Journal of Electronics and Electrical Engineering
https://ojs.wiserpub.com/index.php/JEEE

Review

A Comprehensive Review of Direction of Arrival (DoA) Estimation Techniques and Algorithms

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Received: 15 October 2024; **Revised:** 12 December 2024; **Accepted:** 27 December 2024

Abstract: Direction of Arrival (DOA) estimation is a pivotal aspect of Array Signal Processing (ASP) with significant implications for the performance of modern communication systems. This paper provides a comprehensive review of DOA estimation techniques, encompassing both classical and advanced algorithms. Key methods such as MUSIC, ESPRIT, and their variations as well as Beamforming techniques are analysed for their theoretical foundations, computational complexity, and performance under various conditions. MATLAB simulations are conducted to evaluate the impact of critical parameters such as array element spacing, array geometry, number of snapshots, and signal incidence angle differences on estimation accuracy. Special attention is given to the challenges and enhancements in DOA estimation for MIMO systems, highlighting future directions in adaptive and machine learning-based approaches. The study emphasizes the potential of DOA estimation in applications spanning radar, sonar, wireless communication, and beyond, aiming to bridge gaps between current methodologies and emerging requirements.

***Keywords*:** Direction of Arrival (DoA), Array Signal Processing (ASP), MUSIC, Root-MUSIC, ESPRIT, beamforming techniques

Nomenclature

Term	**Description**
DoA	Direction of Arrival
ASP	Array Signal Processing
RADAR	Radio Detection and Ranging
RDF	Radio Detection Finding
RF	Radio Frequency
MUSIC	Multiple Signal Classification
ESPRIT	Estimation of Signal Parameters via Rotational Invariance Techniques
TLS ESPRIT	Total Least Squares Estimation of Signal Parameters via Rotational Invariance Techniques

DOI: https://doi.org/10.37256/jeee.4120255708

TR	Transmit-Receive
ATR	Anti Transmit-Receive
TDoA	Time Difference of Arrival
ML	Maximum Likelihood
LCMV	Linearly Constrained Minimum Variance Beamforming
DNN	Deep Neural Networks
CNN	Convolutional Neural Networks
UMTS	Universal Mobile Telecommunication System
MIMO	Multiple Input Multiple Output
UCA	Uniform Circular Array
ULA	Uniform Linear Array

1. Introduction

Direction of Arrival (DoA) technology plays a pivotal role in the discipline of signal processing by determining the angle at which signals arrive at an antenna array. This technique is paramount for applications that require precise localization and tracking of signal sources, leveraging the spatial information obtained from multiple sensors oriented in a particular configuration [1]. The effectiveness of DoA estimation significantly influences the performance of smart antennas, which are designed to optimize communication and detection capabilities in dynamic environments. In the realm of smart antenna technology, DoA estimation algorithms are essential, as they enable systems to deliver accurate data regarding the location and direction of wireless signals. Such precision is critical across various applications, including search and rescue operations, law enforcement activities, sonar systems, seismology studies, and emergency response scenarios. The ability to effectively ascertain the direction of incoming signals allows for timely and informed decision-making, thereby enhancing operational efficiency and safety in multiple domains. As a result, DoA estimation remains a vibrant and rapidly evolving area of research, with ongoing advancements aimed at improving algorithm performance and applicability in diverse signal-processing tasks [2, 3].

