



Research Article

Impact of Human Capital Development on Economic Progress: A Panel Data Analysis by Using Classical and Bayesian Estimates

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Received: 17 June 2022; **Revised:** 2 November 2023; **Accepted:** 2 November 2023

Abstract: This comparative research debates a magnified role of human capital development in selected developed, developing economies and less developing economies, including Pakistan. Between 1990 and 2019, two-panel data analysis techniques were employed, one classical and the other Bayesian. The key objective of this study was to investigate the impact of human capital development on economic growth, and also suggest the best statistical techniques for developing an appropriate model. GDP is the dependent variable, whereas the independent variables are human capital, physical capital, physical and human capital growth, health expenditure, education spending, and life expectancy rate. For the optimal model, both Bayesian and classical approaches were used to analyze the appropriate model. Empirical evidence has shown that human capital is the most important factor and is directly correlated with growth in any economy. Furthermore, the findings suggested that Bayesian techniques produce more appropriate models for policy implications than classical techniques.

Keywords: human capital development, classical techniques, Bayesian techniques, panel data, economic progress

JEL Codes: C1, C4

1. Introduction

The world in the twenty-first century is divided into two hemispheres: developing countries and developed countries, as predicted by the World System Theory. Whilst the analogous agencies are all-inclusive to posturize the global governance system. Governance is a slippery term, encapsulating the economic, physical, financial, human resources and eminence institutions that accelerate the machinery of the country depending on the input and output ratios, growth can be acclaimed as both positive and negative, and it has direct or indirect impact on economic prosperity, determining the nation's direction. The prosperity and progression pace of a nation solely depends on the economic growth installed by the human capital development which is the main factor in attaining growth and development, as it leads an economy to adopt new technologies and build a large efficient facility for utilization of resources at a national and international scale.

Economic growth measures through the per capita income of a certain nation are the foremost determinant of the national progress along with the physical capital extracted through the industrial, agricultural, and service sectors. Ostensibly, the signified nexus of public and private sector collaborates and outcomes as a national income source which

Copyright ©2023 Nayab Minhaj, et al.

DOI: <https://doi.org/10.37256/ges.5120241633>

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further leads to economic growth and public welfare. Pakistan has used a variety of planning tactics to grow its economy since the eighteenth century, although has only focused on the construction of physical capital as a viable way of defining progress while neglecting human capital development. Most planning in Pakistan focuses on capital formation exclusively for intrinsic growth while ignoring the immense benefits of capital formation among individuals. As a result, the expansion of investment among Pakistan's workforce is mismanaged, exacerbating the national evolution process to malfunction. In this scenario, physical capital and human capital constitute the expansion and glean the economic output humanized by sound research and development, technical innovation, and administrative posture. The essence of this study is to examine economic progress by panel data regression models through different economies of developing countries, less developing and developed world along with Pakistan. In developed countries, the study incorporated Singapore, South Korea, and Japan. Whereas, developing countries include China, Turkey, and Malaysia while less developing countries comprised India, Bangladesh, and Pakistan. Countries around the world are often categorized into various development stages based on economic growth, industrialization, infrastructure, and other socio-economic factors. The classification of countries into developed, developing, and less-developed categories is a simplified way to understand their economic and social progress. It's important to note that these classifications can change over time as countries evolve. Here's an explanation of the three broad categories you mentioned:

Developed Countries: Developed countries exhibit a range of distinguishing characteristics that set them apart from less developed nations. One of the most prominent features of these nations is their high-income levels, reflecting a superior standard of living for their inhabitants. This affluence is underpinned by advanced infrastructure, encompassing state-of-the-art transportation systems, well-equipped healthcare facilities, and top-tier educational institutions. Furthermore, developed countries are recognized as pioneers in technological innovation and invest significantly in research and development, propelling them to the forefront of global technological advancement.

In addition to their economic prowess, developed nations boast stable and effective political systems, ensuring law and order, the protection of property rights, and the enforcement of contracts. This political stability contributes to a secure environment for businesses and individuals alike. The Human Development Index (HDI), a comprehensive measure of well-being, consistently ranks developed countries high due to their impressive performance in indicators such as life expectancy, education, and income. Moreover, these nations tend to maintain low poverty rates, thanks to comprehensive social safety nets that provide support to vulnerable populations, further cementing their reputation as beacons of prosperity and progress in the global landscape.

Developing Countries: Developing countries exhibit several notable characteristics that distinguish them from both developed and less-developed nations. One key feature is their moderate income levels, positioning them in an intermediate economic category. However, within these countries, there can often be substantial income disparities among their populations, highlighting the ongoing challenges they face in achieving widespread prosperity.

Another hallmark of developing nations is their rapid industrialization and expansion of various industries, including manufacturing, services, and technology. This growth represents a significant driver of economic development and job creation, albeit with varying degrees of success and challenges.

Moreover, developing countries are actively investing in improving their infrastructure, recognizing its crucial role in supporting economic growth and urbanization. These investments encompass transportation networks, energy systems, and communication technologies, among others.

Lastly, while political stability in developing countries can be tenuous at times, there are instances of emerging political stability in some regions. This stability is essential for fostering an environment conducive to economic development and attracting foreign investment, further propelling these nations on their path toward advancement.

The study's main goal is to assess if human development is a significant growth determinant as well as to analyze the association between human capital development and economic growth. Using both classical and Bayesian methodologies, The Bayesian approach provides several advantages over the classical estimating approach, particularly with limited samples. Bayesian models have been recommended for panel data by various authors including Hsiao and Koop (2000). As a result, the study has attempted to identify guidelines to look at the empirical influence of human capital on economic development in various economies, by employing additional statistical tools for optimal modeling, which helps the policymakers to encourage the foremost factor to escalate the wheels of the economy.

1.1 Objective of the study

This study delves into the assessment of economic progress in diverse countries, spanning developed, developing, and less developed nations. It scrutinizes the impact of human capital on economic growth. Employing a variety of statistical tools, including classical and Bayesian methods, the research explores these relationships comprehensively. The primary goal is to create human capital indices for selected countries and utilize a blend of Bayesian and classical techniques to analyze panel data models. This aids in pinpointing the most effective model for policymaking. Ultimately, the research aims to provide valuable guidance to policymakers, emphasizing the significance of human capital development in fostering economic growth across a broad spectrum of economies.

1.2 Contribution of the study

Many economists are actively engaged in the field of human capital development, employing a combination of classical and statistical techniques in their research. While classical methods often yield insignificant results when data is limited, this study innovatively incorporates statistical techniques alongside traditional approaches to compare variable outcomes. The findings demonstrate that Bayesian analysis tends to yield more significant results compared to classical methods. This research makes a significant contribution by constructing human capital indices for selected countries and employing both Bayesian and classical methodologies to analyze panel data models. Ultimately, the study aims to determine the most appropriate model for informing policy decisions. The overarching goal of this research is to provide valuable insights for policymakers, assisting them in prioritizing human capital development as a means to bolster economic growth across a range of economies.

