Research Article

Parameter Design in Production by Means of Robust Fuzzed PMOO in Case of Desirable Target

Maosheng Zheng1* [,](https://orcid.org/0000-0003-3361-4060) Jie Yu[2](https://orcid.org/0000-0001-6606-5462)

¹School of Chemical Engineering, Northwest University, Xi'an, 710069, China
²School of Life Science, Northwest University, Xi'an, 710060, China ²School of Life Science, Northwest University, Xi'an, 710069, China Email: mszhengok@aliyun.com

Received: 29 September 2024; **Revised:** 23 December 2024; **Accepted:** 3 January 2025

Abstract: A proper parameter design in the production process is critical to guarantee the quality of products and their improvement. The rationality of the previous traditional approaches for robust assessment including the Taguchi method and dual response method is questionable. In this article, the combination of probabilistic multi-objective optimization (PMOO) with membership approach in fuzzy theory is developed to conduct parameter design of production in case of desirable target with robustness deeply, which is furthermore applied to two examples of both parametric design of gas metal arc (GMA) welding process and printing machine's ability. In the new approach, the mean value of "complement" of membership value of a set of test data belonging to its desired target of an objective response is taken as one subobjective response, which is an unbeneficial type of index in the assessment to contribute the first part of the partial preferable probability of the objective. In contrast, the dispersion of a set of test data in terms of membership with respect to the desired target value is taken as the other sub-objective response to contribute the second part of the partial preferable probability of the objective simultaneously, which is an unbeneficial index. Thus, the fuzzed PMOO approach is regulated comprehensively. Besides, the consequences of application examples reflect the reasonability of the approach as an auxiliary measure for PMOO consistently to perform optimal robust design.

*Keywords***:** robust design, desirable target, simultaneous optimization, probability theory, fuzzy theory

1. Introduction

A proper parameter design in the production process is quite important to guarantee the quality of products and their improvement. Usually, the arc welding process is complicated and nonlinear; therefore, optimum parameters can be derived with experimental data from appropriately designed welding tests and optimization algorithms.

In general, experiments of the welding process for optimization are influenced by the actual working environment, resulting in unreliability and uncertainty in the welding quality under corresponding actual welding environments. While the robust optimization of the welding process is to seek a set of input parameters, which are insensible to the external environment so as to guarantee the quality of the welding products with high reliability.

A robust parametric design of the printing machine's ability is another typical design to find a set of specific input parameters that ensure the printing machine works close to the desired status with less fluctuation.

Copyright ©2025 Maosheng Zheng, et al.

DOI: https://doi.org/10.37256/est.6120255823 This is an open-access article distributed under a CC BY license

⁽Creative Commons Attribution 4.0 International License)

https://creativecommons.org/licenses/by/4.0/

Early in the 1950s, Box and Taguchi noticed the importance of robust design [1], [2]. Taguchi once proposed the so-called "Taguchi Method" to determine specific controllable parameters under certain conditions to ensure good quality of products, which is insensitive to the disturbance of environmental factors.

In Taguchi's method, a signal-to-noise ratio (SNR) was proposed to assess the robust design [2].

However, Taguchi's SNR was queried by some statisticians, the main reason is the lack of revealing the actions of both mean value and variation of objective responses separately [3], [4]. Subsequently, many modifications and improvements have been put forward, which attempted to solve the above problem [5]-[15], but the intrinsic shortcoming remained still [16], [17].

Recently, the probabilistic multi-objective optimization (PMOO) was proposed [4], which considers the optimization problem of multiple objectives being the integral/overall optimization of multiple objectives within a system, and makes every objective as one independent event of probability theory analogically; furthermore the simultaneous appearance of all those objectives represents the integrity of the system, and the maximum of the joint probability of the system corresponds to the overall optimum state of the system according to systems theory and probability theory analogically [4]. The utility of every objective is quantified by a new term of "partial preferable probability" according to its preference or role in the optimization; furthermore, the product of all partial preferable probabilities results in the "total preferable probability" of a candidate scheme, which is the unique decisive index of the status of the candidate scheme, the maximum of the total preferable probability of the system corresponds to overall optimum state of the system [4]. As to robust design, the mean value and its variance of an objective are taken as dual objectives of the multi-objective optimization problem, thus the robust design problem seems to be solved. Under conditions of 'the smaller the better' and 'the larger the better', it is no doubt about solving the problems as they are directly. Currently, for the case of desired 'target the best', i.e., "desirable target" or "ideal value as the target" in plan or in mind, the departure of the actually tested values from the target value is considered [16], [17], it is a promising solution, especially the membership approach of fuzzy theory, the mean value of tested result belonging to the desired target value of an objective response is taken as one objective response, which is a brand new method and value to be examined extensively.

