

Research Article

Optimal Allocation of Clean Energy in Terms of Probabilistic Multi-**Objective Optimization Method**

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Abstract: In this paper, the Probabilistic Multi-Objective Optimization method (PMOO) is applied to perform the optimal allocation of clean energy with multiple objectives. A solar photo-thermal system, wind energy, and a comprehensive energy storage system for photo-thermal power generation are involved. In PMOO, a new concept of "preferable probability" is put forward to address the preference degree of an attribute of a candidate and the corresponding evaluation method and attributes of the alternative scheme are divided into two types, i.e., beneficial type and unbeneficial (cost) type of attributes, and the corresponding evaluation algorithms of their partial preferable probability are formulated quantitatively. The total preferable probability of each alternative scheme is the product of all possible partial preferable probability, which is employed as the unique indicator to conduct the ranking of the optimization. In the application of optimum allocation problem of clean energy, the solar energy assurance rate and efficiency index of the heating system are the optimal criteria to be maximized, while the heat collecting area of solar collector, the heating capacity of heat pump and the volume of water tank for heat storage are used as input parameters. Especially, the range analysis of the total preferable probability of each alternative scheme is conducted using orthogonal experimental design. The result indicates the optimum configuration for this allocation of clean energy design. Alternatively, in the application of wind-photo-thermal power generation and storage comprehensive energy system problem, both carbon emissions and total operating costs are the optimization criteria to be minimized for the three scenarios, yielding an optimal configuration.

Keywords: clean energy, optimal allocation, Probabilistic Multi-Objective Optimization method (PMOO), preferable probability, overall optimization

1. Introduction

Under the guidance of the goal of "reducing carbon emissions", the building of low-carbon society has become one of the important tasks of national development. The development and utilization of clean energy (such as, solar energy, wind energy, biomass, hydropower, geothermal, and mixed renewable energy systems) has gradually been put on the agenda. A lot of work has been done on the utilization of single renewable energy. The combined utilization of multiple clean energy sources is in urgent need for certain regions for their benefits. Adopting appropriate allocation scheme can not only bring good social benefits, but also achieve the goal of building a society with high efficiency and low cost.

A society with high efficiency and low cost simultaneously involves Multi-Objective Optimization (MOO) problems.

Early in 1950s, optimization with single objective was directly applied in industrial production, which attracted worldwide attention at that time. Box and Wilson of British Imperial Chemical Industry Company proposed the Response Surface Method (RSM) and robust design [1]-[2], and explained the application of RSM in chemical process, which opened the industrial application of experimental design and attracted the attention of research in this field. Taguchi put forward the "Taguchi method" of experimental design with Orthogonal Table in Japan to deal with optimization problems and robust design, and achieved continuous success [3]. In China, Hua L. K. (Hua Loo Keng) and his team popularized the optimum-seeking method and the overall planning method in the 1960s and 1970s, which covered 28 provinces, municipalities, and autonomous regions, and achieved remarkable results with good economic benefits [4]-[5]. Subsequently, these methods also provide consultation for the long-term planning and major project research of the country and industry. Prof. Hua's work laid a solid foundation both theoretically and practically, and can be called an excellent model of mathematical application [4]-[5].

With the increase of the number of objectives (criteria) that need to be optimized simultaneously in the system, the problem and the corresponding method of "MOO" appear [6]-[7]. For MOO problems, the optimal criteria usually conflict with each other, and it is therefore necessary to adopt appropriate MOO method to solve such problems. So far, some MOO methods have been developed, the so called traditional methods for MOO, but each has its own intrinsic limitations [8]-[11]. For example, in the linear weighting method, the rational selection of weighting factors and normalization factors of "addition" approaches is problematic [8]-[14]. Some methods even introduced artificial factors such as virtual "ideal point", which induces unclear elementary significance to the MOO problem itself [8]-[14]. In addition, fundamentally speaking, from the perspective of set theory and probability theory, "addition" is in fact a union, which indicates the "sum" of events. However, the original intention of MOO is that all these multiple objectives are optimized at the same time, i.e., the key point is the simultaneity of the optimization of the multiple objectives. From the aspect of set theory, it is the "intersection" of these objectives; in probability theory, it should be the "product" of the "probability" of each objective (attribute). Therefore, it can be seen that the operation mode of "addition" fundamentally deviates from the original intention of "simultaneity" of multi-objective optimization. On the other hand, Pareto solution can only give one set of solutions [12]-[14]. Therefore, it can be seen that these MOO methods fail to effectively address the essence of MOO problems.