1.3 Significance of the study

This study holds significant importance due to its unique approach to addressing a critical gap in existing research. It pioneers a comprehensive and comparative analysis of economic prosperity while introducing a diverse range of statistical tools, including both classical and Bayesian techniques. Unlike prior studies, this research not only explores the impact of human capital on economic growth but also provides a nuanced assessment that underscores the vital role of human capital in shaping economic well-being. The novelty of this study lies in its multifaceted approach to evaluating economic prosperity across various countries, employing a wide array of statistical tools. It aims to shed light on the differences in variable outcomes between classical and Bayesian methodologies, particularly when data is limited. By doing so, this research contributes to a more comprehensive and informed understanding of the most effective models for studying economic prosperity and the significance of human capital in this context.

2. Review of studies

Human capital, as well as economic and industrial advancement, walk hand in hand. Therefore, it's crucial to assess their relationship to boost productivity and capabilities, resulting in competitive advantages and surplus value that may be used to progress technology and diversify economic activities, allowing for economic expansion. So, the development of human capital has become essential to attain growth. Numerous researchers concur that government spending on health and education to strengthen human capital had a favorable and considerable impact on the economy (Javed et al., 2013). The development of human capital is equally essential for economic success (Ali et al., 2012). Human potential and economic expansion are inextricably intertwined, according to the assertions (Asghar et al., 2012). The relevance of education in promoting the development of human resources and the influence on GDP in Pakistan was highlighted by Jalil and Idrees (2013) during the period 1960 to 2010. The findings suggest that education is vital for rapid expansion and plays a major role in the formation of intellectual resources, which is required for an economic boom.

The economic process is completely and considerably related to human capital. The relationship between human capital and growth in Pakistan was discovered using time series data (1978-2007) and the Cobb-Douglas production function. Furthermore, the study stated that investments in the health and education sectors greatly aid the economic

process (Qadri & Waheed, 2011). Annual Pakistan time series data from 1970 to 2009 were used by Afzal et al. (2010), and the short and long-term interconnections between financial sector development and economic expansion were investigated. The statistics showed that education had a significant long and short-term impact on the economy. Furthermore, the study argued that pro-human capital investment is critical. Whereas, Khan and Rehman (2012) aimed to figure out human capital in different regions of Pakistan from 1979 to 2008 such as rural, urban, and inclusively four provinces of Pakistan. The study noticed detectable contrasts in human capital conditions among rural and urban areas of Pakistan. Building on human resources strengthens the skilled labor force, which causes expansion in the marginal productivity of capital. Human potential and economic advancement have long-term mutual relationships (Chani et al., 2012). Human capital is a mandatory source in achieving economic prosperity (Abbas, 2001; 2000). Likewise, Acemoglu and Robinson (2010) claimed that the supremacy of physical capital over other significant determinants of economic growth, such as human resources and technology, is one of the reasons why certain countries grow more slowly than others. Human capital had a beneficial impact on economic development, Siddiqui (2008) described human capital and physical capital through the simultaneous equation model using panel data from 64 nations for the years, 1996 and 2004. The objective is to identify the significance of primary needs (health and education) in human development techniques in those countries. The findings revealed that in terms of development, income per capita prioritizes over basic expenditures. However, the significance of health and education expenditure cannot be denied to boost productivity. Whereas, physical capital in developing countries was enticed by dynamic human capital.

Affluence is intrinsically tied to human resource efficacy, which is crucial for enhancing competitiveness in practically every economy. As a result, human resources must be considered. According to recent literature, human resources seem to be the most crucial feature of any economy. Minhaj (2021) favored human resources and was utilized to investigate the link between human capital and growth using the Vector Error Correction Model (VECM) technique. The study's findings revealed that human capital and growth are intimately linked to the analysis of numerous policy challenges, and also suggested that the government continue to invest heavily in the health and education sectors.

3. Data and methodology

This research study employs panel data analysis over the full period 1990-2019. The following investigation includes both factors that vary and those that don't. Data for several periods are unavailable in the sample period under review, whereas the collection of various variables for which data is available in developing, less developing, and developed nations is considered. In addition, developed countries include Singapore, South Korea, and Japan, while developing countries include China, Turkey, and Malaysia, with Pakistan, Bangladesh, and India falling into the less developed countries. Variables have been collected based on world development indicators, the World Economic Forum, and the United National Development Programme. Multiple variables, along with human capital, which is the study's main focus, have been explored in a cross-country study to assess growth. Furthermore, the impact of human capital development on growth is observed by employing comparative modeling using both Classical and Bayesian techniques.

The following statistical model is utilized in a proposed framework for quantifying the effect of human capital on economic expansion. The model can also be represented in this way:

$$y_{it} = \beta_0 + \beta_1 h_{it} + \beta_2 k_{it} + \beta_3 Gh_{it} + \beta_4 gk_{it} + \beta_5 Le_{it} + \beta_6 He_{it} + \beta_7 Ee_{it} + \mu_{it} \quad (1)$$

In this model y_{it} is the dependent variable which represents GDP, β_0 is the intercept of the model, and explanatory variables include, h_{it} for the level of human capital development, k_{it} for the level of physical capital, Gh_{it} is the growth rate of human capital, gk_{it} is the growth of physical capital, Le_{it} is Life expectancy rate, He_{it} is expenditure on health, Ee_{it} is expenditure on education, And μ_{it} is the error term of the model.

3.1 Methodology of analysis for classical framework

3.1.1 Panel data regression models

Periods and cross-sectional units separate the three basic categories of data. A collection of information known as

time-series data is data that changes over time. There are many ways to measure time, from a second to an hour to a year. At the same time, data for a certain variable was obtained from a variety of sources.

$$y_{it} = F(x_{it}) + \mu_{it} \quad (2)$$

These assumptions are used to estimate the model in question. The “common effect model”, “fixed-effect model”, and “random effect model” are all based on these assumptions. According to Classic Non-linear Regression Model (CNLRM), a common effect model is one in which all the model’s parameters indicate a common effect for both time and cross-sectional units. Estimation of a common impact parameter from limited follow-up data is used by Greenland and Robins (1985). It is calculated using the least squares method. Instrumental variable techniques, such as 2SLS or GMM, can be used to address endogeneity concerns. At least one model parameter in a fixed-effect model changes over time or across cross-sectional units. The Least Squares Dummy Variable (LSDV) model compensates for heterogeneity by assigning an intercept value to each of its possible entities. Consider the model in (3) of a common effect

$$y_{it} = \beta_{0i} + \beta_1 h_{it} + \beta_2 k_{it} + \beta_3 Gh_{it} + \beta_4 gk_{it} + \beta_5 Le_{it} + \beta_6 He_{it} + \beta_7 Ee_{it} + \mu_{it} \quad (3)$$