However, since the maximum value of membership *μ* is 1, a finite value, instead of infinite, an alternative appropriate manner to deal with this problem can be put forward by introducing a "complement" of the membership value, i.e., $\eta = 1 - \mu$, to conduct the evaluation logically. Additionally, in the condition of robust assessment, the dispersion of test data must be taken into account properly.

In this article, a rational robust parameter design of production in case of desired value as target is developed in terms of the "complement" value of the membership; moreover, two examples are represented to illuminate the procedure, which includes gas metal arc (GMA) welding process and parametric design of printing machine's ability.

2. Regulation of fuzzed probabilistic robust design in case of desired value as target 2.1 *Combination of PMOO with membership approach of fuzzy theory*

In the case of the desired value as the target of an objective response, the actual closeness of each test data *y* to the desired target value y_0 can be used to measure its closeness to the target. Thus, the greater the distance of the test data *y* from the desired target value y_0 is, the smaller the closeness or degree of proximity to the desired target. This is something like the membership in fuzzy theory [17]. The assessment of membership *μ* of a test data *y* belonging to the desired target value y_0 is as follows rationally,

$$
\begin{cases} \mu = 1, \ y = y_0; \\ \mu = 1 - |y - y_0| / \delta, \ |y - y_0| \le \delta; \\ \mu = 0, \ |y - y_0| > \delta. \end{cases}
$$
 (1)

In Eq. (1), μ is the membership function of the test data *y* belonging to the desired target value y_0 , and δ is the preassigned value, over which the value of the membership function becomes 0.

However, since the limit value of membership of *y* belonging to y_0 is "1", which is a finite value instead of an infinitely large one. It seems improper to consider this optimization problem as "the larger the better" type, i.e., beneficial indicator. Since, in the latter type, the value of the objective response has the possibility to take a value of infinitely large value instead of a finite one.

Alternatively, the "complement" *η* of the membership value μ can be used as the appropriate index to deal with this problem with Eq. (2),

$$
\eta = 1 - \mu \tag{2}
$$

The limit value of η is 0, which corresponds to μ taking its maximum value of 1. So, the optimization problem of μ taking its maximum value is equivalent to *η* approaching its minimum value of 0.

Furthermore, as to robustness assessment, since the unavoidability of dispersion of a set of test data in the same experimental conditions due to the effects of external uncertain factors, such as environments, testing, raw materials, etc., the dispersion of a set of test data must be taken into account in the evaluation surely [4], [16], [17].

In the light of Lin and Tu's discussion [7], [16], [17], the dispersion of a set of test data in terms of membership *η* of fuzzy theory can be characterized as follows,

$$
\mathbf{s}_{\mu} = (\overline{\eta}^2 + \sigma_{\mu}^2)^{0.5} \tag{3}
$$

In Eq. (3), σ_{μ} is the standard error of membership value μ of the set of test data in the same experimental conditions; $\overline{\eta}$ is the mean value of "complement" *η* of the membership value in the corresponding set; s_n reflects the dispersion of a set of test data in term of membership with respect to the desired target value to join the assessment of the second part of partial preferable probability. Meanwhile, $\bar{\eta}$ is an unbeneficial index to join the assessment of the first part of partial preferable probability.

2.2 *Assessment of preferable probability*

Furthermore, the parts P_{s_μ} and $P_{\overline{r}_\alpha}$ of the partial preferable probability, corresponding to both s_μ and $\overline{\eta}$ for an objective, can be conducted as unbeneficial indicators to perform the assessment [4], [16], [17]. As a result, the partial preferable probability P_{ij} of the *j*-th objective of the *i*-th candidate scheme (alternative) is the product of both P_{s} and $P_{\overline{s}}$; subsequently, the total preferable probability P_i of the *i*-th candidate scheme is [4], [16], [17],

$$
P_i = \prod_{j=1}^{m} P_{ij}, \ i = 1, 2, ..., n; \ j = 1, 2, ..., m
$$
\n⁽⁴⁾

In addition, the evaluations for partial preferable probability P_{ij} of an objective in both beneficial and unbeneficial conditions were proposed in [4], [16], [17], which are cited as follows.