In view of the above situation, from the viewpoint of systems theory, the MOO problem is an optimization problem within a system that consists of multiple objectives to be optimized simultaneously. According to systems theory, the "optimal status of multiple objectives" in the system corresponds to the "optimum state of the whole system". Furthermore, set theory and probability theory can be adopted to deal with such MOO problems. Subsequently, the new concept of "preferable probability" was introduced to reflect the degree of preference of an attribute in the optimization process, leading to the establishment of the Probabilistic Multi-Objective Optimization (PMOO) method [12]-[14]. In PMOO, the objectives (attributes) of the alternative schemes in the optimization task are preliminarily divided into two basic types, namely beneficial attributes and unbeneficial (cost) attributes. Thereafter, quantitative evaluations of partial preferable probabilities corresponding to both beneficial and unbeneficial (cost) attributes are formulated and standardized [12]-[14]. In principle, the simultaneous optimization of a system with multiple objectives is an optimization problem of the system, and the "simultaneous optimization" of multiple objectives is analogous to the "simultaneous occurrence" of multiple events in probability theory. Therefore, the total preferable probability of each alternative is the product of partial preferable probabilities of all possible attributes of that alternative, in accordance with probability theory. Finally, all candidate schemes are ranked according to their total preferable probability, which serves as the unique decisive index for determining the optimal candidate scheme in this optimization.

Obviously, there is a significant difference between the probabilistic method and the previous methods [8]-[14]. PMOO has not only an algorithm but also a clear conceptual framework. From the viewpoint of systems theory, it defines the "optimum point of a system" as the "optimal status of MOO"; thereafter, this optimum point is obtained directly by using probability theory. However, in traditional MOO methods, there is no explicit definition of the "optimum point"; that is, what constitutes an "optimum point" is undefined, only the algorithm is provided. In summary, the advantage of PMOO is that it provides both a clear theoretical framework for multi-objective optimization and an algorithm, while avoiding any subjective or artificial scaling factors.

In this paper, the PMOO method is first expounded, and then its application to the optimal allocation of clean energy for multi-objective design of the solar photo-thermal system and the wind-photo-thermal power generation and storage comprehensive energy system is presented.

2. Brief introduction of PMOO

2.1 Outline of PMOO

Because the original intention of MOO is "simultaneous optimization" of multiple objectives, only by taking this intention into account can an appropriate method be established.

From the perspective of probability theory, it is the "product" of the probabilities of all attributes, while in set theory, it belongs to the "intersection" of all attributes. In addition, in order to make the problem operable in PMOO, a new concept of "preferable probability" is introduced to address the preference degree of an attribute of an alternative candidate in the optimization. The evaluation objectives (attributes) of candidate schemes in optimization tasks are preliminarily divided into two basic types: i.e., both the beneficial type of attributes and the unbeneficial (cost) type of attributes; thereafter, quantitative evaluations of partial preferable probability corresponding to both beneficial attributes and unbeneficial (cost) attributes are formulated [12]-[14]. Furthermore, it takes the "simultaneous optimization of multiple attributes" as the overall optimization of a system, the total preferable probability of each candidate is the product of partial preferable probabilities of all possible attributes of the candidate scheme. Starting from simplicity, it is also assumed that each partial preferable probability is linearly positively correlated with the utility index of its unbeneficial attributes, and each partial preferable probability is linearly negatively correlated with the utility index of its unbeneficial attributes [17]-[18]. Finally, the total preferable probability of each candidate scheme is the uniquely decisive index for the candidate scheme to win this optimization competition. The procedure of PMOO evaluation is shown in Figure 1 [12]-[14].