The subscript “*i*” in the above equation shows that we can allow intercepts to vary or differ among countries because each country has its characteristics. As a result, the overhead model is referred to as a fixed effect model because each country has its specific intercept value that does not change over time, making it time-invariant. If the variable fluctuates across time, we can include time dummies in the model for all periods. How can we allow for differences in fixed-effect intercept between countries? Using the dummy variable method, we can effectively deal with the situation. We can now write the model as follows:

$$y_{it} = \beta_1 + \beta_{01}D_1 + \beta_{02}D_2 + \beta_{03}D_3 + \beta_{04}D_4 \dots + \beta_{08}D_{08} + \beta_1 h_{it} + \beta_2 k_{it} + \beta_3 Gh_{it} + \beta_4 gk_{it} + \beta_5 Le_{it} + \beta_6 He_{it} + \beta_7 Ee_{it} + \mu_{it} \quad (4)$$

Where, $D_2 = 1$ for country 2, Otherwise 0; $D_3 = 2$ for country 3, Otherwise, zero and so on.

The research involves nine distinct nations, eight dummies can be used. By introducing an error term that represents random fluctuations in one or more parameters, we may simply express REM as though the parameters of the model are predicted to change randomly over time units or periods. This is an example of a random-effect panel data model.

$$\beta_{0i} = (\beta_0 + \varepsilon_i) \quad (5)$$

Where, $i = 1, 2, 3, \dots N$.

Instead of interpreting $0i$ as a random variable with a mean value and an intercept value for every individual, we might assume that 0 is fixed. We can assume that β_0 is fixed, and β_{0i} is treated as a random variable with a mean value of β_{0i} and the intercept value for an individual country. Where ε_i is a random error term with a mean value of zero and variance $\sigma^2\varepsilon$

$$y_{it} = \beta_0 + \beta_1 h_{it} + \beta_2 k_{it} + \beta_3 Gh_{it} + \beta_4 gk_{it} + \beta_5 Le_{it} + \beta_6 He_{it} + \beta_7 Ee_{it} + \varepsilon_i + \mu_{it} \quad (6)$$

so,

$$w_{it} = \varepsilon_i + \mu_{it} \quad (7)$$

In addition to the time-series and cross-sectional error components, the complex error term includes the time-series error component as well.

3.2 Methodology of analysis for bayesian estimation

Classical Bayes is one of the Bayesian strategies that were used in the research. The Bayesian approach is a wide

one, so let's take a closer look at some specific Bayesian estimating methods first.

3.2.1 An overview of the bayesian methodology

Bayesian theory assumes that data regarding unknown parameters should be represented as densities. The Bayes technique was used to revise the prior information and calculate the resulting probability when all the data had been identified. Formulas for the prior-to-posterior transformation of normal distributions may be identified using these phrases. It comprises all the information about a parameter once the data has been collected and analyzed. This is the best Bayesian estimate for a collection of loss functions containing quadratic loss, which is a one-point summary. Because of this, Bayesian regression parameter estimates may be obtained before the posterior modification equations. The formula is the simplest to utilize once the previous data has been transformed to a normal density.

We use the Bayesian estimation procedure to estimate the model's parameters. As compared to Classical estimating approaches, Bayesian analysis provides significant benefits in random research such as:

- In contrast to classical estimation, Bayesian analysis considers that the parameter being estimated is random and has some prior density. As a result of this characteristic, Bayesian estimation is well matched to panel data with independent model parameters.
- Bayesian analysis is a basic way to integrate previous assumptions (information) with data. In principle, any prior information of one's choosing can be combined with data information. In panel data models, a prior can be comprised of the average of individual parameter estimates.
- Bayesian estimations are more precise than classical estimations. This indicates that Bayesian estimates have a low standard error, making inferences more reliable.
- Bayesian estimates produce reliable results for small samples. Unlike classical estimates, Bayesian estimates do not rely on a single asymptotic finding. As a result, several authors, including Hsiao and Koop, and others, suggest Bayesian models for panel data (2000).

3.2.2 Bayesian statistical modeling approach

Bayesian theory's core concept is just to integrate prior and sample information to effectively obtain subsequent information, which is expressed as the posterior density.

$$f(\theta/y) = \frac{L(\theta/y)g(\theta)}{\int L(\theta/y)g(\theta)d\theta}$$

Or

$$f(\theta/y) \propto L(\theta/y)g(\theta)$$

Where $g(\theta)$ denotes the prior probability density of parameter θ , which indicates about the analysis of unusual parameter prior to the sample x . And $L(\theta/y)$ is the sample x likelihood function, which is the sampling distribution of the samples based on the probability model and parameter specified.

3.3 Bayesian derivation of common effect model

Following is the model's specification:

$$y_{it} = \beta_0 + \beta_n h_{it} + \beta_n k_{it} + \beta_n G h_{it} + \beta_n g k_{it} + \beta_n L e_{it} + \beta_n H e_{it} + \beta_n E e_{it} + \mu_{it} \dots \quad (8)$$

Where $i = 1, 2, 3 \dots N, t = 1, 2 \dots N$.

The model may be expressed as a matrix, and the final structural model can be expressed as follows.

$$y = x\beta + ui \quad (9)$$

The given model's likelihood function is as follows:

$$P(y/\beta, H) = \frac{H^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} \left\{ \exp \left[-\frac{H}{2} (y - x\beta)^T (y - x\beta) \right] \right\} \quad (10)$$

$$\left[(y - x\beta)^T (y - x\beta) \right]$$

Add and subtract $x\hat{\beta}$

$$\left[(y - x\hat{\beta} + x\hat{\beta} - x\beta)^T (y - x\hat{\beta} + x\hat{\beta} - x\beta) \right] \left[(y - x\hat{\beta}) - x(\beta - \hat{\beta}) \right]^T \left[(y - x\hat{\beta}) - x(\beta - \hat{\beta}) \right] \quad (11)$$

$$\left[(y - x\hat{\beta})^T (y - x\hat{\beta}) + (\beta - \hat{\beta})^T x^T x (\beta - \hat{\beta}) \right]$$

As well as the cross-product expressions

$$(\beta - \hat{\beta})^T x^T (y - x\beta) = (\beta - \hat{\beta})^T (x^T y - x^T x (x^T x)^{-1} x^T y) = 0$$

$$(y - x\hat{\beta})^T (y - x\hat{\beta}) = SSE$$

$$(y - x\hat{\beta})^T (y - x\hat{\beta}) + (\beta - \hat{\beta})^T x^T x (\beta - \hat{\beta}) (y - x\hat{\beta})^T (y - x\hat{\beta}) = SSE + (\beta - \hat{\beta})^T x^T x (\beta - \hat{\beta}) \quad (12)$$