Propagating fields refer to the energy or wave patterns of various waves that travel through space or a medium. These waves include acoustic waves, which are used in sonar arrays and microphone applications, mechanical waves applied in seismic exploration, and electromagnetic (EM) waves, commonly employed in wireless communication and radar, especially for localization. The localization process involves using multiple sensors arranged in specific geometric configurations for estimation of angular directions for multiple sources. This is classified into two main cases, with the overdetermined case involving fewer sources than sensors, and the underdetermined case with the number of sources equal or greater than the number of sensors [4, 5]. Multiple transducers and sensors are arranged in a specific particular arrangement in a sensor array, wherein each transducer is used to transform mechanical or electrical waves into electrical signals, such as voltage. Incoming waves are identified and measured using this array of sensors. These waves travel from the source/radar transmitter, reflect off targets and are received by the radar receiver [6, 7]. Multiple sensors (i.e., an array of sensors) are used for this purpose, as they gather more detailed information than using only one sensor. Sensor arrays are in diverse fields such as SONAR, RADAR and Wireless Communication. In Radar, an array of sensors detects incoming waves, and the signals from these different sensor locations are processed collectively. Analysing phase and time difference of received signals at different sensors within an array, can help to locate the exact direction from which the signal is arriving. This processing enables the determination of valuable information such as the DoA of the waves, which is crucial for accurately locating and tracking objects. The ability to determine the DoA is essential for navigation, surveillance, and target tracking, significantly enhancing the capabilities of radar systems [8]. DoA estimation methods are subdivided into two primary types. The first consists of classical or conventional methods, which include techniques such as beamforming and delay-and-sum. These methods are relatively simple but often have limited resolution. The second type includes subspace-based or super-resolution methods, such as MUSIC and ESPRIT. These approaches provide higher resolution by exploiting the noise and signal subspaces. In addition to these traditional categories, more recent methods have emerged, incorporating machine learning and sparse signal processing techniques, though the classical and subspace-based methods remain the foundational approaches in DoA estimation. DoA methods are used in various applications, including the design and tuning of antenna arrays. These methods help in identifying the number of incoming signals, receiving signals from specific directions, and blocking or rejecting unwanted or interfering signals. They also play a crucial role in beamforming, where they assist in designing appropriate transmit and receive beamformers and enable the extraction of signals from the desired direction while suppressing signals from unwanted directions [4]. Wireless technology has become prevalent in numerous sectors, including public security, sensor networks, environmental monitoring, and search & rescue. According to recent developments, various technological policies have come into existence to accommodate the needs of current demands. The Federal Communications Commission (FCC) mandates that emergency calls must have location accuracy within 125 m. Similarly, in search and rescue operations, electromagnetic beacons are employed to emit radio signals that assist rescuers in locating individuals in distress. DoA is integral to these applications, as it involves determining the direction from which a radio signal is emitted, thereby enhancing the precision of location tracking [1]. DoA estimation algorithms are particularly important in wireless communication for enhancing the capacity and performance of networks. The ability to determine the DoA is essential for navigation, surveillance, and target tracking, significantly enhancing the capabilities of radar systems [4].

A comprehensive overview of the most significant DoA Estimation Algorithms is conducted in this review paper. A particular focus on their underlying principles, performance characteristics, and application scenarios is aimed. In addition to that, A comprehensive analysis of different algorithms used is presented, to identify the most suitable and efficient approaches for various applications. Applications of DoA in different fields are also explored, along with its challenges and future directions. This review paper aims to highlight the noteworthy role that DoA algorithms make to the continuous advancement of contemporary radar systems, wireless communication, and related technologies.

