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Research Article

Direct and Inverse Problems of the Theory of Energy Motion in Hydrodynamics

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Abstract: This paper compares two approaches to implementing the Theory of Energy Motion in a medium such as a liquid. The first approach derives the laws of energy motion from the hydrodynamic equations. The second assumes that the equations of liquid motion can be obtained if the experimentally established characteristics of energy motion are known. An approach for conducting the relevant experiments is proposed. The feasibility arises from observing liquid behavior relative to a state of stable equilibrium. A thought experiment considering the propagation of mechanical energy in an infinite channel was based on an analysis of known experiments. It was found that the energy flow occurred as a wave transfer, and the associated fluid movement occurred as a wave. The conclusions drawn from the two approaches differ regarding the convective form of energy motion. It is suggested that the theoretical existence of convective energy transport is a consequence due to the mathematical structure of the hydrodynamic equations. Based on the analysis of experiments, the observation supports the possibility of describing liquid motion in an alternative manner, not constrained by existing formulations. Several examples are presented to illustrate how Newton's laws may be observed in liquid behavior.

***Keywords*:** energy transfer, volumetric density of energy, energy flux, solitary wave, long wave, unsteady flow, Newton laws

1. Introduction

In recent years, various aspects of energy-related processes in fluid systems have been studied in connection with wave dynamics, wave–structure interaction, and hydraulic applications [1], [2]. These studies typically focus on specific physical configurations and engineering problems. In contrast, the present work addresses the more general theoretical formulation of energy motion in hydrodynamics within the framework of the Theory of Energy Motion (TEM) [3].

A portion of the material presented in this paper was previously published as a thesis in the proceedings of the International Association for Hydro-Environment Engineering and Research (IAHR) Congress [4]. That work highlighted that, in fluid motion, the nature of the energy flux predicted by theory, i.e., by the equations of hydrodynamics, differs from the energy flux inferred from well-known experiments. This conclusion arises when considering two possible applications of the classical TEM in hydrodynamics. The present paper provides a broader analysis of this issue and substantiates the possibility of an alternative model of fluid motion that deviates from the classical hydrodynamic model. A key feature of this model is the purely wave-driven nature of the motion of the fluid as

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a continuous medium. Its possible application is important, first of all, for the study of unsteady fluid motion [5].

The paper presents a brief overview of two approaches to applying TEM, which can be referred to as the direct and inverse problems, for fluid systems. It also addresses the manifestation of fundamental mechanical principles within this framework.

TEM is a classical physical theory proposed by Umov in 1874 [3]. Subsequent analyses of the theory emphasized one of its most important features, namely its treatment of energy as a substance-like quantity [6]. This view implies that energy exists not only temporally but also spatially. Within a given medium, energy is distributed in the medium with a volumetric energy density e and moves in the form of an energy flux at velocity c, which generally differs from the medium velocity υ. The velocity of the medium defines its momentum and kinetic energy. The law of energy conservation is interpreted locally and expressed as a continuity equation for an element of the medium:

$$\frac{\partial e}{\partial t}+\frac{\partial\left(ec_x\right)}{\partial x}+\frac{\partial\left(ec_y\right)}{\partial y}+\frac{\partial\left(ec_z\right)}{\partial z}=0, \tag{1}$$

where c_x, c_y, c_z–components of the energy flow velocity along the x, y, and z coordinates.

Umov's work considered only the mechanical forms of energy, namely kinetic and potential energy. Kinetic energy is defined as the energy associated with the motion of particles in a volume of the medium. Potential energy is understood as equivalent to the work that these particles can perform when returning from a given state to an initial state corresponding to stable equilibrium.

The work performed by a force acting on a surface area S defines the energy flux $\mathbf{J} = e\mathbf{c}\cdot S$. Umov's second theorem establishes a relationship between the energy flux density $j = ec$ passing through an infinitesimal surface element in the medium and the mechanical work per unit time performed by the elastic stress σ acting on that element. This relationship is expressed by the equation

$$ec_n=-\sigma\upsilon_\sigma, \tag{2}$$

where c_n is the velocity of energy movement through the surface element and along the normal to the element, υ_σ is the velocity of movement of the element in the direction of the elastic force.

Within the framework of TEM, the concept of two forms of energy transfer was introduced. Convective energy transfer occurs when the velocity of energy motion coincides with the medium velocity ($\mathbf{c} = \mathbf{v}$). If the velocities of energy flow and medium motion differ ($\mathbf{c} \neq \mathbf{v}$), wave-based transfer occurs, where the wave velocity c_n effectively represents the velocity of energy propagation. According to this interpretation, the mechanical work of internal stresses in the medium, as described by Eq. (2), is always associated with the transfer of energy in wave form.

The original goal of TEM was to provide a method for deriving the equations of motion of the medium itself. This derivation is based on known relationships between the distribution of energy, its motion within the medium, and the displacement of its particles. It was emphasized that the laws of motion and energy distribution must be established experimentally. This approach can be referred to as the direct problem of TEM. An alternative application of the theory involves deriving the equations for energy transport based on the already known equations of motion of the medium; this is referred to as the inverse problem.