As to the beneficial attribute, its partial preferable probability P_{ij} can be quantitatively written as [4], [16], [17],

$$
P_{ij} = A_j X_{ij}, i = 1, 2, 3, ..., n; j = 1, 2, 3, ..., m
$$
\n(5)

$$
A_j = 1/(n\overline{X_j})\tag{6}
$$

In Eq. (5), *n* is the total number of alternatives in the system; *m* represents the total number of attribute (objective) indicators of each alternative; X_{ij} is the value of utility value of the *j*-th objective (attribute) indicator of the *i*-th alternative; X_i is the averaged value of the utility of the *j*-th objective indexes in the attribute group.

Equivalently, for attributes of the unbeneficial type, its partial preferable probability P_{ij} can be quantitatively written as [4], [16], [17],

$$
P_{ij} = B_j (X_{j\max} + X_{j\min} - X_{ij}), \ i = 1, 2, 3, ..., n; \ j = 1, 2, 3, ..., m
$$
 (7)

$$
B_j = 1/ [n(X_{j\text{max}} + X_{j\text{min}} - \overline{X_j})]
$$
\n⁽⁸⁾

In Eqs. (7) and (8), $X_{j_{\text{max}}}$ and $X_{j_{\text{min}}}$ express the maximum and minimum values of utility of the objective indicators in the *j-*th attribute group, respectively.

Finally, the overall optimal option of the system is the specific scheme with the highest total preferable probability [4], [16], [17].

3. Applications

3.1 *Application in GMA welding process*

As an application of robust parameter optimization for cases where the desired value serves as the target of an objective, the parametric optimization for the gas metal arc (GMA) welding process is employed to illuminate the procedure in detail.

Kim and Rhee once studied the GMA welding process with a dual response approach [18]. Here in this article, this problem is restudied by using the newly developed probabilistic robust design of a product to demonstrate its utilization in case of desired value as a target.

The base metal used in the welding material for Kim and Rhee's study was mild steel [18], welded as an I-groove type joint with a thickness of 5.8 mm. The AWS ER 70S-6 brand electrode wire was used, with a diameter of 1.2 mm. CO₂ was used as the shielding gas, flowing at a rate of 20 liter/min. A constant voltage welding power source was used. The variation range of the root opening was $0.4 \sim 1.2$ mm [18]. The target value for penetration y_0 was 3.5 mm.

In Kim and Rhee's study, 9 groups of tests were conducted according to the experimental design. The mean value and standard deviation of the output response were then caused from the test data of 5 specimens for each designed experiment. The input controllable variables include the rate (x_1) of wire-feed and welding rate (x_2) . The output response is the penetration, which includes the mean value \bar{y} and standard error *σ* of the penetration. The level of each input controllable variable for $2³$ factorial designs is cited and shown in Table 1. Subsequently, their experimental results are cited in Table 2.

The evaluation results for the membership values *μ* and the corresponding errors of GMA welding process are shown in Table 3, in which the pre-assigned δ takes a value of 0.8 mm, i.e., $\delta = 0.8$ mm.

The results in Table 4 show that test scheme No. 2 gains the maximum total preferable probability P_t . Therefore, the robust status from these test data is test scheme No. 2, with a mean value of penetration of 3.42 mm and a standard error of $\sigma = \left[\sum_{i=1}^{5} (f_i - \overline{f})^2 / 5 \right]^{0.5}$ $\sum_{i=1}^{1}(f_i - f)^2 / 5$ = 0.3487 mm $\sigma = \frac{\sum (f_i - f)}{\sum (f_i - f)}$ $=\left[\sum_{i=1}^{5} (f_i - \overline{f})^2 / 5\right]^{1/3} = 0.3487$ mm at a wire-feed rate of $x_1^* = 75$ mm/s and a welding rate of $x_2^* = 6$ mm/ s. Similarly, when the value of *δ* was taken as 1.0 mm [19], the same result was obtained, which indicates that the preassigned value of δ falls within a reasonable range.