2. Evolution of DoA estimation

Driven by the need for better and more effective means of communication and radar systems, there have been significant amount of reforms in the DoA estimation field. This concept began to gain extensive traction during World War I, when the necessity to determine the location of enemy artilleries using acoustic methods was necessary. These initial efforts laid the groundwork for more sophisticated technologies. Initial footsteps in the advancement of the DoA were with the development of RDF (Radio Detection Finding) systems. Systems were developed having antennas in order to capture radio signals and estimate their source's direction. The beginning of World War II accelerated progress in DoA techniques, particularly with the introduction of radar technology. Researchers developed new algorithms and array processing methods, allowing more accurate estimation of incoming signal directions. Tho et al. presented a novel technique for determining the DoA in use-cases where more sources than sensors are available, which is known as under-determined scenarios. They determine the dominating source in a time-frequency bin by combining coherence tests, onset detection, and noise floor monitoring. Then, the significant eigenvectors of the covariance matrix for these bins are grouped into clusters, and the DoA is determined based on the centroids of these clusters [9]. Dey et al. developed a smart headphone application that selectively allows speech sounds from the environment to pass through. Their approach consists of two main parts i.e., a source localization procedure and an effective far-field speech identification algorithm customized for noisy situations. This technique enables users to listen to music through headphones while simultaneously hearing speech from a specific direction. The structure of the array is necessary in DoA estimation [10]. Shi et al. demonstrated that the degree of freedom improves when a coprime array with a unique co-array structure is used. They suggested an approach for predicting sparse reconstruction based DoA. They altered the sliding window technique to get rid of misleading peaks in the reconstructed sparse spatial spectrum in order to enhance power estimation. Their study yielded prominent results in both DoA and power estimation, achieving significant degree of freedom [11]. A coprime array using compressive sensing was proposed by Zhou et al. They reduced the dimensionality of the incoming signals by compressing them using a random compressive sensing kernel. Subsequently, high-resolution DoA estimate was then carried out on these compressed measurements. The work confirmed the computational effectiveness of this approach [12]. However, a few DoA estimation techniques ignore the spatial significance in the partitioned co-array statistics. A recent research proposed a technique for 2D DoA estimation based on coupled co-array tensor canonical polyadic decomposition (CPD) to address this issue. The approach employed shifting co-array concatenation to decompose the partitioned fourth-order co-array statistics into multiple coupled co-array tensors for the coprime L-shaped array, resulting in an increase in number of degrees of freedom [13]. Coprime sensor arrays were employed for far-field DoA estimation of uncorrelated radar signals to enhance the degree of freedom [14]. The study utilized the Cuckoo search algorithm, which effectively increased the Degree of Freedom even at low SNR values. A DoA technique was created by Hioka et al. particularly for speech recognition and angular resolution in array structures and human-machine interactions. Their algorithm outperformed traditional methods in terms of efficiency [15]. To reduce spatial noise covariance and improve the degree of freedom, Basikolo et al. used a non-uniform circular array for DoA estimation using the Khatri Rao (KR) subspace approach. By combining the non-uniform circular array with the KR subspace approach, they achieved an increase in the degree of freedom, facilitating overdetermined and underdetermined DoA estimation [16]. Xu et al. investigated the use of a rectangular array for DoA estimation. They applied the real-valued propagator method for estimating 2D DoA, resulting in improved angle estimation performance with their algorithm [17]. Zhai et al. tackled the ambiguity issue by employing an unfolded coprime linear array. Signals from two sub-array were merged to form the full signal subspace. Additionally, they developed a reduced-dimensional MUSIC (RD-MUSIC) algorithm tailored for noncircular signals from these sub-arrays, improving the precision of noncircular signal estimation. Feng et al. introduced a DoA estimation method for wideband signals that employs fast chirplet-based adaptive signal decomposition to create a time-frequency covariance matrix. They subsequently used subspace fitting techniques like those in traditional MUSIC and ESPRIT algorithms. The time-frequency analysis provides a comprehensive view of both time and frequency domains [18].

One approach employed spatial time-frequency (TF) distributions in wideband scenarios, utilizing the spatial pseudo-Wigner Ville distribution to analyse signals in both the domains. In time-frequency analysis, signal representation in both domains is integrated into a time-frequency energy density function according to the study by Gupta et al. [19]. This method demonstrated superior performance compared to other techniques for frequency-modulated (FM) signals and showed significant improvements for wideband signals [20]. Bouri developed a methodology that employs sample cross-spectral matrix factorization for detecting and localizing sources. This method avoids eigenvalue decomposition, thereby reducing computational costs and enhancing performance [21]. Mohan et al. proposed a methodology for localizing multiple speech sources by using small arrays by applying a coherence test. The two approaches suggested were narrowband spatial spectrum estimation at each bin, followed by summing directional spectra over frequency and time and clustering low-rank covariance matrices and averaging them within clusters [22]. Nishiura et al. developed two alternative methods for DoA estimation beyond classical approaches. The first method uses cross power spectrum phase, and the second utilizes a statistical sound source identification algorithm based on a Gaussian mixture model. These approaches enhance multiple sound signals to localize the source, with a microphone array steered using the delay-and-sum beamformer for source localization [23]. Sawada et al. proposed a DoA estimation approach using independent component analysis (ICA). It was found that ICA effectively identifies source signals from their mixture and highlighted its key advantage over the MUSIC algorithm, it works even when the number of sources equals the number of sensors [24]. Matsuo et al. applied a histogram mapping approach for estimating the DoA of multiple speech signals, highlighting its low computational complexity and the absence of requirements for initial DoA estimates. They also introduced a method to eliminate narrowband components from vector analysis [25]. Swartling et al. advanced the steered response power with phase transform (SRP-PHAT) for DoA estimation, employing second-order statistics from cross-power spectra to direct a beamformer towards the maximum power output. They also discovered that while fourth-order statistics were more effective at differentiating speech from noise in comparison with second-order statistics, they required twice the computational resources [26]. Wang and Zhang developed an iterating positional algorithm to address link blockage in mm-Wave communication systems. Their method used randomised beamforming and maximum likelihood estimation to determine the angle of arrival and departure, achieving centimetre-level positioning accuracy [27, 28, 29, 30]. Kase et al. formulated a DNN-based method for estimation of DoA of two targets, using a correlation matrix Rxx as input. They analysed their DNN with uncorrelated narrowband signals of equal power and found that performance heavily depended on the quality of training data, which was generated by varying the SNR between 0 and 30 dB. Their results highlighted the high performance of DNN-based methods but also their susceptibility to overfitting [31]. Liu et al. proposed a DoA estimation method for underwater acoustic signals using a CNN architecture. They used the covariance matrix Rxx as input but separated it into two channels that are real and imaginary parts respectively, in order to avoid dealing with complex numbers. The proposed CNN-based DoA estimation method is comparatively better than the traditional MUSIC algorithm and is better suited for underwater acoustic environment [28]. Asano et al. developed a two-stage technique for handling multiple types of noise. The initial stage reduced ambient noise by removing noise-dominant subspaces, while the subsequent stage extracted the target source spectrum from multi-directional components [29]. Visser et al. proposed a technique for enhancing speech in noisy environments. Independent component analysis was employed for the adaptive denoising of separated signals [30]. Mitianoudis et al. also utilized blind source separation for audio source separation [31]. Dai et al. proposed a deep learning approach for estimating the DoA of multiple narrowband signals in a coherent environment using a uniform linear array. LogECNet, a logarithmic eigenvalue-based classification network, was introduced in order to improve signal number detection in challenging scenarios like low SNR and limited snapshots [32]. More recent scenarios include application of compressed sensing techniques with deep learning [33, 34]. Several preprocessing techniques used prior to DoA estimation include speech enhancement using the blind source separation, subspace method, sub-band clustering, and the ADTFD method. Khan et al. found that the ADTFD method excelled at analysing closely spaced signal components compared to other preprocessing techniques. ADTFD optimized kernel direction at each point in the time-frequency domain to achieve a clear representation used for DoA estimation [35]. Postprocessing methods, such as post-filtering algorithms, are used after DoA estimation. Habets et al. introduced a post-filtering algorithm that enhances speech signal spectra and reduces interference [36]. Gu et al. proposed using QR Decomposition—Recursive Least Squares (QRD-RLS) algorithm for postprocessing. QRD-RLS estimates DoA from autoregressive sources, which are modelled using the Kalman filter, providing excellent temporal information for accurate DoA estimation [37]. Table 1 summarizes the significant contributions to the field of signal processing, specifically in the context of Direction of Arrival (DoA) estimation, highlighting key advancements and methodologies:

Table 1. Significant contributions in DoA

Time/Research	Event/Technique	Key Contribution
WWI	Acoustic methods for locating enemy artillery	Early groundwork for DoA estimation.
WWII	Radar technology and RDF (Radio Direction Finding) systems	Enhanced accuracy in signal direction estimation.
Mid 1900s	Advancements in digital signal processing	Enabled development of modern DoA algorithms.
Shi et al.	Coprime arrays with sparse reconstruction	Increased degrees of freedom and improved power estimation.
Zhou et al.	Compressive sensing with coprime arrays	High-resolution DoA estimation with reduced computation.
Xu et al.	Real-valued propagator method with rectangular arrays	Improved 2D angle estimation performance.
Zhai et al.	Unfolded coprime linear array with reduced-dimensional MUSIC algorithm	Enhanced noncircular signal estimation precision.
Feng et al.	Time-frequency covariance matrix using chirplet-based decomposition	Effective wideband signal DoA estimation.
Kase et al.	DNN for estimation of DoA	High performance for narrowband signals, but sensitive to training data.
Liu et al.	CNN for underwater DoA estimation	Superior performance in underwater environments compared to traditional MUSIC.
Swartling et al.	Phase Transform for Steered Response Power (SRP-PHAT)	Improved noise differentiation using second- and fourth-order statistics.
Khan et al.	Adaptive Directional Time-Frequency Distributions (ADTFD)	Optimized kernel direction for clearer time-frequency representation, enhancing DoA estimation.
Habets et al.	Post-filtering algorithm	Enhanced speech signal spectrum and reduced interference.
Gu et al.	QRD-RLS with Kalman filter	Provided excellent temporal information for autoregressive source DoA estimation.