One application of TEM considered by Umov involved liquids. As the hydrodynamic equations were already available, the example addressed the inverse problem.

The direct problem of TEM had never been examined until very recently. As will be shown below, the prerequisites for solving it existed from the very inception of the theory. Analyzing and potentially solving this problem is of clear scientific interest based on its formulation alone. It is also important to assess how its conclusions align with the established equations of hydrodynamics. Both the mathematical description of fluid motion and the specific characteristics of energy behavior could serve as criteria for this evaluation.

2. The inverse problem of TEM for liquids

Umov considered what is referred to in this paper as the inverse problem, for both inviscid and viscous liquids. Specifically, for an inviscid liquid, the energy flux density in vector form is given by:

$$\mathbf{j} = \mathbf{v}\left(\rho\frac{\upsilon^2}{2} + p\right), \tag{3}$$

where $\mathbf{v}$ is the vector of liquid velocity, p is the hydrostatic pressure, ρ is the density of liquid, $\upsilon^2 = \upsilon_x^2 + \upsilon_y^2 + \upsilon_z^2$. For the purpose of the present paper, it is sufficient to limit ourselves to this equation.

From Eq. (3), it was concluded that "the energy flux equals the product of the liquid velocity and the sum of the hydrostatic pressure and the kinetic energy density". A fundamental conclusion was also drawn, namely, that the direction of energy motion coincides with the direction of liquid motion. Later, the same conclusion was reached by Wien [7], also based on an analysis of the hydrodynamic equations. Although Umov's results are rarely used or cited in modern fluid mechanics, for example, in [8], it may be assumed that these conclusions reflect a well-established and generally accepted view of energy transfer in fluid flows [9].

The structure of Eq. (3) is of particular interest. Without loss of generality for the subsequent analysis, we consider one-dimensional energy motion aligned with the liquid flow velocity υ. Expanding the brackets, we obtain:

$$j = \rho\frac{\upsilon^2}{2}\upsilon + p\upsilon. \tag{4}$$

Taking into account Eq. (2), where for liquid $p = -\sigma$, we obtain Eq. (4) in the form

$$j = \rho\frac{\upsilon^2}{2}\upsilon + ec. \tag{5}$$

Thus, instead of Umov's formulation, which treats Eq. (3) as representing the energy flux as a single process of energy transfer, Eq. (5), it follows that the energy flux consists of two components. The first term represents the convective transport of kinetic energy at the velocity υ. This component is not associated with the work done by forces in the liquid at a given instant. The second term, according to Eq. (2), corresponds to the pressure work per unit time. It is related to the wave-based transfer of energy with density e and velocity c. Thus, the results obtained within the framework of the inverse problem suggest the presence of both possible forms of energy transfer in a liquid.

3. The direct problem of TEM for liquids

It should be noted in advance that the solution to the direct TEM problem must be based on empirically established laws of energy motion. Furthermore, the concept of energy used here is restricted to mechanical forms only, namely kinetic and potential, as considered in Umov's work and commonly accepted in fluid mechanics. Thermal energy, as well as the energy associated with the thermal motion of fluid particles, is not taken into account.

3.1 *Localization of energy*

A key idea for analyzing the direct problem of TEM is the localization of energy. This issue was considered, for example, in [7], [10] independently of Umov's earlier work. However, the principles according to which energy localization in a liquid should be defined have never been clearly formulated. It can be assumed that energy was associated with liquid particles, since the initial analysis was based on the hydrodynamic equations. For this reason, the problem of deriving the equations of liquid motion, referred here as the direct problem of TEM, did not even arise.

In contrast, in Umov's work, energy was associated with a volume of the medium, which in our case corresponds to a volume of liquid.

In this paper, the conclusion regarding energy localization is based on a fundamental physical principle stating that the energy of a physical system in a state of a stable equilibrium is minimal. The potential energy of such a system can be taken as zero. The choice of the state regarded as stable equilibrium is relative and often selected for convenience in solving a particular problem. A simple example is water at rest in a channel. If this state is taken as the stable equilibrium, the mechanical energy of the water can be considered zero. This implies that, at any point in the channel, the energy density is equal to zero ($e = 0$). Consequently, any change in the state of water, such as directed motion or deformation of a volume of water, indicates that energy has appeared in this volume ($e > 0$). Based on this reasoning, it becomes possible to determine the location of energy and to describe its motion in the most general form.

Obviously, energy in the water within the channel can appear only from external sources, through the work of external forces. For example, this occurs when such a force moves a vertical plate (a wavemaker) along the channel, thereby generating water motion (Figure 1a), as has been repeatedly studied in various works [11], [12]. As a result of the wavemaker's movement, volumes of water near it deform, causing a rise or fall in the water level, and the particles within these volumes come into motion. Motion implies the presence of momentum and kinetic energy, while deformation corresponds to potential energy, since the water can perform work to return to its equilibrium state. These volumes of water constitute the region of energy localization.