Welding rate (mm/s), x_2 6 8 10

Table 1. Parameter design of GMA welding process experiment [18]

No.	Input variable					Experimental result of penetration (mm), f	Mean value, (mm)	Standard error, (mm)	
	x_1 , (mm/s)	x_2 , (mm/s)	f_1	f_2	f_3	f_4	f_5	\overline{f}	σ
$\mathbf{1}$	60	6	2.7	2.9	3.1	3.1	3.4	3.04	0.2332
$\overline{2}$	75	6	3.0	3.2	3.3	3.6	4.0	3.42	0.3487
3	90	6	3.6	3.8	4.2	4.3	4.5	4.08	0.3311
$\overline{4}$	60	8	2.4	2.6	2.6	2.7	2.8	2.62	0.1327
5	75	8	2.5	2.7	3.0	3.5	3.8	3.10	0.4858
6	90	8	2.9	3.3	3.6	3.7	4.1	3.52	0.4020
7	60	10	1.9	1.9	2.2	2.2	2.5	2.14	0.2245
8	75	10	1.8	2.4	2.7	2.9	3.2	2.60	0.4775
9	90	10	2.6	2.7	3.2	3.5	3.9	3.18	0.4874

Table 2. Experimental results of GMA welding process [18]

Table 3. Membership values μ and the corresponding errors of GMA with δ = 0.8 mm

No.			Membership function μ		Mean value $\bar{\eta}$	$\bar{\eta}$	$\sigma_{\!\scriptscriptstyle\mu}$	S_μ	
$\mathbf{1}$ 1	$\boldsymbol{0}$	0.25	0.5	0.5	0.875	0.425	0.575	0.3453	0.6707
$\mathfrak{2}$	0.375	0.625	0.75	0.875	0.375	0.6	0.4	0.3124	0.5075
3	0.875	0.625	0.125	$\mathbf{0}$	$\boldsymbol{0}$	0.325	0.675	0.3687	0.7692
$\overline{4}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	0.125	0.025	0.975	0.0559	0.9766
5	$\boldsymbol{0}$	$\boldsymbol{0}$	0.375	1	0.625	0.4	0.6	0.4008	0.7216
6	0.25	0.75	0.875	0.75	0.25	0.575	0.425	0.3211	0.5327
7	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	1	$\mathbf{0}$	\pm
8	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	0.25	0.625	0.175	0.825	0.2627	0.8658
9	$\boldsymbol{0}$	$\boldsymbol{0}$	0.625	1	0.5	0.425	0.575	0.3982	0.6994

Table 4. Evaluation results of the preferable probability

L,

No.	$P_{\overline{\eta}}$	P_{s_μ}	$P_t \times 10^2$	Rank
6	0.1489	0.1429	2.1264	2
$\overline{7}$	0.0611	0.0744	0.4542	9
8	0.0878	0.0940	0.8255	7
9	0.1260	0.1184	1.4916	4

Table 4. (cont.)

3.2 *Application in the parametric design of printing machine's ability*

Vining and Myers once raised the problem of parametric design of printing machine's ability [20], [21]. Three factors were involved, i.e., speed x_1 , pressure x_2 , and distance x_3 , which affect the printing machine's ability to use ink to package labels. A $3³$ complete factorial design was used in the experiments which were with three replicates at each design point. The goal was to seek the optimal location at which the printing number is around 500 with minimum variance. Table 5 cited the experimental results.

From Table 5, it can be seen that the sample standard deviation of zero falls at two locations, alternatives 10 and 14, but their printing numbers are significantly far from the target value $y_0 = 500$. Table 6 provides the fuzzification evaluation results with $y_0 = 500$ and $\delta = 100$. Table 7 shows the evaluation results using PMOO, which shows that the scheme No. 23 of experimental alternative gains the highest overall preferable probability. Thus, scheme No. 23 can be selected as the optimal design, which corresponds to the coded input variables speed $x_1 = 0$, pressure $x_2 = 0$, and distance $x_3 = 1$. At scheme No. 23, the mean printing number is 485.33 with a standard error of 44.64. Analogically, when the value of δ was taken as 200 in [19], it got the same result.

This approach avoids the effect of the puzzled standard deviation of zero, such as in this example at two locations, alternative schemes No. 10 and No. 14.