3. Incorporation of DoA in RADAR and smart antenna systems

The incorporation of DoA estimation techniques in smart antenna and radar systems has become an essential aspect of modern signal processing, image processing and communication technologies [38, 39]. Antennas dynamically adjust their radiation pattern for optimizing signal reception and transmission by enabling adaptive beamforming using estimation. In radar applications, interference from antennas can be effectively suppressed through array pattern shaping achieved via array processing. This process involves beamforming, where spatial data samples from an array with controllable elements are coherently integrated, and is utilized to manage interference [40]. Such capabilities significantly improve performance and reliability of wireless communication systems across various applications, including mobile telephony and data networks. In radar systems, the application of DoA estimation is important for detecting and tracking moving objects. It aids in determining the precise angle from which radar signals are received, leading to more accurate information regarding object speed, direction, and position. Furthermore, advanced algorithms, such as the MUSIC algorithm, are commonly employed in these systems to enhance estimation accuracy and resolution.

3.1 *Application of DoA in radar system*

RADAR is used to determine an object's location, direction and velocity. Radar is a remote-sensing system frequently utilized for military and civilian surveillance, tracking, and imaging purposes [41, 42]. The function of radar is closely related to physical objects, Radar is an EM wave-assisted target detection system that utilises RF waves to determine target range, azimuth, and velocity, as demonstrated by Kumawat et al. [42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52] in their work. Its primary task is the positional estimation of a moving target and predict its trajectory, as studied by Garg et al. [53]. By detecting reflected waves, radar systems determine the presence of a target, thus performing the crucial task of detection [54]. A RF sensor uses EM waves to detect objects, referred to as targets. Continuous-wave (CW), pulsed-wave, and frequency-modulated continuous-wave (FMCW) sensors are the three primary types of radio frequency (RF) sensors. CW and FMCW sensors utilise less power compared to pulsed radar sensors due to continuous modulation or unmodulated transmission, allowing them to be designed with comparatively simpler circuits [55]. The two most important functions of radar are target detection and target tracking. Hypothesis testing is conducted in radars performing target detection wherein, H_0 denotes target absence, and H_1 denotes target presence. According to Vaishnavi et al., two of the most commonly used algorithms for target detection are Detect Before Track (DBT) and Track Before Detect (TBD) [56].

The basic principle of radar, as illustrated in Figure 1, involves transmitting electromagnetic waves and detecting their reflections from objects. The radar system emits high-frequency pulses, which travel through the medium and reflect off targets. These reflected signals, or echoes, are received by the radar, and their analysis provides crucial information such as range, velocity, and direction of the target [57, 58, 59]. The range (R), is estimated based upon the time (Δt), taken by EM waves to propagate to and fro from the target at the speed of light (c). As, Distance is the product of speed and time; the distance the EM waves travel to the target and back is *2R* [57, 60].

$$2R = C\Delta t$$

$$R = \frac{C\Delta t}{2}$$

where, R = range from the radar to the object in meters, c = speed of light, 3×10^8 m/s, Δt = elapsed time, seconds [60]. The received target signals consist of interference along with the information.

3.1.1 *Radar classification*

Radar are various types of radars, and each is classified according to specific characteristics. These characteristics include the purpose of the radar, the kind of antenna it uses, the frequency range it works in, the precise measurements it needs for performing, the waveforms it utilizes, it's operational environment, and the interferences it is expected to encounter. Primary radars transmit a signal to illuminate the target, acting as a passive reflector, and then receive the returned echo to extract information. This method faces challenges in long-range detection and accurately determining an aircraft's altitude. Secondary radars address these issues by triggering an active response from cooperative targets, allowing for the extraction of both altitude and identification information. They require less transmitter power due to one-way transmission. However, the high installation cost of electronic equipment is a drawback. Primary Radar is classified further as Continuous wave (CW) radar and Pulsed wave radar. De et al. states that in CW radar there is continuous transmission of EM waves, whereas, in Pulse wave radar, there is a sequence of finite pulsed waveforms [61]. CW Radars continuously transmit and receive EM energy using separate antennas for transmission and reception. High-Energy Waveform makes CW radars suitable for very long-range applications. It is not used to determine the target's direction, because of constant transmission and reception in CW radars i.e., it is not possible to determine if the target is approaching or receding. As CW radar cannot find the direction of the target in an open environment, the direction of target is found by using various parameters such as received power. However, various factors such as environmental attenuation, clutter, RCS fluctuation or variation due to range and aspect angle variation affect the received power and hence, its power is not used for direction finding. Doppler frequency shift is another parameter that is used. All CW radars use superheterodyne receivers for target detection and extraction of Doppler frequency, employing in-phase and quadrature-phase signals as noted by Kumawat et al. [62]. Unmodulated CW Radars accurately measure the radial velocity of a target and angular position. By modulating the waveform, CW radars also extract range information, allowing them to measure the distance to the target. One of the major applications of CW radar include classification of different flight modes of an ornithopter, as proposed by Akella et al. [63, 64].