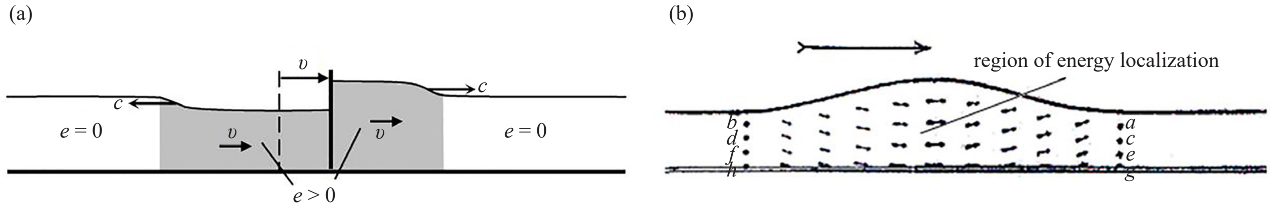

Figure 1. Localization of mechanical energy in a liquid: a) near the source of motion; b) in a solitary wave based on Russell's original drawing

3.2 *Movement of energy in a channel*

In the example shown in Figure 1a, it is evident that the deformation and motion of the water propagate along the channel away from the wavemaker in the form of a wave traveling at velocity c, indicating the direction of energy motion. On the right, energy propagates in the form of a wave of elevation; on the left, in the form of a wave of depression.

As the wavemaker moves, it imparts momentum to the water directed along the channel. The horizontal velocity of the water particles corresponds to this momentum due to the boundary conditions, although displacement also occurs in the vertical direction, causing deformation of the water volumes.

The horizontal motion of the water on both sides of the plate occurs in the same direction, coinciding with the direction of the plate's displacement. However, the propagation of energy to the right and to the left occurs in opposite directions. Thus, on the right-hand side, the directions of water motion and energy motion coincide, whereas on the left-hand side, they are opposite. This example provides grounds to conclude that the result obtained within the framework of the inverse problem, that energy motion and water motion are always co-directed, is not consistent with the effects observed in practice.

3.3 *The nature of energy movement in liquids*

Since the motion of the liquid from its equilibrium state begins in the form of a wave, it is evident that the nature of energy motion is also wave-like. How, then, does the subsequent energy transfer occur?

In the case of a brief displacement of the wavemaker, when the external force performs a limited amount of work and imparts a finite quantity of energy to the water, a solitary wave forms in the channel (Figure 1b). Russell's experiments [13] demonstrated that the region of motion is confined within the boundaries of the wave disturbance and occupies the volume from the free surface to the channel bottom, which also corresponds to the region of energy localization. This region moves along the channel, sequentially passing through volumes of water together with the wave, at the wave velocity. This is precisely how Longuet-Higgins [14] described the localization and movement of energy by a solitary wave. The only peculiarity was that the longitudinal profile of the solitary wave had a theoretical extent from -∞ to +∞.

Continuous movement of the wavemaker generates a flow in the water, which takes the form of a wave elevation on one side and a wave of depression on the other. If the wavemaker's motion becomes steady, the water flow on both sides tends toward a steady state. However, the establishment of the steady flow does not imply the disappearance of the wave.

In the theory of unsteady flow, a wave is considered the region with variable characteristics at the leading edge of such long waves [15]. However, it is straightforward to show that even within the region of steady flow, energy motion remains a wave-based process. To illustrate this, consider the example in which the wavemaker moves in discrete short displacements, generating a series of solitary waves with finite energy quantities (Figure 2a). If the intervals between wavemaker displacements shrink to zero, the motion becomes continuous, and the individual waves merge into a single long wave with an elevated water level (Figure 2b). In this case, the energy quantities of the individual waves combine, forming a continuous sequence, i.e., an energy flow. Clearly, energy motion within this flow occurs at the corresponding wave velocity c, which exceeds the average velocity of water particle motion in each volume. Considering the continuity of the energy flow, the water motion can be represented as a sequence of elementary plane waves associated with discrete energy quantities (Figure 2c). Overall, the flow constitutes a long wave.

The non-uniform distribution of energy along the wavelength causes unsteady flow. When the energy distribution is uniform, steady flow occurs in accordance with the conservation law expressed by Eq. (1).

