 b_2}\tanh\left(\sqrt{-b_0 b_2}(\xi + \xi_0)\right)}{b_2},\quad b_0 b_2 < 0. \tag{19}$$

Step 5: A system of equations is produced by substituting Eq. (15) into Eq. (13) and setting the coefficients of powers of $f^j(\xi)$ to zero. After the set of equations is solved, the constant parameter values are found. The solution of Eq. (13) is obtained by carrying these constant values along with the $f(\xi)$ values in Eq. (16).

Now to find the values of N, apply the homogeneous balance principle to Eq. (13). By balancing u^2u'' and $u^{(4)}$, one gets $N + 4 = 2N + N + 2$, then $N = 1$. Thus $u(\xi)$ has the form that is given below:

$$u(\xi) = \frac{B_{-1}}{f(\xi)} + B_0 + B_1 f(\xi),\ B_1 \neq 0. \tag{20}$$

By using Eq. (23) and their derivatives in Eq. (13) along with $b_1 = 0$, we get:

$$b_0 = \sqrt{-\frac{\sigma_4+\sigma_5}{8\sigma_1}},\ b_1 = 0,\ B_{-1} = 0,\ B_0 = 0,\ B_1 = b_2\sqrt{-\frac{20\sigma_1}{\sigma_4+\sigma_5}}. \tag{21}$$

Consequently, a dark soliton solution comes out as

$$\psi_1(x,t) = \left\{\sqrt{\frac{20\sigma_1 b_0 b_2}{\sigma_4+\sigma_5}}\tanh\left(\sqrt{-b_0 b_2}\left(x - k^2\{3c_2\sigma_7 - 8kc_1\sigma_1\}t\right)\right)\right\}\exp^{i(-kx+\omega t+\theta_0)}, \tag{22}$$

where

$$b_0 b_2 < 0,\ \sigma_1(\sigma_4+\sigma_5) < 0.$$

4.2 *Tanh-coth method*

Assume $u = u(\xi)$, by using the ansatz [8, 9]:

$$Y = \tanh(\xi), \tag{23}$$

that leads to the change of variables:

$$\frac{du}{d\xi} = (1-Y^2)\frac{du}{dY}, \tag{24}$$

$$\frac{d^2u}{d\xi^2} = -2Y(1-Y^2)\frac{du}{dY} + (1-Y^2)^2\frac{d^2u}{dY^2}, \tag{25}$$

and

$$\frac{d^4u}{d\xi^4} = 4\left[3a_1Y - (a_1+9b_1)Y^{-3} + 17b_1Y^{-5} + (a_1+b_1)Y^{-1} - 9b_1Y^{-7} - 3a_1Y^3\right]. \tag{26}$$

For the next step, assume that the solution for Eq. (13) is expressed in the form

$$u(Y) = \sum_{i=0}^{p} a_i Y^i + \sum_{i=1}^{p} b_i Y^{-i}, \tag{27}$$

where the parameters p can be found by balancing the highest-order linear term u^5 with the nonlinear terms u^2u'' or uu'^2 in Eq. (13). Therefore, one gets

$$5p = 2p + p + 2, \text{ then } p = 1. \tag{28}$$

The tanh-coth method admits the use of the finite expansion for

$$u(Y) = \left(a_0 + a_1 Y + \frac{b_1}{Y}\right). \tag{29}$$

Here a_0, a_1, and b_1 are constants to be determined. Substituting Eq. (29) with their derivatives into Eq. (13), we get

$$a_{0,1} = \sqrt{3\left[\frac{12\sigma_1 5\sigma_6 - (\sigma_2+\sigma_3)^2}{5\sigma_6(\sigma_2+\sigma_3)}\right]},\ a_{0,3} = \sqrt[4]{-\frac{(\sigma_2+\sigma_3)^2}{5\sigma_6{}^2}},\ a_{0,2} = \sqrt{-\frac{2(\sigma_2+\sigma_3)}{5\sigma_6}},\ a_1 = 0,\ b_1 = \sqrt{\frac{\sigma_2+\sigma_3}{5\sigma_6}}.$$