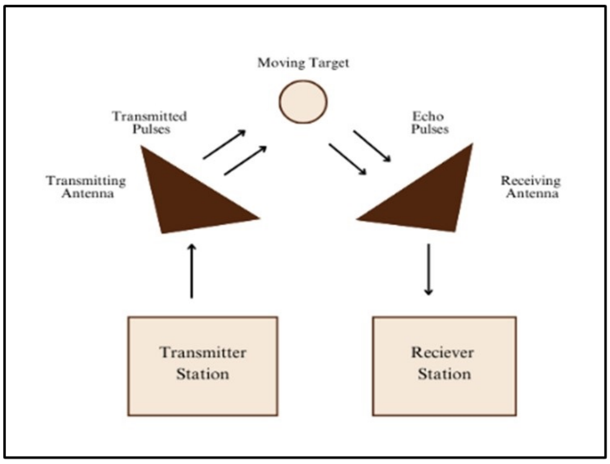

Figure 1. Basic principle of radar

Pulse-Doppler radar systems operate by transmitting short pulses at specific intervals known as pulse repetition intervals (PRI) and receive backscattered echo signals from targets. The rate at which pulses are transmitted vary, affecting the radar's resolution and range capabilities is called the pulse rate. The utilization of pulse compression technique allows for achieving the energy of a long pulse while maintaining the resolution of a short pulse, thus eliminating high peak power requirement. This results in a short-duration pulse [65, 66, 67]. The measurement of time delay between transmission of the pulse and the reception of its echo, facilitates accurate estimation of the distance to a target, which is called range measurement. This time-of-flight measurement is fundamental to determining the range of the target. Pulsed radars are used in a wide spectrum of applications due to their ability to adjust pulse rates and modulation techniques for specific needs, and hence are known for their versatility [68, 69]. Mono-pulse tracking radar, a type of pulsed radar, requires high accuracy in angular measurement. The target's location, including the azimuth and elevation angles, are determined using mono-pulse processing technique [70].

Based on the position of the antenna with respect to the receiver the radar is classified as monostatic, bistatic and multi-static radar. Monostatic radars use the same antenna for transmission and reception of signals or have separate antennas that are located in close proximity. This setup simplifies the radar system, as transmitted and received paths share the same location. The monostatic configuration is predominantly used in pulse radar systems and benefits due to the straightforward integration of transmit and receive functions, increasing cost-effectiveness and ease of implementation. In bistatic radars, the transmit and receive antennas are positioned at two separate geographic locations. This arrangement reduces spillover interference and can enhance detection capabilities by observing targets from different angles as stated in the work of Kumawat et al. to create a simple and efficient algorithm for data acquisition and signal processing in radar systems [71]. Bistatic radars are particularly useful in CW or frequency-modulated continuous-wave (FMCW) applications. FMCW radar is a frequently used configuration [72, 73]. FMCW radars are predominantly employed in applications involving short ranges, including, automotive radars, airplane altimeters, range detectors and naval industries [74]. The main difference from monostatic radars is the absence of a duplexer, as the separate locations negate the need for one. Synchronization between the transmitter and receiver is crucial to ensure accurate timing and phase reference, maximizing the receiver's ability to interpret the transmitted signal. Bistatic images typically result in direct-scattering mechanisms [75, 76]. Multi-static radars extend the concept of bistatic radars by utilizing more than two geographically separated antennas. These systems offer improved target detection and tracking by observing targets from multiple perspectives, allowing for better resolution and accuracy. Like bistatic radars, multi-static systems require synchronization links to maintain coherence across the different antennas. Multi-static configurations are beneficial for applications requiring extensive coverage and enhanced target discrimination. MIMO Radars are a subset of multi-static radar. It is an emerging technology capturing the attention of practitioners, scientists, and researchers. It offers the flexibility to select diverse waveforms based on specific needs and applications [77]. These radar systems consist of multiple transmit and receive antennas. Each transmit antenna radiates a unique waveform independently, and each receive antenna captures these signals. This setup allows the radar system to reassign the received echo signals to individual transmitters, providing significant advantages in target detection, parameter estimation, and clutter suppression. MIMO radar systems over enhanced spatial resolution and robustness against interference and noise due to the diversity of transmitted waveforms and receiving paths. Multi-static systems face the challenge of choosing appropriate weights for signal fusion from multiple receivers to satisfy pre-specified performance goals. Addressing this, the multi-static ambiguity function is utilized [78]. MIMO radar offers several advantages over conventional radar systems, including enhanced resolution, improved angle estimation accuracy, the capability to detect a large number of targets, an increased array aperture, and greater waveform diversity [79].