Put equation (12) in equation (10) we have

$$P(y/\beta, H) = \frac{H^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} \left\{ \exp \left[-\frac{H}{2} SSE + (\beta - \hat{\beta})^T x^T x (\beta - \hat{\beta}) \right] \right\} \quad (13)$$

Non informative prior for linear model:

The uniform prior is defined as follows:

$$p(\beta) \propto c \text{ and } p(Y) = 1 \quad (14)$$

The posterior distribution of the model is calculated using the likelihood function (11) and the prior distribution (14).

$$P(y/\beta, H) = \frac{H^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} \left\{ \exp \left[-\frac{H}{2} (y - x\beta)^T (y - x\beta) \right] \right\} \quad (15)$$

By applying rules of the Ordinary Least Squares (OLS) method

$$\hat{\beta} = (x^T x)^{-1} x^T y \quad \delta^2 = \frac{(y - x\hat{\beta})^T (y - x\hat{\beta})}{(n - k)}$$

Hence,

$$= \frac{H^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} \left\{ \exp \left[-\frac{H}{2} SSE + (\beta - \hat{\beta})^T x^T x (\beta - \hat{\beta}) \right] \right\}$$

As a result, the joint posterior distribution function can be written as,

$$p(\beta, y/X, Y) \propto p(\beta, y/X, Y) p(\beta) p(y)$$

$$\propto \frac{H^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} \left\{ \exp \left[-\frac{H}{2} SSE + (\beta - \hat{\beta})^T x^T x (\beta - \hat{\beta}) \right] \right\} \quad (16)$$

By employing the kernel density methodology, we get the following equation

$$\propto H^{\frac{N}{2}} \exp^{-\frac{N}{2}(SSE)} \times \exp \left[-\frac{1}{2}(\beta - \hat{\beta})^T H x^T x (\beta - \hat{\beta}) \right]$$

$$\propto H^{a^* - 1} \exp^{-Hb^*} \times \exp \left[-\frac{1}{2}(\beta - \hat{\beta})^T Q^* (\beta - \hat{\beta}) \right] \quad (17)$$

The posterior parameters are

$$a^* = \frac{N+2}{2}, b^* = \frac{SSE}{2}, \hat{\beta} = (x^T x)^{-1} x^T y, Q^* = H(x^T x)$$

3.4 Normal linear regression model through independent normal gamma prior

By using natural conjugate prior whereas $p(\beta/H)$ is a normal density and $p(H)$ is the gamma density function. Here we use the same prior for the independence of β and H .

When $p(\beta/H)$ is a normal density and $p(H)$ is a gamma density function, a natural conjugate prior is used. For the interdependence of β and H , we utilise the same prior here.

Specifically, we adopt $p(\beta/H) = p(\beta)p(H)$ with $p(\beta)$ existence normal distribution and $p(H)$ being the pdf of the gamma distribution.

$$p(\beta) = \frac{1}{(2\pi)^{\frac{k}{2}}} |Q|^{-\frac{1}{2}} \exp \left[-\frac{1}{2}(\beta - \underline{\beta}) \underline{Q}^{-1} (\beta - \underline{\beta}) \right] \quad (18)$$

And

$$p(H) = C_G^{-1} H^{a-1} \exp(-Hb) \quad (19)$$

Where, C_G^{-1} suppose to the integrating constant which is used for the gamma pdf. Whereas $\underline{\beta} = E(\beta/Y)$ is still the prior mean of β and \underline{Q} is the variance and covariance matrix of β . Where $\text{var}(\beta/H) = H^{-1}\underline{Q}$. Where in $p(\beta)$, “ a ” is the scale parameter and “ b ” shape parameter. The parameter in $p(\beta)$ and $p(H)$ written as follows.

By compiling equation (18) and equation (19) we obtain the following results of normal gamma prior

$$p(\beta, H) \propto \exp \left[-\frac{1}{2}(\beta - \underline{\beta}) \underline{Q}^{-1} (\beta - \underline{\beta}) \right] \left[H^{a-1} \exp(-Hb) \right] \quad (20)$$

3.5 Posterior distribution under normal gamma prior

$$p(\beta, H/Y) \propto p(\beta, H)L(\beta, H) \quad (21)$$

$$p(\beta, H/Y) \propto \exp\left[-\frac{1}{2}(\beta - \underline{\beta})\underline{Q}^{-1}(\beta - \underline{\beta})\right] \left[H^{a-1} \exp(-Hb) \right] \times \left\{ \exp\left[-\frac{H}{2}SSE + (\beta - \hat{\beta})^T x^T x(\beta - \beta)\right] \right\} \quad (22)$$

$$= H^{a+\frac{1}{2}-1} \exp\left(-H\left(b - \frac{SSE}{2}\right)\right) \exp\left[-\frac{1}{2}(\beta - \underline{\beta})\underline{Q}^{-1}(\beta - \underline{\beta}) + (\beta - \hat{\beta})^T Hx^T x(\beta - \beta)\right]$$

$$= H^{a_1-1} \exp(-Hb_1) \exp\left[-\frac{1}{2}(\beta - \underline{\beta})\underline{Q}^{-1}(\beta - \underline{\beta}) + (\beta - \hat{\beta})^T Hx^T x(\beta - \hat{\beta})\right] \quad (23)$$

Now taking a part of equation 23

$$\left[(\beta - \underline{\beta})\underline{Q}^{-1}(\beta - \underline{\beta}) + (\beta - \hat{\beta})^T Hx^T x(\beta - \hat{\beta}) \right]$$

We get the following equation by simplifying the preceding equation

$$H^{a_1-1} \exp(-Hb_1) \exp\left[-\frac{1}{2}(\underline{Q}^{-1} + Hx^T x) \left(\beta - \frac{\underline{Q}^{-1}\beta + Hx^T x\hat{\beta}}{\underline{Q}^{-1} + Hx^T x} \right)^2\right] \quad (24)$$

$$a_1^* = a + \frac{N}{2} \quad \text{And} \quad b_1^* = b + \frac{SSE}{2}$$

As we know that

$$\bar{Q}^{-1} = (\underline{Q}^{-1} + Hx^T x)$$

$$\bar{Q} = (\underline{Q}^{-1} + Hx^T x)^{-1}$$

$$\bar{\beta} = \frac{\underline{Q}^{-1}\beta + Hx^T x\hat{\beta}}{\underline{Q}^{-1} + Hx^T x} = \frac{\underline{Q}^{-1}\beta + Hx^T x\hat{\beta}}{\underline{Q}^{-1}}$$

$$\bar{\beta} = \bar{Q}(\underline{Q}^{-1}\beta + Hx^T x\hat{\beta})$$

Where $\bar{\beta}$ is the mean and \bar{Q} is the posterior distribution's var-cov matrix. However, it should be noted that the probability function and the distribution of normal Gamma prior do not correspond to the posterior, and the posterior simulator is known as Gibbs' sampler, which follows multivariate normal and gamma distributions. Then we'll move on to Monte Carlo integration.