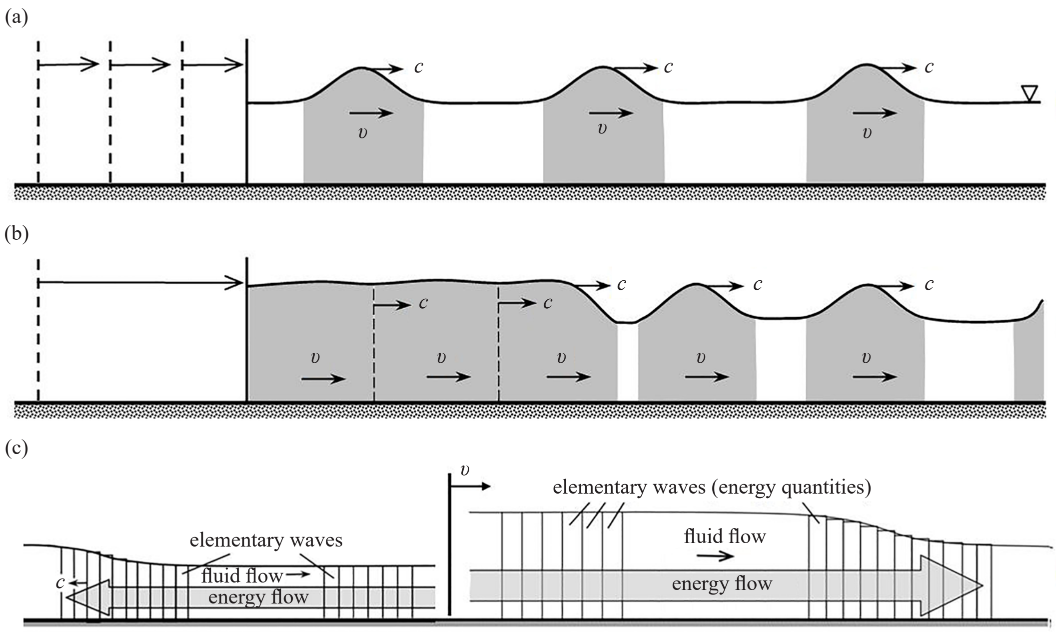

Figure 2. Energy flow in a liquid as a sequence of individual quantities of energy: (a) transport of individual quantities of energy in the form of wave pulses; b) the merging of individual portions of energy into a continuous flow; c) energy flow in the form of energy of elementary waves that make up the waves of elevation and depression

This raises the question: if the inverse problem of TEM allows for the possibility of convective energy transfer, under what conditions could this occur? By definition, this transfer arises when the velocity of energy motion coincides with the velocity of the water flow itself and is interpreted as the transfer of energy within water volumes. Suppose that the water flow, which always initiates as a wave process, eventually reaches a state in which the velocity of energy motion decreases to the average velocity of the flow. Clearly, such a scenario can only occur in the case of a wave of elevation, where the energy displacement aligns with the direction of water motion. It is also reasonable to assume that the transformation of the wave-based energy motion into a convective form would need to occur for each discrete energy portion, for instance, the energy of a solitary wave. This assumption would imply that a solitary wave, moving along the channel at velocity c, gradually dissipates (Figure 3a), and its energy inevitably decreases, but not entirely. The remaining energy would need to be distributed and conserved within a specific mass of water of finite volume and nonzero energy density $e = \rho \upsilon^2/2$ (Figure 3b). If convective energy transfer were to arise, this mass would have to move along the channel at velocity υ, while the surrounding water remains stationary with zero energy. Clearly, such motion is impossible. Thus, the assumption of a transformation from wave-based to convective energy transfer is incorrect. Consequently, we conclude that water motion, emerging from equilibrium as a wave, retains its wave-based nature even when it appears as a steady flow.

The choice of a vertical plate as the generator of energy motion in the water does not affect the conclusions drawn. The wave may also be generated by a sudden water release, as in numerous laboratory studies. Photographic recordings of experiments with dyed water [16] have shown that the mechanism by which the front of a release wave passes through an arbitrary channel cross-section is analogous to that observed when the wave is generated by a wavemaker. This is also consistent with the experiments of Russell, in which different methods of wave generation produced essentially the same water motion.

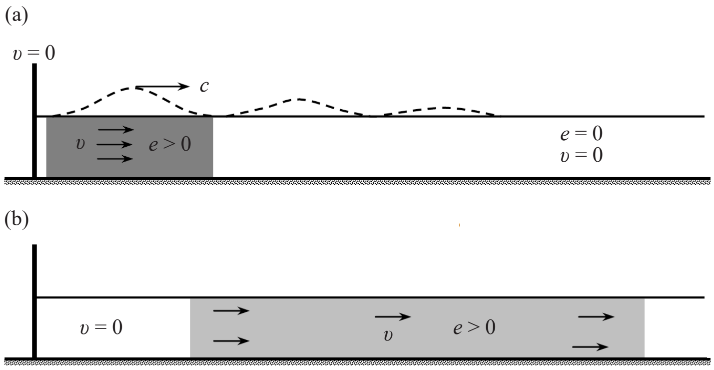

Figure 3. Sketch of the possible transformation of wave energy transport into convective one: a) reduction of the energy carried by a solitary wave and its localization in a certain mass of liquid; b) completely convective energy transfer in a liquid at rest

Does the wave character of energy motion persist in the region behind the front of a propagating long wave if the liquid motion there acquires the character of steady uniform flow?

The proposed approach to addressing the direct TEM problem allows the motion of energy to be observed only within the region of unsteady flow, while the remaining analysis relies on theoretical considerations, conservation laws, and the fundamental principles of the theory. When a long wave is considered independently of the conditions under which it is generated, the impression may arise that the work associated with the propagation of the wave front initiates the motion of the liquid behind it, after which the liquid continues to move independently. Such an interpretation, however, neglects the work performed by the wavemaker, which serves as the actual source of energy (Figure 2c). In reality, the wave front does not generate energy but only transmits the energy supplied by the source. The wavemaker itself represents only the most illustrative example of such a source.