Accordingly, a singular soliton solution shapes up as:

$$\psi_{2,j}(x,t) = \left\{a_{0,j} + \sqrt{\frac{\sigma_2+\sigma_3}{5\sigma_6}} \coth\left(x - k^2\{3c_2\sigma_7 - 8kc_1\sigma_1\}t\right)\right\} \exp^{i(-kx+\omega t+\theta_0)},\ j = 1, 2, 3 \tag{30}$$

where

$$\sigma_1 = \frac{(\sigma_2+\sigma_3)^2}{5\sigma_6},\ (\sigma_4+\sigma_5) = -\frac{\sigma_2+\sigma_3}{2}.$$

4.3 *Improved modified extended tanh-function method*

This method offers a simpler and more condensed way for the exact optical soliton solution than the other existing schemes. Several authors created the IMETF technique to figure out the soliton solution to several model equations in the deferential sense of derivative, including Jumarie's modified Riemann-Liouville derivatives, conformable derivatives, and Kerr law nonlinearity [10-14].

In this section, the (IMETF) method is introduced to obtain the soliton solutions.

Step 1: Consider the form of the solution to Eq. (13) [26-37]:

$$u(\xi) = \sum_{j=0}^{j=N} A_j f^j + \sum_{j=1}^{j=N} B_j f^{-j}. \tag{31}$$

Here, A_j, and B_j are real constants.

Step 2: Find the positive integer N that appeared in Eq. (13) by employing the balance rule between non-linear terms and the highest-order derivative.

Step 3: Suppose that $f(\xi)$ satisfies the following differential equation:

$$f'(\xi) = \sqrt{g_0 + g_1 f(\xi) + g_2 f^2(\xi) + g_3 f^3(\xi) + g_4 f^4(\xi)}, \tag{32}$$

where g_0, g_1, g_2, g_3, and g_4 are arbitrary constants.

Step 4: For different values of g_i, the solutions of Eq. (13) are given below.

Step 5: A system of equations is produced by substituting Eq. (32) into Eq. (13) and setting the coefficients of powers of $f^j(\xi)$ to zero. After the set of equations is solved, the constant parameter values are found. The solution of Eq. (13) is obtained by carrying these constant values along with the $f(\xi)$ values in Eq. (32).

With the integrity of homogeneous evaluating of the highest order derivative terms u^2u'' and $u^{(4)}$, one gets $N + 4 = 2N + N + 2$, then $N = 1$. Thus, our technique permits us to use the supplementary solution of the form:

$$u(\xi) = \left[A_0 + A_1 f(\xi) + \frac{B_1}{f(\xi)}\right]. \tag{33}$$

By using Eq. (33) and their derivatives in Eq. (13), we get the following cases:

Case I

Taking $g_0 = g_1 = g_3 = 0$, one secures the results:

$$A_0 = 0, B_1 = 0,\ A_1 = \mp g_2\sqrt{\frac{2\sigma_1}{3(\sigma_2+\sigma_3)}},\ g_4 = -\frac{{g_2}^2}{12},\ \sigma_4 = -\sigma_5,\ \sigma_6 = -\frac{(\sigma_2+\sigma_3)^2}{4}. \tag{34}$$

When $g_2 > 0$, and $g_4 < 0$, we acquire a bright soliton solution:

$$\psi_3(x,t) = \mp g_2\sqrt{\frac{2\sigma_1}{3(\sigma_2+\sigma_3)}}\operatorname{sech}\left\{\sqrt{g_2}(x-\gamma t)\right\}\exp^{i(-kx+\omega t+\theta_0)}. \tag{35}$$