No.	Coded input variable				Output response		Mean value	Standard error
	x_1	x_2	x_3	y_1	\mathcal{Y}_2	\mathcal{Y}_3	\overline{y}	σ
$\mathbf{1}$	-1	-1	-1	34	$10\,$	$28\,$	24.00	12.49
$\mathfrak{2}$	$\boldsymbol{0}$	-1	-1	115	116	130	120.33	8.39
\mathfrak{Z}	$\mathbf{1}$	-1	-1	192	186	263	213.67	42.83
$\overline{4}$	-1	$\boldsymbol{0}$	-1	82	$88\,$	$88\,$	86.00	3.46
5	$\boldsymbol{0}$	$\boldsymbol{0}$	-1	44	178	188	136.67	80.41
6	$\mathbf 1$	$\boldsymbol{0}$	-1	322	350	350	340.67	16.17
τ	-1	$\mathbf{1}$	-1	141	110	86	112.33	27.57
8	$\boldsymbol{0}$	$\mathbf{1}$	-1	259	251	259	256.33	4.62
9	$\mathbf{1}$	$\mathbf{1}$	-1	290	280	245	271.67	23.63
$10\,$	-1	-1	$\boldsymbol{0}$	$8\sqrt{1}$	81	$8\sqrt{1}$	81.00	0.00
11	$\boldsymbol{0}$	-1	$\boldsymbol{0}$	90	122	93	101.67	17.67

Table 5. Cited experimental result of printing ink data [20], [21]

No.		Coded input variable			Output response		Mean value	Standard error
	$x_{\scriptscriptstyle 1}$	x_2	x_3	y_1	y_2	y_3	\overline{y}	σ
12	$\mathbf{1}$	-1	$\boldsymbol{0}$	319	376	376	357.0	32.91
13	-1	$\mathbf{0}$	$\boldsymbol{0}$	180	180	154	171.33	15.01
14	$\boldsymbol{0}$	$\mathbf{0}$	$\boldsymbol{0}$	372	372	372	372.0	0.00
15	$\mathbf{1}$	$\boldsymbol{0}$	$\boldsymbol{0}$	541	568	396	501.67	92.50
16	-1	$\mathbf{1}$	$\boldsymbol{0}$	288	192	312	264.00	63.50
17	$\boldsymbol{0}$	$\mathbf{1}$	$\boldsymbol{0}$	432	336	513	427.00	88.61
18	$\mathbf{1}$	$\mathbf{1}$	$\boldsymbol{0}$	713	725	754	730.67	21.08
19	-1	-1	$\mathbf{1}$	364	99	199	220.67	133.82
20	$\boldsymbol{0}$	-1	$\mathbf{1}$	232	221	266	239.67	23.46
21	$\,1$	-1	$\mathbf{1}$	408	415	443	422.00	18.52
22	-1	$\boldsymbol{0}$	$\mathbf{1}$	182	233	182	199.00	29.44
23	$\boldsymbol{0}$	$\boldsymbol{0}$	$\mathbf{1}$	507	515	434	485.33	44.64
24	$\,1$	$\boldsymbol{0}$	$\mathbf{1}$	846	535	640	673.67	158.21
25	-1	$\mathbf{1}$	$\mathbf{1}$	236	126	168	176.67	55.51
26	$\boldsymbol{0}$	$\mathbf{1}$	$\,1$	660	440	403	501.00	138.94
27	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	878	991	1,161	1,010.00	142.45

Table 5. (cont.)

Table 6. Fuzzification evaluation results with $y_0 = 500$ and $\delta = 100$

No.		Membership function μ		Mean value $\bar{\mu}$	$\sigma_{\!\scriptscriptstyle\mu}$	$\bar{\eta}$	S_μ
$\mathbf{1}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\mathbf{0}$	$\boldsymbol{0}$	1	1
$\overline{2}$	θ	θ	$\mathbf{0}$	θ	θ	1	
3	θ	$\mathbf{0}$	$\boldsymbol{0}$	$\mathbf{0}$	$\boldsymbol{0}$		
$\overline{4}$	θ	$\boldsymbol{0}$	$\mathbf{0}$	$\overline{0}$	$\boldsymbol{0}$		
5	θ	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	1	
6	θ	$\mathbf{0}$	$\boldsymbol{0}$	$\mathbf{0}$	$\boldsymbol{0}$		
7	$\overline{0}$	$\mathbf{0}$	$\mathbf{0}$	$\overline{0}$	$\boldsymbol{0}$		
8	θ	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	1	
9	Ω	$\mathbf{0}$	$\mathbf{0}$	Ω	$\mathbf{0}$		
$10\,$	$\mathbf{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$		1