The velocity of the liquid flow υ within the body of the wave corresponds to the velocity of the wavemaker at the moment when a particular wave element is generated. At the same time, the wavemaker moves at a velocity lower than that of the wave front. Consequently, the continuously increasing distance between the wave front and the wavemaker indicates that the energy content of the liquid in this region (the energy flow) propagates from the wavemaker at a velocity greater than that of the liquid itself ($c > \upsilon$), i.e., in a wave-based form. Experiments with dyed water [16] confirm this conclusion. This difference in velocities indicates that the motion of energy cannot be interpreted as convective transport by the moving liquid.

Thus, the model of energy motion represented as a sequence of elementary waves appears physically plausible.

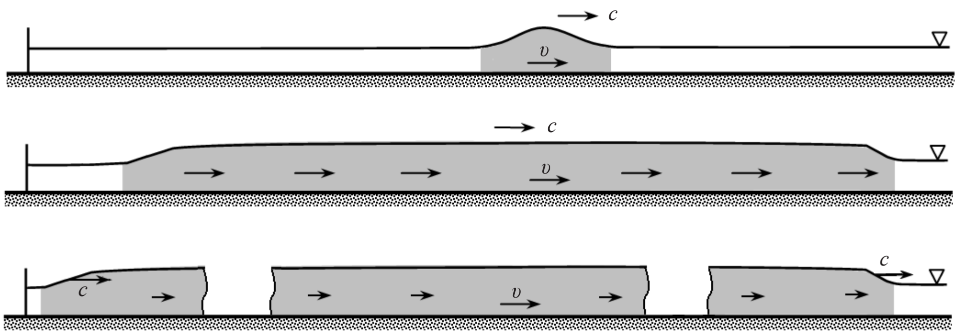

Figure 4. Uniform liquid flow in the region of a long wave after the wavemaker has stopped

When the wavemaker stops operating, the liquid in its immediate vicinity comes to rest, while the region of motion in the channel becomes isolated and, remaining a long wave, continues to propagate at a velocity determined by the motion of its trailing and leading fronts (Figure 4). The cessation of liquid motion in the volumes near the wavemaker demonstrates that these volumes did not possess their own energy of motion.

The length of the channel does not affect the wave character of liquid motion. In a channel of infinite extent, the motion will always occur in the form of some wave propagating into quiescent liquid. If the channel has a finite length, characteristic wave effects such as complete or partial reflection (refraction) [17] arise at the boundaries; however, these effects do not alter the wave nature of the liquid motion. Nevertheless, the superposition of incident and reflected long waves, each carrying its own momentum and energy fluxes, may substantially modify the motion within that region. The formation of a flow with convective energy transfer, that is, motion occurring without the action of forces, is not assumed in any case.

Regarding the flow that occurs in the form of waves of depression generated from the equilibrium state, convective energy motion in these cases is impossible by definition, regardless of the nature of the liquid motion.

The character of energy motion and liquid motion does not fundamentally change if they occur in a pipe rather than in an open channel. The principal difference is that, in this case, the liquid can no longer be treated as incompressible. Waves in a pipe take the form of compression and rarefaction waves, whose propagation velocity is substantially higher than in an open channel.

The manifestation of energy is indicated by a change in pressure relative to the initial equilibrium state. The pressure variation, in turn, is caused by deformation of liquid volumes. Accordingly, the potential energy of the liquid in a pipe is represented by the potential energy of volumetric elasticity.

If a hypothetical piston is considered as the wavemaker, a compression wave is generated on one side and a rarefaction wave on the other, propagating in opposite directions from the piston, while the liquid motion occurs in a single direction.

In a theoretically infinite pipe, the motion would propagate without spatial limitation. The analysis of the motion does not fundamentally differ from that in an open channel, and energy motion occurs at a high propagation velocity.

In a pipe of finite length, wave-reflection effects arise at its ends, and the overall motion is formed as a result of the superposition of incident and repeatedly reflected waves. In many practical situations, the motion of water in a pipe begins with rarefaction waves. In such cases, the direction of wave propagation associated with the energy flux and the direction of the water flow occur in opposite directions. Energy motion therefore, clearly takes place in wave-based form.

3.4 *A special case of wave-based energy motion*

The motion of mechanical energy in which the liquid flow velocity coincides with the energy flux velocity can, in principle, occur under the following conditions. The liquid motion must take the form of a wave of elevation, in which both velocities are collinear. Let us consider the formation of such a wave in a channel (Figure 5). Assuming the liquid to be incompressible, we obtain

$$cS_0 = (c - \upsilon)S, \tag{6}$$

or

$$\frac{\upsilon}{c} = 1 - \frac{S_0}{S}, \tag{7}$$

where S_0 is the cross-sectional area of the liquid in the initial state, and S is the cross-sectional area of the volume of liquid disturbed by the displacement of the wavemaker.