Case II

Choosing $g_0 = g_1 = g_3 = 0$, the outcomes are:

$$A_0 = 0,\ \sigma_4 = -\sigma_5,\ \sigma_6 = 0,\ g_2 = 8g_4. \tag{36}$$

Family I

$$\text{For } A_1 = B_1 = \Delta_1 = \sqrt{\frac{4\sigma_1(4g_4+1)}{3(\sigma_2+\sigma_3)}},\ A_2 = B_2 = \Delta_2 = \sqrt{-\frac{12\sigma_1 g_4}{(\sigma_2+\sigma_3)}}.$$

$$\text{For } A_3 = B_3 = \Delta_3 = \sqrt{-\frac{2\sigma_1}{(\sigma_2+\sigma_3)}},\ A_4 = B_4 = \Delta_4 = \sqrt{\frac{-64\sigma_1 g_4}{7(\sigma_2+\sigma_3)}}.$$

$g_2 > 0$, and $g_4 < 0$, we attain a straddled bright-singular soliton solution:

$$\psi_{4,j}(x,t) = \sqrt{\frac{4\sigma_1(4g_4+1)}{3(\sigma_2+\sigma_3)}}\left[\operatorname{sech}\left\{\sqrt{g_2}(x-\gamma t)\right\} + \cosh\left\{\sqrt{g_2}(x-\gamma t)\right\}\right]\exp^{i(-kx+\omega t+\theta_0)},$$

$$j = 1, 2, 3, 4. \tag{37}$$

Case III

Choosing $g_1 = g_3 = 0$, we have the solution set:

$$A_0 = 0,\ B_1 = 0,\ A_1 = 2\sqrt{\frac{(3g_4 - 2g_2)\sigma_1 g_4}{(\sigma_2 + \sigma_3)(g_2 - g_4)}},\ \sigma_4 = -\sigma_5. \tag{38}$$

Taking $g_2 < 0$, $g_4 > 0$, and $g_0 = \dfrac{(g_2^{\,2} + 12g_4)\left\lceil g_4 - g_2 \right\rceil}{4g_4(3g_4 - 2g_2)}$, a dark soliton solution is acquired:

$$\psi_5(x,t) = -2\sqrt{\frac{(3g_4 - 2g_2)\sigma_1 g_4}{(\sigma_2 + \sigma_3)(g_2 - g_4)}} \tanh\left\{\sqrt{g_2}\,(x - \gamma t)\right\} \exp^{i(-kx + \omega t + \theta_0)}. \tag{39}$$

Case IV

Assuming $g_1 = g_3 = g_4 = 0$, we get the findings:

$$\sigma_4 + \sigma_5 + \sigma_2 + \sigma_3 = 0,\ \sigma_6 = 0,\ A_0 = 0,\ B_1 = 0,\ A_1 = \mp g_2\sqrt{\frac{\sigma_1}{-(\sigma_2 + \sigma_3)g_0}}.$$

Thus, a bright soliton solution comes out as:

$$\psi_6(x,t) = g_2\sqrt{\frac{\sigma_1}{-(\sigma_2 + \sigma_3)g_0}} \operatorname{sech}\left\{\frac{\sqrt{g_2}}{2}(x - \gamma t)\right\} \exp^{i(-kx + \omega t + \theta_0)},\ g_2 > 0. \tag{40}$$

Case V

Setting $g_1 = g_3 = g_4 = 0$, one arrives at the consequences:

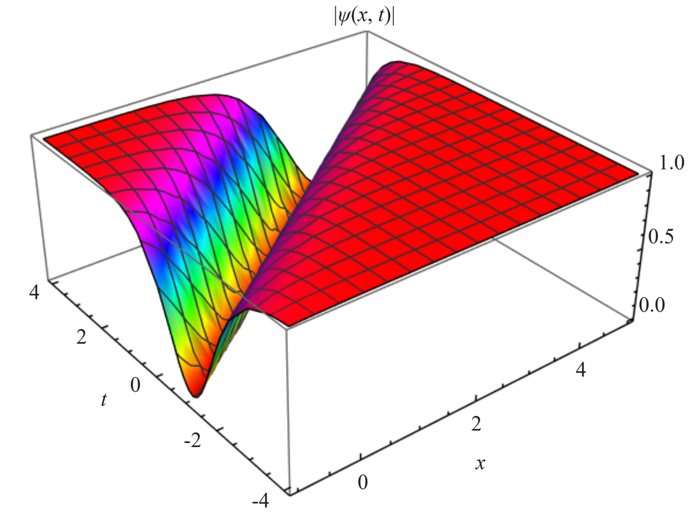
$$A_0 = 0,\ A_1 = 0,\ B_{1,1} = \sqrt{\frac{2\sigma_1(3g_0 - 1)}{(\sigma_2 + \sigma_3)}},\ B_{1,2} = \sqrt{\frac{\sigma_1(g_2^{\,2} + 12g_0 g_4)}{(\sigma_2 + \sigma_3)g_4}},\ \sigma_4 = -\sigma_5,\ g_2 = \sqrt{-2(1 + 3g_0)g_4}.$$