Search Radars are designed to continuously scan a large area of space to detect the presence of targets. They provide key information about detected objects, such as range, angular position, and velocity. Search radars predominantly use different scanning patterns to cover the search area effectively. Tracking Radars are specialized for following the movement of a specific target, allowing for precise measurement of its position and velocity over time. These radars predict a target's path and future position, which is critical in applications like military defence, missile guidance, and air traffic control. Infrared Search and Track (IRST) systems are used for defence applications due to their advantage of passive surveillance [80].

Human Activity Recognition (HAR) is a technology that involves identifying and classifying human actions or behaviour from sensor data or video footage. It typically relies on algorithms that analyse patterns and movements to determine specific activities or behaviour. HAR is crucial in fields such as healthcare, military surveillance systems, protest-crowd assessment, rescue missions, and national security. However, vision-based HAR methods face several limitations, including blurred images from IR or night-vision cameras, short-range operation, ambient vision requirements, high computational complexity for micro-motion analysis, dependence on weather conditions [81, 82].

Synthetic Aperture Radar (SAR) is a radar technique using the movement of the radar antenna to generate high-resolution images of a target area. By synthesizing the effects of multiple radar returns over time, SAR achieves much finer spatial resolution than conventional radar systems. SAR is a technology that uses RF waves to create images under various circumstances and is applicable in civilian and defence sectors [83, 84, 85, 86]. Inverse Synthetic Aperture Radar (ISAR) is similar but focuses on imaging moving targets, such as ships or aircraft. ISAR utilizes relative motion between radar apparatus and target to produce detailed images and identify targets based on the characteristics of the backscattered signals [87, 88]. SAR image formation is often considered an ill-posed linear inverse problem, where traditional imaging techniques, such as the matched filter (MF), face limitations due to data bandwidth availability, causing resolution constraints. To overcome this, sparse SAR imaging using Compressed Sensing (CS) has been formulated, offering improvements like super-resolution and feature enhancement. Machine learning (ML) and deep learning (DL) algorithms are explored for sparse SAR imaging, demonstrating significant potential in enhancing imaging performance [89, 90]. Kumawat et al. stated that apart from this, an effective data acquisition system and signal processing model is also important for radar effectiveness [91].

3.1.2 *Radar architecture and working*

As shown in Figure 2, a functional pulse radar system comprises fundamental components, including a transmitter, duplexer, antenna, receiver, and gauge, as explained in Garg et al.'s work [92, 93] on radar.

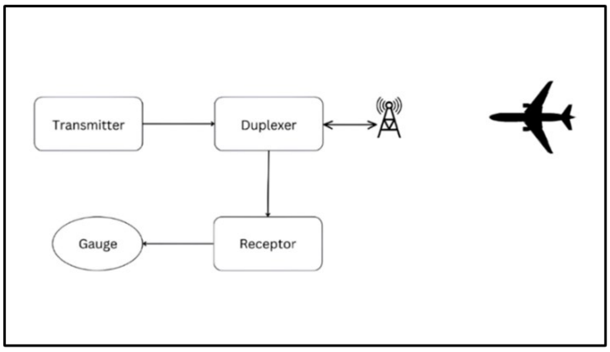

Figure 2. Radar architecture

The transmitter is responsible for generating the RF power signal that illuminates the target. The RF signal waveform ranges from an unmodulated CW to a complex frequency according to system requirements. Primarily, there are two methods for generating RF power: the power oscillator approach, where the signal is produced with the necessary power level for direct application to the reflective antenna, and the master oscillator approach, where a lower-power RF signal is generated by the oscillator and then amplified to the required level [58, 59]. Radar systems typically use highly directional antennas. Primary function of the radar antenna is to take EM waves generated by the transmitter and introduce them to a propagation medium. It serves as a transducer that efficiently transfers electromagnetic energy between the radar's transmission line and surrounding propagation medium. It also provides beam directivity as well as gain for transmission and reception of EM energy [58, 60]. The radar receiver is used to receive weak target signals, amplify them, and translate the RF to baseband information incorporated in them. Different types of radar configurations such as crystal detectors, RF amplifiers, homodyne and superheterodyne are available; but the most widely used configuration is the superheterodyne receiver [58]. The superheterodyne receiver comprises a low-noise RF amplifier, a mixer, an intermediate frequency (IF) amplifier, a video amplifier, and a display unit. The components in the receiver amplify the received signal, convert the RF signal to an intermediate frequency (IF), and subsequently apply the signal to an ADC and then to the data processor. The detector removes the carrier from the modulated target return signal so that target data can be sorted and analysed by the signal processor [57, 59]. The duplexer enables a single antenna in monostatic radars to handle both signal transmission and reception. It comprises two devices: one is the transmit-receive (TR) device, and the other is the anti-transmit-receive (ATR) device. This device allows the transmitter and receiver to be attached to the antenna simultaneously while isolating the receiver from high-powered transmitted signal to protect its frequency sensitive components. The transmitter is connected to the antenna via a transmit/receive (T/R) device, like a circulator or switch. During reception, an ATR device directs the echo signal towards the receiver.