			Mean value $\bar{\mu}$	$\sigma_{\!\scriptscriptstyle\mu}$	$\bar{\eta}$	S_μ
$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\mathbf{1}$	$\mathbf{1}$
$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\mathbf{1}$	$\,1$
$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\,1$	$\,1\,$
$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\,1\,$	$\,1$
0.59	0.32	$\boldsymbol{0}$	0.3033	0.3875	0.6967	0.7972
$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\,1$	$\mathbf{1}$
0.32	$\boldsymbol{0}$	0.87	0.3967	0.5352	0.6033	0.8065
$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\,1$	$\mathbf{1}$
$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\,1$	$\mathbf{1}$
$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\,1$	$\,1$
$0.08\,$	0.15	0.43	0.22	0.2670	0.78	0.8244
$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\,1\,$	$\,1$
0.93	0.85	0.34	0.7067	0.7534	0.2933	0.8085
$\boldsymbol{0}$	0.65	$\boldsymbol{0}$	0.2167	0.3753	0.7833	0.8686
$\overline{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\,1\,$	$\mathbf{1}$
$\boldsymbol{0}$	0.4	0.03	0.1433	0.2316	0.8567	0.8874
$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\mathbf{1}$	$\mathbf{1}$
No. 11 12 13 14 15 16 17 $18\,$ 19 20 $21\,$ $22\,$ 23 24 25 26 27			Membership function μ			

Table 6. (cont.)

No.	$P_{\overline{n}}$	P_{s_μ}	$P_{\rm r} \times 10^4$	Rank
11	0.0296	0.0372	11.0128	
12	0.0296	0.0372	11.0128	
13	0.0296	0.0372	11.0128	
14	0.0296	0.0372	11.0128	
15	0.0602	0.0365	22.0130	3
16	0.0296	0.0372	11.0128	
17	0.0697	0.0364	25.3184	\overline{c}
18	0.0296	0.0372	11.0128	
19	0.0296	0.0372	11.0128	
20	0.0296	0.0372	11.0128	
21	0.0518	0.0367	19.0303	$\overline{4}$
22	0.0296	0.0372	11.0128	-
23	0.1009	0.0357	36.0286	$\mathbf{1}$
24	0.0515	0.0367	18.9104	5
25	0.0296	0.0372	11.0128	-
26	0.0441	0.0369	16.2599	6
27	0.0296	0.0372	11.0128	

Table 7. (cont.)

4. Discussion

In the present approach, the option of the value of the parameter δ is quite important, it should be chosen properly not only to cover an appropriate number of test data but also to keep the test data close to the desired target value for the specific problem. If the value of the parameter δ is chosen too big, the difference of the test data cannot be revealed in terms of membership function in the manner of fuzzy theory distinctly; otherwise, if the value of the parameter δ is too small, there is no enough number of test data to participate the evaluation by means of probabilistic multi-objective optimization. As to an intensity type of physical quantity, it could suffer a 10% fuzziness (*δ*) in actual treatment approximately in general [4], [22]. Here in our studies for the GMA welding process and parametric design of printing machine's ability problems, the desired target values for penetration (3.5 mm) and printing number (500), belong to the intensity type of physical quantities. The only exception here is that a 20% fuzziness is employed so as to cover more data from the experiments.

5. Conclusion

Disturbances and changes of unavoidable factors, such as environments, testing, raw materials, etc., lead to uncertainty of product quality. In this paper, a fuzzified approach of robust design for the case of desired value as a target is developed in terms of probabilistic multi-objective optimization in this paper. The consequences of application examples for robust designs in GMA welding process parameters and the printing machine's ability indicate the reasonability of the fuzzification approach as an auxiliary measure.

Conflict of interest

The authors declare no competing interest.