Accordingly, a singular soliton solution is obtained from the analysis as follows:

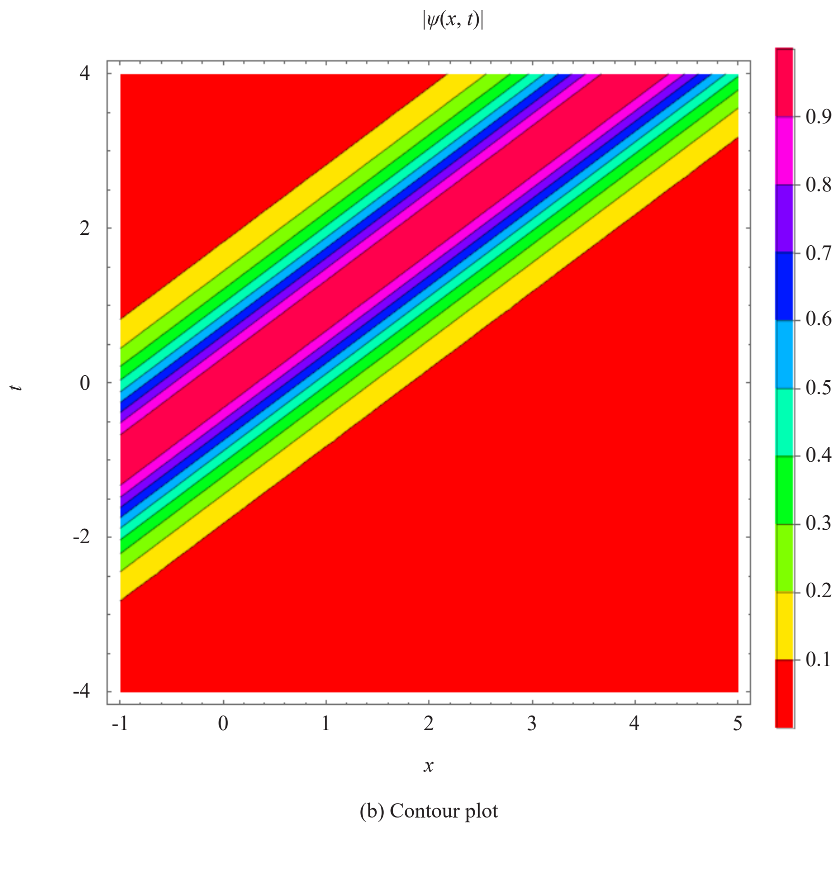
$$\psi_{7,j}(x,t) = -B_{1,j} \coth\left\{\sqrt{\frac{(1 + 3g_0)g_4}{2}}(x - \gamma t)\right\} \exp^{i(-kx + \omega t + \theta_0)},\ j = 1, 2. \tag{41}$$