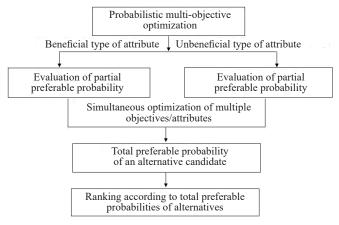


Figure 1. Evaluation procedure of PMOO method

2.2 Formulae in PMOO evaluation

(1) Evaluations of partial preferable probability in beneficial type of attribute case [12]-[14],

$$P_{ij} = A_j Y_{ij}, A_j = 1/(k\overline{Y_j}), i = 1, 2, ..., k, j = 1, 2, ..., l.$$
 (1)

(2) Evaluations of partial preferable probability in unbeneficial type of attribute case [12]-[14],

$$P_{ij} = B_j \left(Y_{j \max} + Y_{j \min} - Y_{ij} \right), B_j = 1 / \left[k \left(Y_{j \max} + Y_{j \min} - \overline{Y_j} \right) \right], i = 1, 2, ..., k; j = 1, 2, ..., l.$$
 (2)

(3) Total preferable probability of an alternative candidate [12]-[14],

$$P_i = P_{i1} \cdot P_{i2} \cdots P_{il} = \prod_{j=1}^{l} P_{ij}, i = 1, 2, ..., k; j = 1, 2, ..., l.$$
(3)

In Eq. (1)-Eq. (3), P_{ij} reflects the preferable probability of the j-th performance attribute of the i-th alternative [12]-[14], k is the total number of alternative candidates, and l is the total number of performance attributes; P_i indicates the total preferable probability of the i-th candidate; Y_{ij} reflects the utility index value of the j-th performance attribute of the i-th alternative candidate; A_j is the normalization factor of the j-th beneficial type of performance attributes index; $\overline{Y_j}$ is the arithmetic average of j-th utility index of the performance attribute in the evaluated group; $Y_{j\min}$ and $Y_{j\max}$ represent the minimum and maximum values of the utility index Y_{ij} of the j-th objective in the evaluated group, respectively.

Besides, the evaluations of normalization factors A_i and B_j are obtained from the general principle of normalization of probability theory for P_{ij} over all alternative candidates i [12]-[14], i.e.,

$$\sum_{i=1}^{k} P_{ij} = \sum_{i=1}^{k} A_j Y_{ij} = 1,$$
(4)

and

$$\sum_{i=1}^{k} P_{ij} = \sum_{i=1}^{k} B_j \left(Y_{j \max} + Y_{j \min} - Y_{ij} \right) = 1.$$
 (5)

Thus, it leads to the following results for normalization factors A_i and B_i ,

$$A_j = 1/\left(k\overline{Y_j}\right),\tag{6}$$

and

$$B_{j} = 1 / \left[k \left(Y_{j \max} + Y_{j \min} - \overline{Y_{j}} \right) \right]. \tag{7}$$

As to tackling MOO, the PMOO emphasize the simultaneity of the multiple attributes in the optimization process of a system, and the irreplaceability of each attribute. Therefore, it is analogical to the "intersection" in set theory and "joint probability" of two independent events in probability theory, which is against to any type of "additive algorithm", accordingly a new concept of "preferable probability" is proposed, and the corresponding evaluations were formulated. However, in traditional methods for MOO evaluations, such as Simple Additive Weighting (SAW) method, additive algorithm is applied to weighted attribute, obviously this kind of "additive algorithm" contradicts to irreplaceability of each attribute. Therefore, in principle, PMOO conforms to the essence of MOO properly.

The remarkable feature of this PMOO method is to optimize multiple objectives in a system at the same time in terms of preferable probability, which is without any subjective or artificial scaling factor, and opens up a new way to solve MOO problems with broad application prospects [8]-[14]. Here "subjective or artificial scale factor" means that it is unnecessary to introduce any nonobjective factor artificially in general cases, but it is not absolutely to repel necessary subjective factor in special cases. Actually, PMOO gained many applications in some fields, such as material selection, shortest path, engineering, experimental design, medical scheme, robust design of industrial processes and products, planning problems, investment optimization, and so on. The possible limitation of the PMOO approach might be a lack of the introduction of modern searching methods.

3. Applications of PMOO method in optimal allocation problems of clean energy

In this section, two examples are provided to show the optimal allocations in clean energy.