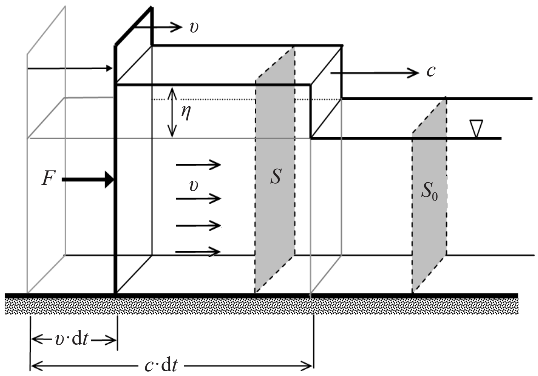

Figure 5. Sketch of wave formation by a wave-maker. *F* is the applied force, *h* is the wave elevation

The ratio υ/c on the left-hand side of Eq. (7) tends toward unity as the ratio S_0/S on the right-hand side of the equation approaches zero. For $S \gg S_0$, the two velocities become close in magnitude, but nevertheless remain distinct. Only in the case $S_0 = 0$, i.e., when liquid propagates over a dry bed, does the wave velocity coincide with the velocity of the liquid itself.

3.5 *Differences between the direct and inverse problems of TEM*

It is evident that the conclusions of the direct and inverse problems of TEM for liquids differ with respect to the possibility of convective energy transfer. Hydrodynamic equations assume the existence of such a transfer, whereas the logic of a possible experiment does not permit it. Analysis of the problem indicates that the cause of these discrepancies may arise from features of the mathematical model represented by the hydrodynamic equations, which serve as the basis for analyzing the inverse problem. The postulate of the model is the assumption that the fluid consists of particles. The hydrodynamic equations describe the behavior of such a particle in accordance with the principles of mechanics of a point of constant mass ($\rho \frac{d\upsilon}{dt}$). Under the action of an external force, the particle moves with proportional acceleration at each instant, and its velocity satisfies the general equation

$$\upsilon = \upsilon_0 + a_t t, \tag{8}$$

where υ_0 is the initial velocity of the particle; a_t is the instant acceleration; and t is time. Similar to Eq. (4), the term $a_t t$ corresponds to the action of an external force, while υ_0 represents the velocity acquired by the particle at a given moment. In the absence of an external force, the energy of the particle is kinetic, moving convectively through space at the velocity of the particle itself, corresponding to the first term in Eq. (5). It is clear, therefore, that convective energy transfer is associated with the velocity the particle has acquired at the current moment and maintains thereafter.

4. Newton's laws

The differences between the conclusions of the direct and inverse problems of TEM concern not only the seemingly specific issue of the form of energy motion. In the case of liquid motion involving wave-based energy motion, certain fundamental principles of mechanics also exhibit distinctive features.

4.1 *Newton's first law*

According to Newton's formulation [18], the first law manifests itself as the preservation of rest or uniform motion of a material point until it is acted upon by a force. For example, after a brief application of a force, the point continues to move at the acquired velocity υ (Figure 6), thereby providing convective transfer of its kinetic energy. In a liquid under an analogous situation, however, it is not a specific mass of liquid that propagates, but rather a wave impulse. The wave velocity c in this case exceeds the velocity υ acquired by the liquid at the moment when the force ceases. Whereas the velocity of the body is determined by the work done by the force, the wave velocity depends on the conditions of the channel through which the wave propagates, such as its cross-sectional geometry [19]-[24]. Nevertheless, in both cases, it is appropriate to speak of the conservation of momentum and energy.

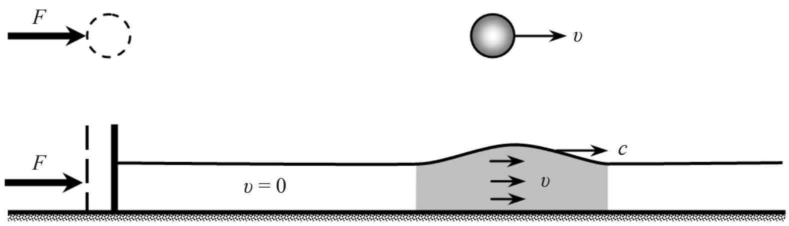

Figure 6. After the work of the force, the kinetic energy of a point with constant mass is transferred together with the point. In a liquid, kinetic energy, being part of the wave energy, is transferred together with the wave

4.2 *Newton's second law*

In the equations of hydrodynamics, the sum of forces acting on a liquid particle determines its acceleration in the same way as for a point with constant mass. A similar relation is assumed to apply not only to abstract particles but also to finite volumes of liquid.

Here, we formulate Newton's second law for liquid motion where the flow occurs in the form of a long wave. Let the wave be generated by a wavemaker, as in the examples above, under the action of a constant external driving force. The channel is assumed to be horizontal, infinitely long, and of uniform cross-section. In general, the cross-section shape is arbitrary. Let the mass of the wavemaker be denoted as M.