3.6 Bayesian derivation of fixed effect model

Given hyperparameters $\alpha_1, \tau_1, \alpha_0, \tau_2, \beta_0, V_0$, the prior Distributions are: $f_i \sim N(0, \Phi)$, $\Phi = \text{diag}(\sigma_1^2, \dots, \sigma_k^2)$ and

$\sigma_k^{-2} \sim \text{Gamma} \left(\frac{\alpha_1}{2}, \frac{2}{\tau_1} \right)$; $\beta \sim N(\beta_0, V_0)$, $\varepsilon_i \sim N(0, \Psi)$, with $\Psi = \text{diag}(\sigma_1^2, \dots, \sigma_K^2)$ and $\sigma_k^{-2} \sim \text{Gamma} \left(\frac{\alpha_1}{2}, \frac{2}{\tau_1} \right)$, Allowing heteroskedasticity and independence across all pairs, $(\varepsilon_{it}, \varepsilon_{it})$; factor loading vector for everyone $\lambda_i \sim (0, I_k)$.

The likelihood of the data $y_i = (y_{i1}, \dots, y_{iN})'$

$$P(y_i | \beta, X_i, \Lambda, \Psi) = (2\pi)^{-\frac{N}{2}} |\Psi|^{-\frac{1}{2}} \exp \left(-\frac{1}{2} (y_i - X_i \beta - \Lambda f_i)' \Psi^{-1} (y_i - X_i \beta - \Lambda f_i) \right)$$

In matrix form, we could write is:

$$P(Y | \beta, X, F, \Lambda, \Psi) \propto (2\pi)^{-\frac{N}{2}} |\Psi|^{-\frac{1}{2}} \exp \left(-\frac{1}{2} \sum_{i=1}^T (y_i - X_i \beta - \Lambda f_i)' \Psi^{-1} (y_i - X_i \beta - \Lambda f_i) \right)$$

The Posterior Distribution for factor is:

$$\begin{aligned} P(f_i | \beta, X_i, \Lambda, \Psi, y_i) \\ \propto P(y_i | \beta, X_i, f_i, \Lambda, \Psi) \cdot P(f_i) \\ \propto \exp \left(-\frac{(y_i - X_i \beta - \Lambda f_i)' (y_i - X_i \beta - \Lambda f_i) + f_i' \Phi^{-1} f_i}{2} \right) \end{aligned}$$

The term $(y_i - X_i \beta - \Lambda f_i)' \Psi^{-1} (y_i - X_i \beta - \Lambda f_i) + f_i' \Phi^{-1} f_i$ can be written as:

$$(f_i - m) \Sigma^{-1} (f_i - m)' + (y_i - X_i \beta) \Psi^{-1} (y_i - X_i \beta) - m' \Sigma^{-1} m$$

With $\Sigma = (\Phi^{-1} + \Lambda' \Psi^{-1} \Lambda)^{-1}$ and $m = \Sigma \Lambda^{-1} \Psi^{-1} (y_i - X_i \beta)$. The Posterior Distribution of $f_i \sim N(m, \Sigma)$. The Posterior Distribution for the Variance matrix Φ and F is derived as:

$$\begin{aligned} P(\Phi | Y, X, F, \Lambda, \Psi, \beta) \\ \propto P(Y, X, \Lambda, \Psi, \beta | F) \cdot P(F | \Phi) \cdot P(\Phi | \alpha_1, \tau_1) \\ \propto P(F | \Phi) \cdot P(\Phi | \alpha_1, \tau_1) \end{aligned}$$

According to assumptions, $P(F | \Phi^{-1}) = \prod_{k=1}^K \prod_{t=1}^T (\sigma_k^{-2})^{\frac{1}{2}} \exp \left(-\frac{1}{2} \sigma_k^{-2} (f_{t,k}^2) \right)$ and then

$$\begin{aligned} P(F | \Phi^{-1}) \\ \propto \prod_{k=1}^K (\sigma_k^{-2})^{\frac{\alpha_1 + T}{2} - 1} \exp \left(-\frac{\tau_1 + \sum_{t=1}^T f_{t,k}^2}{2} \sigma_k^{-2} \right) \end{aligned}$$

And so, the Posterior Distribution of $\sigma_k^{-2} \sim \text{Gamma} \left(\frac{T + \alpha_1}{2}, \frac{2}{\tau_1 + \sum_{t=1}^T f_{t,k}^2} \right)$.

The Posterior Distribution of λ_i is proportional to $\propto P(Y | \beta, X, \Lambda, F, \Psi) \cdot P(\lambda_i)$ and can be written as:

$$P(\lambda_i | Y, X, \Lambda, F, \Psi)$$

$$P(Y|\beta, X, \Lambda, F, \Psi).P(\lambda_i)$$

$$\propto \exp\left(-\frac{(y_i - X_i'\beta - F\lambda_i)'(y_i - X_i'\beta - F\lambda_i) + \Psi_i^{-2}\lambda_i'\lambda_i}{2}\right)$$

Together with the assumptions of Λ as a lower triangle matrix, we can see the Posterior Distribution of λ_i is for $i = 1, \dots, k$, $\lambda_i \sim N(m_i, \Sigma_i)$ with $\Sigma_i = (F_{(i)}' \Psi_i^{-2} F_{(i)} + I_i)^{-1}$ and $m_i = (F_{(i)}' \Psi_i^{-2} F_{(i)} + I_i)^{-1} \Psi_i^{-2} F_{(i)}' (y_i - X_i'\beta)$; $K + 1 = 1, \dots, N$, $\lambda_i \sim N(m_i, \Sigma_i)$ with $\Sigma_i = (F'\Psi_i^{-2}F + I_K)^{-1}$ and $m_i = (F'\Psi_i^{-2}F + I_K)^{-1} \Psi_i^{-2} F'(y_i - X_i'\beta)$. The diagonal elements of λ_{ii} of are assumed to be 1.

The Posterior Distribution of β can be written as:

$$\propto |\Psi|^{\frac{T}{2}} \exp\left(-\frac{1}{2} \sum_{t=1}^T (y_t - X_t'\beta - \lambda f_t)' \Psi^{-1} (y_t - X_t'\beta - \lambda f_t)\right)$$

$$|V_0|^{-\frac{1}{2}} \exp\left(-\frac{1}{2} (\beta - \beta_0)' V_0^{-1} (\beta - \beta_0)\right)$$

After matrix computations, the Posterior Distribution of β is $\beta \sim N(\bar{\beta}, \bar{V})$, with $\bar{V} = \left(\sum_{t=1}^T X_t^T \Psi^{-1} X_t + V_0^{-1}\right)$ and $\bar{\beta} = \bar{V} \cdot \sum_{t=1}^T X_t^T \Psi^{-1} (y_t - \lambda f_t) + V_0^{-1} \beta_0$.