5. Results and discussion

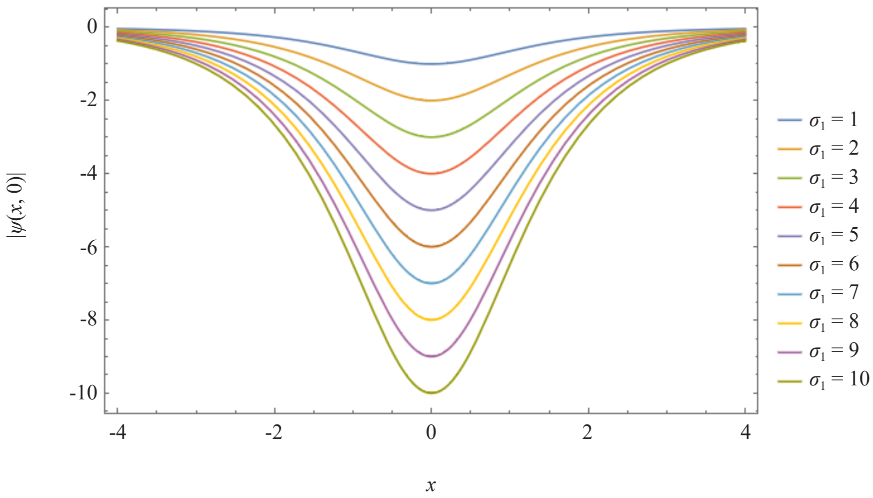
With the help of Eqs. (1), we have successfully derived optical soliton solutions for the concatenation model. Three methods are used: the ESEM, the Tanh-Coth method, and the Improved Modified Extended Tanh-Function method (IMETF). The system's optical soliton solutions are denoted by $\psi(x, t)$. Equations (4) and (5) are the system of real and imaginary equations that are derived from the concatenation model. Next, we investigate this system's optical soliton solutions utilizing the three techniques. Some restrictions were assumed. Surface, contour, and 2D plots of dark and bright soliton solutions described by Eqs. (22) and (35) are shown in Figures 1 and 2, respectively. In Figure 1, the parameters that have been chosen are $k = 1$, $c_1 = 1$, $c_2 = 1$, $\sigma_7 = 1$, $b_0 = 1$, $b_2 = -1$, and $\sigma_5 = 1$, while the parameters $g_2 = 1$, and $\sigma_3 = 1$ are addressed in Figure 2.