We assume the wavemaker moves without friction and maintains a vertical orientation. Its motion is initially accelerated but, due to increasing resistance from the liquid, tends toward uniform motion with a certain maximum velocity. Accelerated motion implies that, at each moment, an increasing amount of momentum is transferred to the volume of liquid immediately adjacent to the wavemaker, which then propagates freely as a wave along the channel. During the accelerated motion of the wavemaker, the flow in the channel is unsteady and is characterized by a traveling wave with an elevated free-surface level. Once the wavemaker reaches a constant velocity, the flow in the channel transitions to a steady and uniform flow at the same average velocity. In general, the flow can be regarded as a sequence of elementary plane waves, as shown in Figure 2c.

At the wavemaker boundary, liquid particles are imparted with a velocity υ that is uniform across the entire cross-section. This velocity determines the momentum and kinetic energy of each individual wave element moving through the channel and passing through successive volumes of liquid. As the wave propagates along the channel, the distribution of local velocities across the cross-section changes. However, assuming momentum conservation, the average (cross-sectional) velocity must remain constant, as observed in Russell's experiments.

The dynamics of the wavemaker are governed by the resultant of two forces: a constant external driving force F_1 and a resistance force F_2 exerted by the liquid in the channel:

$$F_1 - F_2 = M\frac{d\upsilon}{dt}. \tag{9}$$

The work done by force F_1 contributes to changes in both the kinetic energy of the wavemaker and the energy of the liquid. The work done by force F_2, which opposes the motion of the wavemaker, reduces the wavemaker's energy and correspondingly increases the energy of the liquid; that is, it is equivalent to the action of a driving force on the liquid. Unlike the wavemaker, the liquid also acquires potential energy due to deformation of the volumes relative to the initial equilibrium state.

Let us rewrite Eq. (9) in the following form

$$F_1 = M\frac{d\upsilon}{dt} + F_2. \tag{10}$$

Using Newton's second law, we express F_1 in terms of the change in momentum of the system comprising the wavemaker and the liquid in the channel.

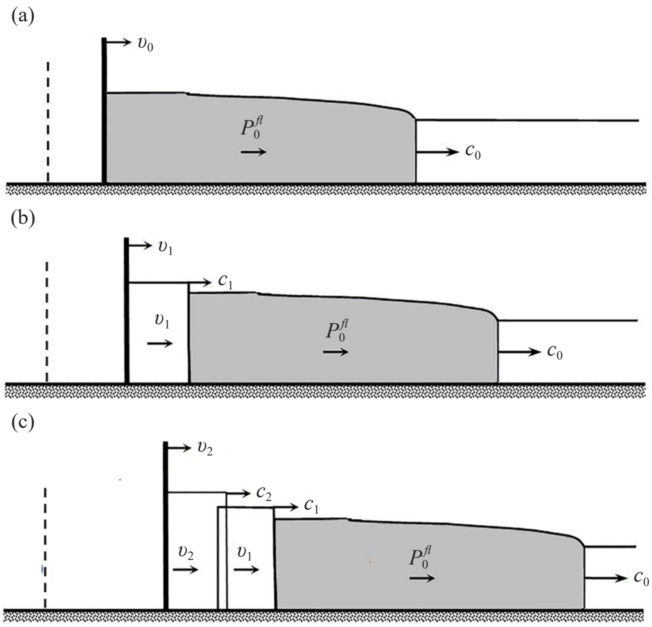

Figure 7. Changing the momentum of the wave-maker and liquid: (a) at the initial moment of time t_0; (b) $t = t_0 + \mathrm{d}t$; (c) $t = t_0 + 2\mathrm{d}t$

Suppose a wave is already propagating in the channel (Figure 7a). At a given moment t_0, the velocity of the wavemaker is υ_0. The total momentum of the liquid over the entire length of the moving region at that moment is denoted as P_0^{fl}. The total momentum P of the entire system can then be expressed as

$$P(t_0) = M\upsilon_0 + P_0^{fl}. \tag{11}$$

When moving with acceleration over a time interval dt (Figure 7b), the velocity of the wavemaker increases to υ_1, and a new elementary wave is generated at its surface in the liquid, traveling at the velocity c_1 with momentum $\rho\upsilon_1 c_1 S_0 \cdot \mathrm{d}t$, where $c_1 S_0 \cdot \mathrm{d}t$ represents the volume of liquid occupied by this wave (according to Eq. (6)). The total momentum of the system at time $t_1 = t_0 + \mathrm{d}t$ then becomes

$$P(t_1) = M\upsilon_1 + P_0^{fl} + \rho\upsilon_1 c_1 S_0 \cdot \mathrm{d}t. \tag{12}$$

After an additional time interval dt (Figure 7c), the velocity of the wavemaker becomes $\upsilon_2 = \upsilon_1 + \mathrm{d}\upsilon$. Assuming that the velocity of wave impulses may vary, a new elementary wave is generated in the liquid with momentum $\rho\upsilon_2 c_2 S_0 \cdot \mathrm{d}t$, where $c_2 = c_1 + \mathrm{d}c$. At the moment $t_2 = t_0 + 2\mathrm{d}t$, the total momentum of the system then becomes

$$P(t_2) = M\upsilon_2 + P_0^{fl} + \rho\upsilon_1 c_1 S_0 \cdot \mathrm{d}t + \rho\upsilon_2 c_2 S_0 \cdot \mathrm{d}t - \rho\upsilon_1 (c_2 - c_1) S_0 \cdot \mathrm{d}t. \tag{13}$$

The last term on the right-hand side of the equation corresponds to the momentum lost in the region where the new wave element overtakes and overlaps the previous one.