The Posterior Distribution of the covariance term of the error term is derived in below.

The Prior Distribution of Ψ^{-1} is Gamma $\left(\frac{\alpha_1}{2}, \frac{2}{\tau_1}\right)$ and the Posterior of Ψ^{-1} is

$$\propto \prod_{i=1}^N (\Psi^{-1})^{\frac{\alpha_2+T}{2}-1} \exp\left(-\frac{\sum_{t=1}^T (y_{it} - x_{it}\beta - \lambda_i' f_t)^2 + \tau_2}{2} \sigma_k^{-2}\right)$$

And hence $\sigma_k^{-2} \sim \text{Gamma}\left(\frac{T + \alpha_2}{2}, \frac{2}{\sum_{t=1}^T (y_{it} - x_{it}\beta - \lambda_i' f_t)^2 + \tau_2}\right)$ for $i = 1, \dots, N$.

3.7 Bayesian derivation of random effect model

Consider the same model for random effect in bayesian derivation, which is used above in classical derivation

$$y_{it} = \beta_0 + \beta_1 h_{it} + \beta_2 k_{it} + \beta_3 G h_{it} + \beta_4 g k_{it} + \beta_5 L e_{it} + \beta_6 H e_{it} + \beta_7 E e_{it} + \varepsilon_{it} + \mu_{it} \quad (25)$$

Where $\mu_{it} = \beta_0 + \mu_{it}$

$$y_{it} = \mu + \Sigma \beta_j x_j + \varepsilon_{it} \quad (26)$$

$$y_{it} = \beta_0 + \mu_i + \Sigma \beta_j x_j + \varepsilon_{it} \quad (27)$$

$$y_{it} = \beta_0 + \Sigma \beta_j x_j + \varepsilon_{it} + \mu_i \quad (28)$$

$$y_{it} = \beta_0 + \Sigma \beta_j x_j + \omega_{it} \quad (29)$$

$$y = x\beta + \omega$$

$$y \sim N(x\beta, \omega)$$

$$\Psi = E(\omega'\omega) = I_N \times (\delta^2 \varepsilon I_t + \delta^2 u e e')$$

$$\delta^2 \varepsilon (I_N \times I_t) + \delta^2 u (I_N \times e e')$$

Replace I_t by $(E_t + J_t)$ and $e e'$ by T_{JT} , where $J_t = \frac{1}{T} e e'$ and $E_t = I_t + J_t$, then

$$\Psi = \delta^2 \varepsilon [(I_N \times (E_t + J_t))] + \delta^2 u + (I_N \times T_{JT})$$

$$= \delta^2 \varepsilon (I_N \times I_t) + \delta^2 \varepsilon (I_N \times J_t) + T \delta^2 u (I_N \times J_t) \quad (30)$$

Where

$$Q = (I_N \times E_t)$$

$$\delta^2_1 = (\delta^2 \varepsilon + T \delta^2 u)$$

$$P = (I_N \times J_t)$$

Replace this in equation 5

$$= \delta^2 \varepsilon Q + \delta^2_1 P$$

$$\Psi^{-1} = \left(\frac{Q}{\delta^2 \varepsilon} + \frac{P}{\delta^2_1} \right)$$

$$|\Psi| = (\delta^2 \varepsilon)^{N(T-1)} (\delta^2_1)^N$$

Now likelihood function is the joint density of the y 's that is

$$\begin{aligned} L(y; \varphi, \Psi) &= \prod_{i=1}^{NT} \frac{1}{(2\pi)^{\frac{1}{2}}} |\Psi|^{-1/2} \exp \left\{ -\frac{1}{2(y-x\beta)' \Psi (y-x\beta)} \right\} \\ &= (2\pi)^{-\frac{NT}{2}} (\delta^2 \varepsilon)^{-\frac{N(T-1)}{2}} (\delta^2_1)^{-\frac{NT}{2}} \exp \left\{ -\frac{1}{2} (y-x\beta)' \left[\frac{Q}{\delta^2 \varepsilon} + \frac{P}{\delta^2_1} \right] (y-x\beta) \right\} \end{aligned} \quad (31)$$

The prior information:

A prior distribution on $(\beta, \delta^2 \varepsilon, \delta^2_1)$ is required to specify a complete Bayesian model. We'll take the vector parameters' uniform distribution $u(0, 1)$ and assume that the previous distribution on $\delta^2 \varepsilon$ and δ^2_1 are invers gamma with parameters, $\alpha_\varepsilon, \beta_\varepsilon, \alpha_1$ and β_1 respectively.

$$P(\delta^2 \varepsilon) = \frac{\beta_\varepsilon^{\alpha_\varepsilon}}{\sqrt{\alpha_\varepsilon}} (\delta^2 \varepsilon)^{-(\alpha_\varepsilon+1)} \exp \left(\frac{-\beta_\varepsilon}{\delta^2 \varepsilon} \right)$$

And

$$P(\delta^2_1) = \frac{\beta_1^{\alpha_1}}{\sqrt{\alpha_1}} (\delta^2_1)^{-(\alpha_1+1)} \exp\left(\frac{-\beta_1}{\delta^2_1}\right)$$

$$L(y/\varphi, \delta^2\varepsilon, \delta^2_1) = \prod_{i=1}^{NT} t = 1 (2\pi)^{-\frac{1}{2}} |\Psi|^{1/2} \exp\left\{-\frac{1}{2(y-x\beta)^T \Psi (y-x\beta)}\right\}$$

Where $\hat{y} = x\hat{\beta}$

$$[(y-x\beta)^T \Psi^{-1} (y-x\beta)]$$

Adding and subtracting $x\hat{\beta}$

$$\begin{aligned} & \left[(y-x\hat{\beta} + x\hat{\beta} - x\beta)^T \Psi^{-1} (y-x\hat{\beta} + x\hat{\beta} - x\beta) \right] \\ &= \left[(y-x\hat{\beta}) - x(\beta - \hat{\beta}) \right]^T \Psi^{-1} \left[(y-x\hat{\beta}) - x(\beta - \hat{\beta}) \right] \\ &= \left[(y-x\hat{\beta})' (y-x\hat{\beta}) - (y-x\hat{\beta})' x(\beta - \hat{\beta}) - (\beta - \hat{\beta})' x' (y-x\hat{\beta}) + (\beta - \hat{\beta})' x' x (\beta - \hat{\beta}) \right] \end{aligned} \quad (32)$$

The equation now gives the joint posterior density of the coefficient β and variance $\delta^2\varepsilon$ and δ^2_1 .