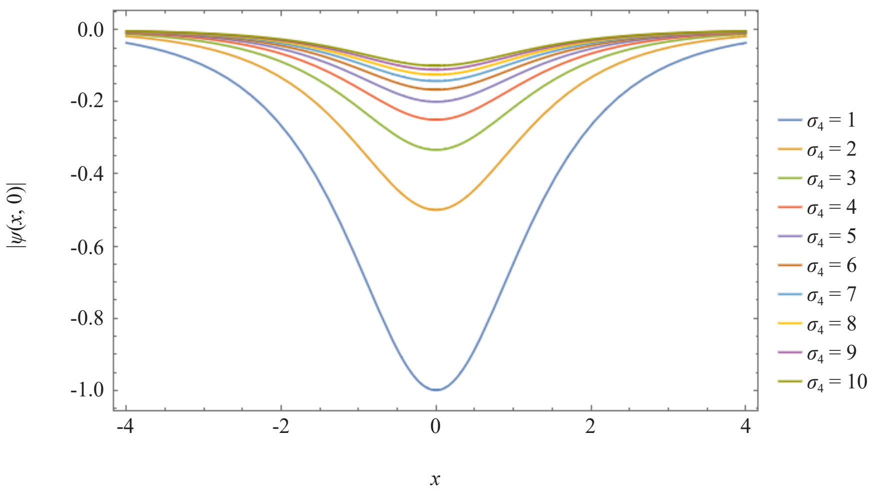
(a) Surface plot



(b) Contour plot



(c) The effect of fourth-order dispersion



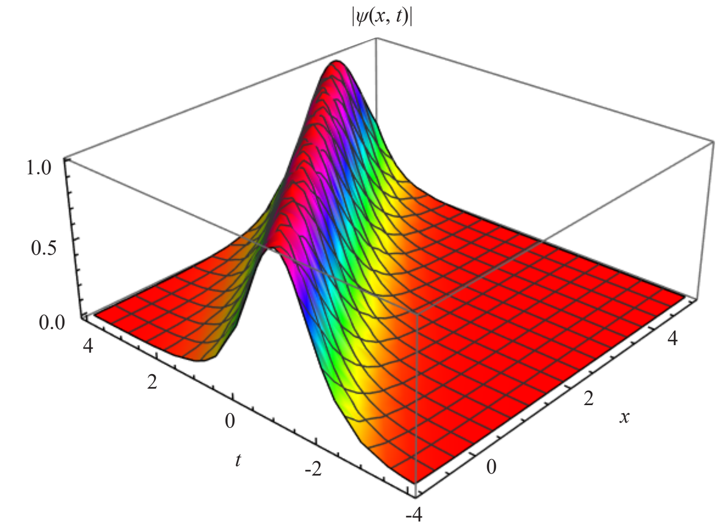
(d) The effect of nonlinear dispersion

Figure 1. Profile of a dark soliton solution (22)

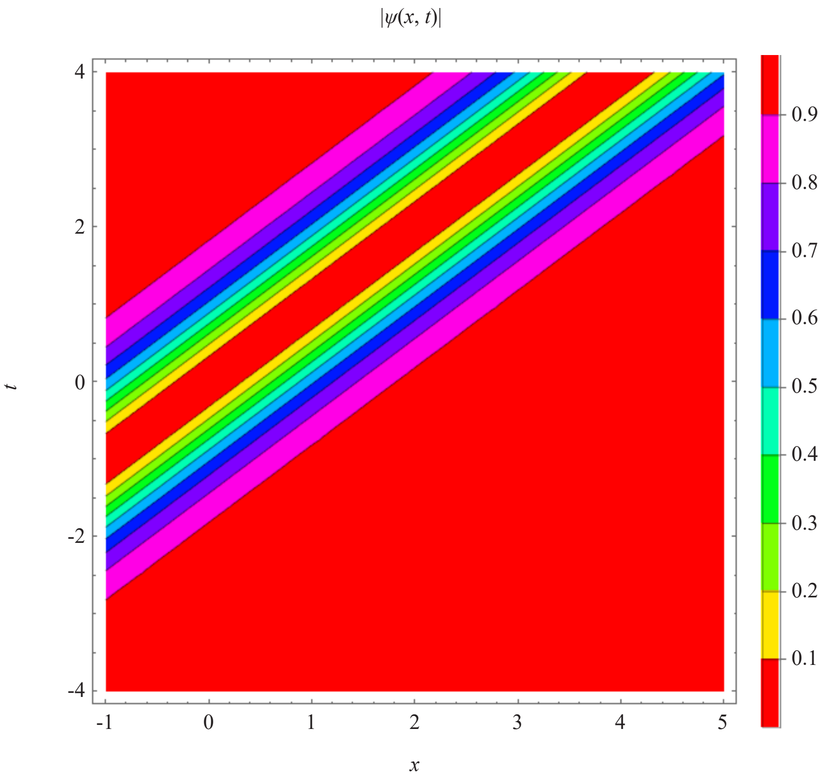
The Figures presented in our analysis illustrate not only the functional dependence of dark and bright soliton solutions on the parameters of fourth-order dispersion (σ_1) and nonlinear dispersions (σ_2 and σ_4) but also provide insights into their intricate properties. By examining the behavior of these solutions across varying parameter values, we uncover nuanced characteristics such as amplitude modulation. Furthermore, through detailed numerical simulations and theoretical analyses, we elucidate the underlying mechanisms driving the observed behaviors. Specifically, we explore how changes in the dispersion parameters influence the formation, propagation, and interaction of dark and bright solitons within the medium. By delving into these properties, our analysis not only establishes the functional dependencies but also enhances our understanding of the dynamic nature of soliton solutions in the context of the studied system.

In our investigation, we have observed that modifying the parameter yields varying outcomes for $\psi(x, t)$. However, it is important to elucidate that these differences extend beyond mere variations in the numerical values of $\psi(x, t)$. Rather, the alterations in the parameter manifest distinct patterns of behavior within the system under study. Thus, the essence of the differences lies not only in the diverse numerical outputs but also in the underlying dynamics and emergent phenomena shaped by the parameter's manipulation.