3.1 Performance optimization analysis of solar photo-thermal system

3.1.1 Evaluation by means of PMOO

Zhang et al. conducted experiments of photo-thermal complementary system to conduct the optimization [15], which is taken to fulfill the analysis here first. In the optimization, solar energy assurance rate (F) and heating system efficiency index (COPs) are taken as optimization objective (attribute) indexes; the heat collecting area (A) of the solar collector, the heating capacity (B) of heat pump and the volume (C) of water tank for heat storage are taken as the input variables of the experiment [11]. The variable level of orthogonal test design L_9 (3^4) is shown in Table 1. Furthermore, the result is generated and obtained by using Statistical Product and Service Solutions (SPSS) software and run simulation [15], as shown in Table 2. While Table 3 shows the evaluation results of this photo-heat complementary system by using the PMOO approach, in which the solar energy assurance rate (F) and the efficiency index (COPs) of the heating system are all beneficial type of attributes, P_F is the partial preferable probability of the solar energy assurance rate, P_{COPs} is the partial preferable probability of the efficiency index of the heating system, and P_t is the total preferable probability of an experimental scheme. The evaluation of partial preferable probabilities and total preferable probabilities are conducted according to the formulae shown in Eq. (1)-Eq. (7).

Table 1. Level table of parametric design for solar photo-thermal complementary system in orthogonal test L₉ (3⁴)

Factor	- Area of heat collector A (m ²)	Composity of heat myone D (IrW)	Volume of water tank for heat storage C (m ³)
Level	Area of fleat coffector A (fif)	Capacity of heat pump <i>B</i> (kW)	
1	195	15	11.7
2	260	20	15.6
3	324	25	19.4

Table 2. Simulation test results for solar photo-thermal complementary system in orthogonal test L₉ (3⁴)

No.		Input variable			Result	
	$A (m^2)$	B (kW)	$C(m^3)$	F (%)	COPs	
1	195	15	11.7	42	5.08	
2	260	20	11.7	51	5.99	
3	324	25	11.7	56	6.67	
4	324	20	15.6	60	7.03	
5	260	15	15.6	52	5.80	
6	195	25	15.6	41	5.13	
7	195	20	19.4	42	5.03	
8	260	25	19.4	53	6.25	
9	324	15	19.4	60	6.72	

In the simulation, Transient System Simulation Program (TRNSYS) model was used. The results in Table 3 show that scheme 4 ($A_3B_2C_2$) has the largest total preferable probability visually, therefore the optimal scheme should be near the configuration of scheme 4. Table 4 shows the results of range analysis for total preferable probability of each alternative scheme, which shows that the impact order of input variables is A > C > B directly from the value of range analysis of the total preferable probability corresponding to parameters A, B and C, and the optimum configuration for

this optimization problem is $A_3B_2C_3$, i.e., a configuration is with the area of heat collector is 324 m², capacity of heat pump 20 kW and volume of water tank for heat storage 19.4 m³, which is close to scheme 4 indeed.

Table 3. Evaluation results of the photo-thermal complementary system by means of PMOO

Scheme	P_F	P_{COPs}	$P_t \times 10^2$	Rank
1	0.0919	0.0946	0.8694	7
2	0.1116	0.1115	1.2448	5
3	0.1225	0.1242	1.5220	3
4	0.1313	0.1309	1.7188	1
5	0.1138	0.1080	1.2290	6
6	0.0897	0.0955	0.8571	9
7	0.0919	0.0937	0.8608	8
8	0.1160	0.1164	1.3498	4
9	0.1313	0.1251	1.6430	2

Table 4. Results of range analysis for total preferable probability

Factor	- A	В	С
Level			
1	0.862438	1.247115	1.212085
2	1.274525	1.274811	1.268264
3	1.627922	1.242959	1.284536
Range	0.765484	0.03185	0.072451
Impact order	1	3	2
Optimal configuration	A_3	B_2	C_3

3.1.2 Discussion

In the evaluation of PMOO, the assessments of partial preferable probabilities are computed based on available data. Therefore, the available data must have representativeness to reflect the substantive and essential characteristic of the experimental results. In order to achieve this goal, statisticians designed the "Designs of Experiment (DOE)" especially, which includes orthogonal experimental design, response surface method, and uniform experiment design, etc. In DOE, though only limited or finite test data are obtained from the designed experiments according to certain rules, which actually could have the typical feature or characteristics to reflect the essential characteristic of the results.