$$\begin{aligned} & \pi_1\left(\beta, \delta^2\varepsilon, \frac{\delta^2_1}{y}\right) \propto L\left(\frac{y}{\beta}, \delta^2\varepsilon, \delta^2_1\right) \pi_0(\beta, \delta^2\varepsilon, \delta^2_1) \\ & \propto (2\pi)^{-\frac{NT}{2}} (\delta^2\varepsilon)^{-\frac{N(T-1)}{2}} (\delta^2_1)^{\frac{NT}{2}} \exp\left\{-\frac{1}{2}(y-x\hat{\beta})^T \left[\frac{Q}{\delta^2\varepsilon} + \frac{P}{\delta^2_1}\right] (y-x\hat{\beta})\right\} \exp\left\{-\frac{1}{2}(\beta-\hat{\beta})^T X^T \Psi^{-1} X (\beta-\hat{\beta}) \times \frac{\beta_1^{\alpha_1}}{\sqrt{\alpha_1}} (\delta^2\varepsilon)^{-(\alpha_1+1)}\right\} \\ & \exp\left(\frac{-\beta_1}{\delta^2_1}\right) \frac{\beta_1^{\alpha_1}}{\sqrt{\alpha_1}} (\delta^2_1)^{-(\alpha_1+1)} \exp\left(\frac{-\beta_1}{\delta^2_1}\right) \\ & \propto (\delta^2\varepsilon)^{-\left(\alpha_1 + \frac{N(T-1)}{2} + 1\right)} \exp\left\{-\frac{1}{2} \frac{(y-x\hat{\beta})^T Q (y-x\hat{\beta}) + \beta_1}{\delta^2\varepsilon}\right\} \times \delta^2_1^{-\left(\alpha_1 + \frac{N}{2} + 1\right)} \exp\left\{-\frac{1}{2} \frac{(y-x\hat{\beta})^T P (y-x\hat{\beta}) + \beta_1}{\delta^2_1}\right\} \exp\left\{-\frac{1}{2} (\beta-\hat{\beta})^T X^T \Psi^{-1} X (\beta-\hat{\beta})\right\} \end{aligned}$$

The conditional and marginal posterior distributions may be deduced from this equation.

$$\begin{aligned} & \pi_i(\beta/\delta^2\varepsilon, \delta^2_1, Y) \propto \exp\left\{-\frac{1}{2} \frac{(y-x\hat{\beta})^T Q (y-x\hat{\beta}) + \beta_1}{\delta^2\varepsilon}\right\} \\ & \pi_i(\delta^2\varepsilon/\beta, \delta^2_1, Y) \propto (\delta^2\varepsilon)^{-\left(\alpha_1 + \frac{N(T-1)}{2} + 1\right)} \exp\left\{-\frac{1}{2} \frac{(y-x\hat{\beta})^T Q (y-x\hat{\beta}) + \beta_1}{\delta^2\varepsilon}\right\} \pi_i(\delta^2_1/\beta, \delta^2\varepsilon, Y) \end{aligned}$$

$$\propto \delta_1^{2-\left(\alpha_1 + \frac{N}{2} + 1\right)} \exp\left\{-\frac{1/2(y-x\hat{\beta})^T P(Y-x\hat{\beta}) + \beta_1}{\delta_1^2}\right\}$$

Therefore, it follows that

$$(\beta/\delta^2 \varepsilon, \delta^2_1, Y) \sim N(\varphi, X^T \Psi^{-1} X)^{-1}$$

$$(\delta^2 \varepsilon/\beta, \delta^2_1, Y) \sim IG\left(\alpha_\varepsilon + \frac{N(T-1)}{2}, \frac{1}{2}(y-x\hat{\beta})^T Q(y-x\hat{\beta}) + \beta_\varepsilon\right)$$

$$(\delta^2_1/\beta, \delta^2 \varepsilon, Y) \sim IG\left(\alpha_1 + \frac{N}{2}, \frac{1}{2}(y-x\hat{\beta})^T P(Y-x\hat{\beta}) + \beta_1\right)$$

4. Result and discussion

4.1 Estimation of panel data models under classical framework

The model's parameters, which contain both time-period and cross-sectional units as well as the error term, are referred to as a common effect model. This model may also be estimated using the conventional least square approach, however only if done properly.

The upper table displays the classical outcomes of 3-panel data models. Various types of diagnostic tests are employed for different models. F-tests are utilized between the common effect model and the fixed-effect model. According to the F test, which model has a low F test score it must be considered a good model. In the following table fixed model has a low value as compared to the common effect model. So, we can claim that the fixed model is the best rather than the common effect model. The Hausman test is used between the fixed effect model and the random effect model. The Hausman test indicates that the random effect is the best model. Because the P-value is 0.31, which is more than 0.05. So, we can only interpret the random effect model. The hausman test is used between the Fixed effect model and random effect model.

The Hausman test is used between the Fixed effect model and the random effect model. The Hausman test indicates that the random effect is the best model. Because the P-value of the Hausman test is more than 0.05. So, we can only interpret the random effect model. According to the study's findings, the unit change in h_{it} was responsible for a 4.73 unit increase in y_{it} , with a standard error of 1.071, and showed a significant effect on the growth, which is the dependent variable, while the unit change in Gh_{it} responded to 0.15 increase in y_{it} with standard error 0.07 and show a significant effect on growth. Moreover, a unit change in gk_{it} drops y_{it} by 0.09, with a standard error of 0.022. Unit Change in Le_{it} will increase to y_{it} by 0.014 with a standard error of 0.80. A unit change in He_{it} strengthens the y_{it} by 0.30 significantly with a standard error of 0.10. Furthermore, a unit change in E_{it} decreases by 2.53 units in y_{it} with a standard error of 0.42. One unit increase in k_{it} 0.70 unit increase in y_{it} value-added with standard error 0.07 and show a significant effect on the dependent variable. Additionally, the R-squared coefficient of determination ranges from 0 to 1. According to the R-squared value, the 0.99 variation in GDP is explained by that model. The adjusted R-squared, also known as the correlation coefficient, is always between -1 and 1. and evaluates the degree of linear relationship between the variables and illustrates the model's excellent f_{it} . A value of 0.98 is found in the data, showing that there is a greater positive linear correlation.

The findings of this model 1 are shown in Table 1.