References

- [1] G. E. P. Box and N. R. Draper, *Empirical Model Building and Response Surfaces*. New York, NY: Wiley, 1987.
- [2] G. Taguchi, *The System of Experimental Design Engineering Methods to Optimize Quality and Minimize Cost*. New York: Productivity Press, 1987.
- [3] V. N. Nair, "Taguchi's parameter design: A panel discussion," *Technometrics*, vol. 34, no. 2, pp. 127-161, 1992.
- [4] M. Zheng, J. Yu, H. Teng, Y. Cui, and Y. Wang, *Probability-Based Multi-Objective Optimization for Material Selection*, 2nd ed., Singapore: Springer, 2023.
- [5] G. G. Vining and R. H. Myers, "Combining Taguchi and response surface philosophies: A dual response approach," *Journal of Quality Technology*, vol. 22, no. 1, pp. 38-45, 1990.
- [6] E. D. Castillo and D. C. Montgomery, "A nonlinear programming solution to the dual response problem," *Journal of Quality Technology*, vol. 25, no. 3, pp. 199-204, 1995.
- [7] D. K. J. Lin and W. Tu, "Dual response surface optimization," *Journal of Quality Technology*, vol. 27, no. 1, pp. 34-39, 1995.
- [8] A. F. C. Karen and P. R. Nelson, "Dual response optimization via direct function minimization," *Journal of Quality Technology*, vol. 28, no. 3, pp. 26-30, 1996.
- [9] K. J. Kim and D. K. J. Lin, "Dual response surface optimization: A fuzzy modeling approach," *Journal of Quality Technology*, vol. 30, no. 1, pp. 1-10, 1998.
- [10] R. Ding, K. J. Lin, and D. Wei, "Dual response surface optimization: a weighted MSE approach," *Quality Engineering*, vol. 16, no. 3, pp. 377-385, 2004.
- [11] J. Kovach, B. R. Cho, and J. Antony, "Development of a variance prioritized multi-response robust design framework for quality improvement," *International Journal of Quality and Reliability Management*, vol. 26, no. 4, pp. 380-396, 2009.
- [12] I. J. Jeong, K. J. Kim, and D. K. J. Lin, "Bayesian analysis for weighted mean squared error in dual response surface optimization," *Quality and Reliability Engineering International*, vol. 26, no. 5, pp. 417-430, 2010.
- [13] D. H. Lee, I. J. Jeong, and K. J. Kim, "A posterior preference articulation approach to dual-response-surface optimization," *IIE Transaction*, vol. 42, no. 2, pp. 161-171, 2010.
- [14] Z. He, Y. H. Ma, and Y. Zhao, "Multi-response robust optimization design based on Taguchi process capability index and entropy weight theory," *Chinese Agricultural Mechanization*, vol. 3, no. 33-36, 2008.
- [15] L. Ouyang, Y. Ma, J. Wang, and F. Wu, "Robust design based on entropy weight and dual response surface," *Journal of Industrial Engineering/Engineering Management*, vol. 28, no. 2, pp. 191-195, 2014.
- [16] M. Zheng and J. Yu, "Probabilistic approach for robust design with orthogonal experimental methodology in case of target the best," *Journal of Umm Al-Qura University for Engineering and Architecture*, vol. 15, no. 1, pp. 55-59, 2024.
- [17] M. Zheng and J. Yu, *Robust Design and Assessment of Product and Production by Means of Probabilistic Multi-Objective Optimization*. Singapore: Springer, 2024.
- [18] D. Kim and S. Rhee, "Optimization of GMA welding process using the dual response approach," *International Journal of Production Research*, vol. 41, no. 18, pp. 4505-4515, 2003.
- [19] M. Zheng and J. Yu, *Systems Theory for Engineering Practice*, *Insights from Physics*. Singapore: Springer, 2024.
- [20] S. M. Pickle, T. J. Robinson, J. B. Birch, and C. M. Anderson-Cook, "A semi-parametric approach to robust parameter design," *Journal of Statistical Planning and Inference*, vol. 138, no. 1, pp. 114-131, 2008.
- [21] G. G. Vining and R. H. Myers, "Combining Taguchi and response surface philosophies: a dual response approach," *Journal of Quality Technology*, vol. 22, pp. 38-45, 1990.
- [22] T. W. Liao, "A fuzzy multicriteria decision-making method for material selection," *Journal of Manufacturing Systems*, vol. 15, no. 1, pp. 1-12, 1996.