Besides, in the traditional MOO or so called "Standard Multi-Criteria Decision-Making (MCDM) technique", such as "SAW method", there exists fundamentally unreasonable treatment of "additive algorithm", which indicates the meaning of union of a series of objectives/attributes/criteria in set theory and the substitutability of objectives instead of their non-substitutability and simultaneity. While PMOO represents the irreplaceability and simultaneity of objectives in MOO, the so-called "Standard MCDM technique" is fundamentally incomparable with the approach of PMOO.

3.2 Optimization of wind-photo-thermal power generation storage comprehensive energy system in terms of PMOO

Han et al. set up three scheduling scenarios to compare and consider the impact of carbon capture on the operation economy, consumption of each case and carbon emission of Integrated Energy System (IES) [16]. Detailed discussion on efficiency of the carbon capture system and its cost can be found in literature [16]. The three scenarios include,

Scenario 1: IES operation optimization considering scenery of wind-photo-thermal power generation; Scenario 2: IES operation optimization considering scenery of wind-photo-thermal power generation with storage; and Scenario 3: IES optimization of scenery of wind-photo-thermal power generation with storage considering carbon capture.

Table 5 gives the results for wind power consumption, carbon emission, and total operating cost of the system for these three scenarios [16]. As can be seen from Table 5, the carbon emission of typical day in scheduling Scenario 3 considering carbon captures is reduced by 1,159 t and 1,013 t, respectively, that is, by 9.58% and 8.47% compared with Scenarios 1 and 2, respectively. Due to the operation of carbon capture, Scenario 3 effectively reduces carbon emissions while improving the permeability of renewable energy in the system. At the same time, as compared with Scenario 1 and Scenario 2, the total operating cost of Scenario 3 decreased by 254,567 yuan and 137,079 yuan, respectively, and the operating cost of the same day decreased by 5.19% and 2.86%, respectively. In PMOO evaluation, it takes carbon emissions and total operating costs as the optimization objectives to be minimizing, that is, unbeneficial type of attribute indicators. The reason for "choosing carbon emissions and total operating costs as unbeneficial attributes" is due to their features of being as smaller as possible.

Utilization rate of Wind power consumption Total operating cost Scenario Carbon emission (t)(CNY) renewable energy (%) rate (%) 1 34.30 94.37 12,102 4,903,524 2 35.10 96.56 11,956 4,786,036 3 36.13 99.38 10,943 4,648,957

Table 5. Wind power consumption, carbon emissions, and total operating costs under different scenarios

Table 6 shows the evaluated results of this problem by using PMOO approach, in which P_{CO2} is the partial preferable probability of carbon emissions, P_{Cost} is the partial preferable probability of the total operating cost of the system, and P_t is the total preferable probability of a certain situation. The results in Table 6 show that the total preferable probability of Scenario 3 is obviously greater than that of the other two scenarios, so the optimal scheme is Scenario 3, i.e., utilization rate of renewable energy 36.13%, wind power consumption rate 99.38%, carbon emission 10,943 t, and total operating cost 4,648,957 CNY. It is obvious that Scenario 3 is with lower cost of 4.65 M CNY and lower carbon emission of 10,943 t, therefore, it is listed as No. 1 in the total preferable probability evaluation comparatively.

 P_{CO2} P_{Cost} Scenario $P_t \times 10^2$ Rank 1 0.3206 0.3247 1.0409 3 2 0.3249 0.3329 1.0814 2 3 0.3545 0.3425 1.2141 1

Table 6. Evaluation results of different scenarios by PMOO

4. Conclusion

From the above discussions, it includes following results:

- 1. The optimal allocations of clean energy with multiple objectives involve the simultaneous optimization of multiple objectives fundamentally, and a proper method that can consider the simultaneity of multiple objectives is adopted;
- 2. The PMOO is a proper approach to conduct the problem of optimal allocations of clean energy with multiple objectives. It provides rational optimum configuration for solar photo-thermal system, and appropriate scheme for wind-photo-thermal power generation storage comprehensive energy system;

3. The simultaneity of multiple objectives in the optimization is reflected by using probability theory rationally in PMOO.

Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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