Table 1. Results of Classical estimation of panel data model 1

Models	Common effect model	Fixed effect model	Random effect model
	Estimates [std. Error] (P-value)	Estimates [std. Error] (P-value)	Estimates [std. Error] (P-value)
Intercept	6.594 3.99 0.006	9.30 4.10 0.030	6.594 3.99 0.009
h_{it}	4.73 1.071 0.000	5.404 1.141 0.000	4.73 1.071 0.000
Gh_{it}	0.15 0.07 0.032	0.20 0.07 0.000	0.15 0.07 0.002
gk_{it}	-0.09 0.022 0.000	-0.09 0.024 0.001	-0.09 0.022 0.000
Le_{it}	0.014 0.80 0.99	-0.46 0.80 0.57	0.014 0.80 0.99
He_{it}	0.30 0.10 0.008	0.11 0.08 0.008	0.30 0.10 0.006
Ee_{it}	2.53 0.42 0.000	-2.71 0.50 0.000	-2.53 0.42 0.000
k_{it}	0.70 0.07 0.000	0.07 0.07 0.000	0.70 0.07 0.000
R ₂	0.99	0.99	0.97
Adjusted R ₂	0.98	0.98	0.96
F test	1,486.24	1,391.77
Hausman test	-	-	0.31

4.2 Estimation of panel data model under bayesian framework

The interval does not contain any 0 value; hence it is significant based on a 95 percent credible interval coefficient estimate of variables, according to the results of Bayesian estimation of panel data Model 1 in Table 2.

Whereas, one unit change in h_{it} causes 0.15 units to increase in GDP. The average effect may vary from 0.090 to 0.21 which is a 95% credible interval for the average effect of h_{it} on GDP. Moreover, one unit change k_{it} causes 0.06 units to increase in GDP with the credible interval of 0.0157 to 0.109 which shows a significant effect on GDP.

While one unit change in Gh_{it} causes 0.013 units to increase in GDP with the credible interval of 0.010 to 0.013 which shows the significant effect on GDP. Also, a change of one unit in gk_{it} generates a 0.04 rise in GDP, with a credible Interval of 0.013 to 0.06, indicating a significant influence on GDP. While one unit changes in Le_{it} causes 0.004 units to increase in GDP with the credible interval of 0.016 to 0.004 which shows the significant effect. Moreover, one unit change in Ee_{it} causes 0.01 units to increase in GDP with a credible interval of 0.012 to 0.010 which shows the significant effect of the dependent variable. One unit change in He_{it} causes 0.03 units to increase in GDP with a credible interval of 0.065 to 0.069 which shows the significant effect of the dependent variable.

Table 2. Results of Bayesian estimation of panel data Model 1

Models	Common effect model	Fixed effect model	Random effect model
	Mean [Std. Error] Cd. I	Mean [Std. Error] Cd. I	Mean [Std. Error] Cd. I
Intercept	2.60 0.041 2.55-2.71	7.60 0.21 7.17-7.94	2.80 0.29 2.59-2.81
h_{it}	0.15 0.031 0.090-0.21	0.45 0.14 0.19-0.71	-0.006 0.043 -0.028-0.15
k_{it}	0.06 0.024 0.01-0.109	-3.032 0.07 -3.16--2.9	0.03 0.20 0.221-0.083
Gh_{it}	0.013 0.012 0.010-0.13	0.06 0.05 0.033-0.147	0.03 0.04 0.019-0.036
gk_{it}	0.04 0.01 0.013-0.06	0.20 0.05 0.064-0.169	0.02 0.04 0.013-0.033
Le_{it}	0.004 0.01 0.016-0.004	0.0601 0.05 0.060-0.154	0.003 0.020 0.004-0.011
Ee_{it}	0.01 0.01 0.012-0.010	0.003 0.05 0.098-0.109	0.005 0.02 0.003-0.012
He_{it}	0.03 0.05 0.065-0.065	0.10 0.08 0.063-0.0678	0.04 0.20 0.244-0.091

5. Conclusion

This research focused on constructing human capital indices for selected countries and utilized both Bayesian and classical methodologies to analyze panel data models. The primary goal was to identify the most effective model for informing policy decisions and providing valuable guidance to policymakers. The study’s findings emphasize the vital importance of prioritizing human capital development to enhance economic growth across diverse economies.

Moving forward, policymakers should heed these findings and recognize the pivotal role of human capital in achieving sustainable economic growth. Investing in education, healthcare, skills development, and technology is crucial for strengthening human capital, ultimately positioning countries for greater economic stability and prosperity on a global scale. Recognizing the significance of human capital and employing various statistical tools for data analysis are essential steps in shaping effective policies for economic advancement.

This research study employs a primary model with GDP as the dependent variable. In the classical approach, three sub-models are utilized to predict GDP. The Bayesian strategy is then applied by adapting the same models used in the classical approach. Subsequently, the outcomes of both the Classical and Bayesian approaches are compared to determine which strategy is more effective and yields more significant results. To further ascertain the best technique, estimations from both Classical and Bayesian panel data models are compared.

In the classical approach, the GDP model incorporates three sub-models: common effect, fixed effect, and random effect. All three models yield favorable results. However, due to a high F-test score, the fixed effect model is preferred over the common effect model. To choose between the fixed and random effect models, the Hausman test is conducted. The Hausman test’s P-value is greater than 0.05 in the case of random effects, indicating that the random effect model is more appropriate. Only one variable shows insignificance among all variables, suggesting it has no influence on GDP. While all other factors exhibit a connection to GDP, the study’s primary focus on human capital reveals a highly significant P-value of 0.000, indicating a robust and significant relationship with economic growth across all countries.

Moreover, Bayesian estimations are incorporated into the model, with credible interval values chosen. Positive or

negative credible interval values indicate the variables' significant impact on the dependent variable. Based on the credible interval (0.090-0.21), it can be concluded that human capital significantly influences GDP in Bayesian estimation.

In summary, both classical and Bayesian approaches support the significant impact of human capital on GDP. However, Bayesian models exhibit lower standard errors and are considered superior. Ultimately, Bayesian approaches reveal that human capital serves as a major determinant of GDP in developed, developing, and less developed nations.

5.1 Limitations

This study has notable limitations. It relies on data quality and availability, potentially affecting the analysis. It may not fully consider regional nuances, and establishing causality between human capital and economic outcomes can be complex. The conclusions are based on a specific time frame, and different statistical techniques could yield varying results. Policy mechanisms and external shocks may not receive in-depth exploration, and the sustainability of economic growth is not extensively examined. Differences within countries or regions might be overlooked, and subjectivity in variable selection could influence findings. These limitations should be considered when interpreting the study's results for policymaking.

6. Recommendations

The research underscores several actionable recommendations to enhance human capital and drive economic growth. Firstly, promoting direct learning and knowledge exchange with Focal Points can accelerate human capital outcomes, which are crucial for economic expansion, particularly in developing and less developed economies. Secondly, the government should incentivize and support research and development activities, fostering innovation and aiding the performance of manufacturing sectors and SMEs.

Additionally, long-term economic progress relies on skilled labor, with various nations requiring different levels. Investments in education, healthcare, and skills development should be prioritized to strengthen human capital. Addressing disparities in access to these services, while focusing on reducing gender and regional inequalities, is also vital. Improving data collection and analysis capabilities is essential for monitoring policy impact effectively, while international collaborations can help exchange best practices in human capital development.

Conflict of interest

The authors declare no competing financial interest.

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