The change in momentum over the time interval dt, given by the difference between Eqs. (12) and (13), is equal the applied force according to Newton's second law

$$F_1 = M\frac{\upsilon_2 - \upsilon_1}{\mathrm{d}t} + \rho\upsilon_2 c_2 S_0 - \rho\upsilon_1 (c_2 - c_1) S_0. \tag{14}$$

Since $\upsilon_2 = \upsilon_1 + \mathrm{d}\upsilon$ and $c_2 = c_1 + \mathrm{d}c$, the difference between the terms $\upsilon_2(c_2 - c_1)$ and $\upsilon_1(c_2 - c_1)$ is $O(\mathrm{d}\upsilon \cdot \mathrm{d}c)$, i.e. of second order in the small increment dt. Within the linear approximation with respect to dt, such terms can be neglected, and we can replace the last term on the right side of Eq. (14)

$$F_1 = M\frac{\upsilon_2 - \upsilon_1}{\mathrm{d}t} + \rho\upsilon_2 c_2 S_0 - \rho\upsilon_2 (c_2 - c_1) S_0. \tag{15}$$

Since $c_2 = c_1 + \mathrm{d}c$ and $\mathrm{d}c \ll c_1$, the last term is asymptotically small compared with the momentum flux term $\rho\upsilon_2 c_2 S_0$:

$$\frac{\rho v_2 (c_2 - c_1) S_0}{\rho v_2 c_2 S_0} \sim \frac{\mathrm{d}c}{c_2} \ll 1.$$

Therefore, within this approximation, the last term can be neglected. As a result, at an arbitrary moment in time, we obtain

$$F_1 = M\left(\frac{\mathrm{d}\upsilon}{\mathrm{d}t}\right)_t + \rho\upsilon_t c_t S_0. \tag{16}$$

The equation represents Newton's second law expressed in terms of the total derivative of momentum with respect to time:

$$F = \frac{\mathrm{d}(m\upsilon)}{\mathrm{d}t} = m\frac{\mathrm{d}\upsilon}{\mathrm{d}t} + \upsilon\frac{\mathrm{d}m}{\mathrm{d}t}.$$

On the right-hand side of Eq. (16), the first term corresponds to the wavemaker, while the second term corresponds to a liquid. Therefore, Newton's second law for the liquid takes the following form:

$$F_2 = \rho\upsilon c S_0 = \upsilon\frac{\mathrm{d}m}{\mathrm{d}t}. \tag{17}$$

It is natural that the resistance of the liquid manifests itself through the wave pressure $\rho\upsilon c$, which, in turn, facilitates the energy motion along the channel, according to Eq. (2), where σ for the liquid represents the pressure.

5. Limitations and scope of the direct problem of TEM approach

The analysis applies to mechanical energy in unsteady 1D flows initiated by localized disturbances.

Claims about "*all fluid motion*" are preliminary hypotheses requiring validation in other regimes.

The paper proposes a conceptual framework for re-examining energy transport but not a replacement hydrodynamic model.

Future work must address: turbulent flows, closed conduits, rotational flows, and compressible regimes.

6. Conclusions

1. This paper presents, for the first time, a practical approach to implementing the direct problem of the Theory of Energy Motion in liquids. It is based on experimentally establishing the principles of mechanical energy motion and localizing energy relative to a state of stable equilibrium.

2. Analysis of well-known experiments and simple examples shows that energy in a liquid moves exclusively via wave propagation. The solitary wave emerges as the simplest form of energy motion, while more complex flows can be represented as sequences of elementary waves.

3. The conclusions of the direct problem differ fundamentally from the inverse problem, which is derived from hydrodynamic equations. Notably, convective energy motion predicted by theory is not observed in the considered examples of flow experimentally.

4. This paper highlights a previously unexplored aspect of energy dynamics in liquids. Combining experimental observation with TEM, it provides a new framework for understanding how mechanical energy moves and can complement traditional theoretical approaches.

5. In the direct TEM framework, observable energy propagation precedes and constrains the mathematical description of fluid motion, suggesting energy transport mechanisms should inform (not merely follow from) hydrodynamic equations.

The Theory of Energy Motion was published about three decades after Russell's solitary wave experiments, which proved to be a key to understanding energy motion in liquids. This indicates that the foundations for analyzing the direct problem of TEM already existed in the 19th century.

Conflict of interest

The author declares no competing financial interest.

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