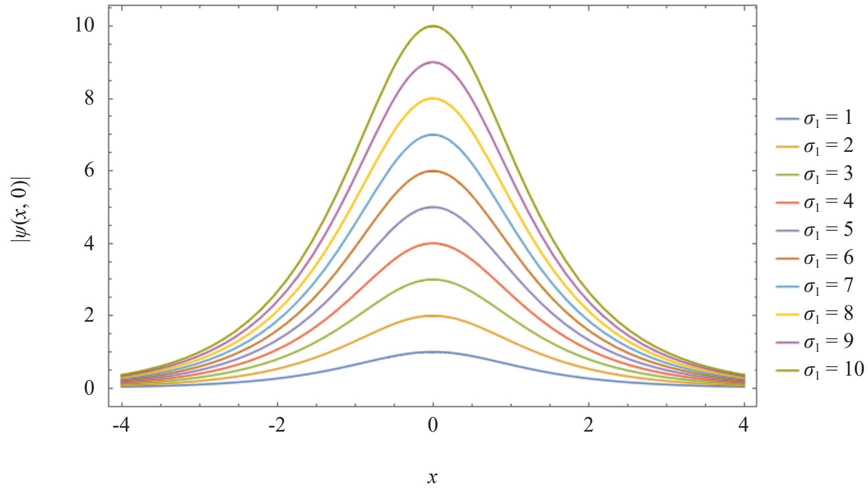
Our analysis, conducted through rigorous application of established methods, consistently yields robust results. The observed consistency and strength of these results are indicative of the reliability and effectiveness of the methodologies employed.



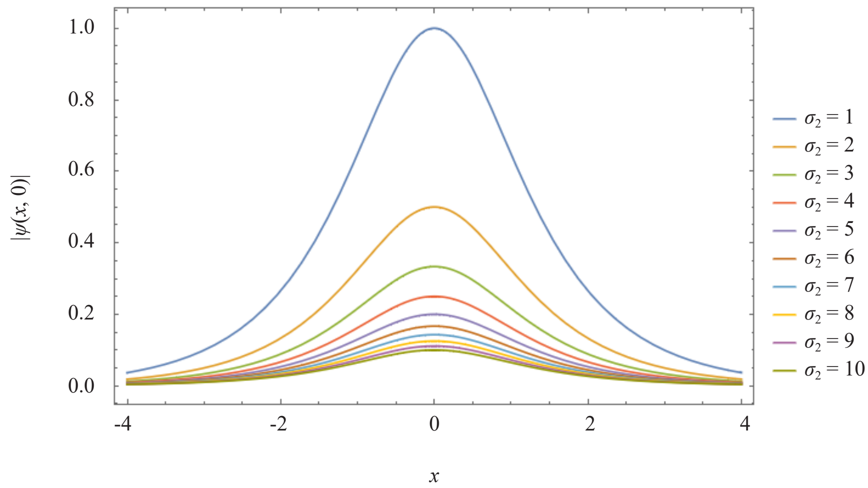
(a) Surface plot



(b) Contour plot



(c) The effect of fourth-order dispersion



(d) The effect of nonlinear dispersion

Figure 2. Profile of a bright soliton solution (35)

6. Conclusion

This paper reports the application of three integration algorithms that were implemented for the first time to the concatenation model. The results align with those reported in previous papers. However, the implementation of the three integration technologies successfully to the concatenation model, with Kerr law of SPM, is being presented for the first time in this paper. The three successfully applied integration algorithms are the extended simple equation approach, the tanh-coth method, and the improved modified extended tanh-function approach. Later these methodologies will be implemented into the concatenation model with polarization-mode dispersion as well as for dispersion-flattened fibers. Additional forms of SPM and the application of these algorithms to the dispersive concatenation model are awaited at this time. These results will be made visible after they are aligned with the previously reported works [8-15].

Conflict of interest

The authors declare no conflict of interest.

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