3.1.3 *Radar equation and principle*

The radar range equation provides a mathematical framework to determine the farthest range at which a target is detected by a radar system. This radar equation can be derived from fundamental principles of radar. The effects of the target, target background, propagation route, and medium are all taken into consideration by the radar equation in addition to those of each main radar system component. This equation is essential for designing and optimizing radar systems to ensure they meet specific detection requirements. The radar range equation is crucial for system design, as it is used to design radar systems with optimal performance characteristics for specific applications, such as air traffic control, weather monitoring, and military defence. Performance optimization refers to adjusting components like antenna gain, transmitted power, and signal processing techniques to maximize detection capabilities. Understanding limitations refers to identifying limitations due to environmental factors or system constraints and finding ways to mitigate them [94]. The Basic Radar Range Equation is given as:

$$R = \left(\frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 P_r L} \right)$$

Where; transmitted power (P_t, in watts), which is the power emitted by the radar system and can increase the detection range (R in meters); antenna gain (G_t, dimensionless), which enhances the effective radiated power by directing or receiving energy in a specific direction; wavelength (λ, in meters) which determines the radar's resolution and penetration ability and is inversely related to frequency (f, in hertz) by the speed of light (c, in meters per second), given as λ = c/f; radar cross-section (σ, in square meters), which measures how detectable an object is by radar, with larger cross-sections reflecting more energy and making objects easier to detect, minimum detectable power (P_r, in watts), the smallest signal power that the radar system can reliably detect, influenced by factors like noise and receiver sensitivity and (L, in dB) represent loss [51].

The strength of the target echo determines the maximum detection range of a radar system. When a signal is transmitted, the echo that returns is weaker than the original signal because it loses strength and gets attenuated over time and distance. Since the strength of the pulse is dependent on peak power and pulse duration, it is essential to evaluate the signal energy, which is a product of the two factors. A low peak power pulse with a longer duration delivers the same energy as a high peak power pulse with a shorter duration. However, using a shorter pulse width improves the resolution range [52].

3.1.4 *Application of radar*

According to the study by Kumawat et al., the modern radar systems have gained popularity due to progressive features such as compact size, ease of use, payload-carrying capability, system integrated surveillance cameras, and Tx–Rx communication/control systems. These radars are now widely employed in fields like surveillance, border patrol, aerial photography and video recording and goods transportation. They are also being used for air-traffic guidance and monitoring, and their applications continue to expand day by day [95]. Radar applications have adapted from being strictly military to today's wide spectrum utility [96]. Radar serves a variety of important functions across numerous sectors. In aviation, both civilian and military, air traffic control radars ensure safe spacing between aircraft and assist with take-offs and landings, while high-resolution radars track aerial systems and monitor vehicle traffic at airports. For navigation, aircraft radars help with weather and terrain avoidance and ground mapping. Military applications, the largest use of radar, include navigation, surveillance, and weapons guidance [97]. In the medical field, radars are being researched for diagnostics like breast tumour detection and remote monitoring of vital signs [98]. Satellite radars are crucial for tasks such as docking and tracking, while remote sensing radars provide valuable data on geophysical objects and support astronomical exploration of celestial bodies. Marine radars enhance safety by preventing collisions and aiding navigation, and law enforcement uses radars for vehicle speed measurement and intruder detection. Radar plays a key role in atmospheric monitoring through the Global Ozone Monitoring Experiment, which tracks ozone and other climate-related variables. Wind scatterometers also use radar to measure ocean surface wind speed with precision. Emerging methods like orthogonal frequency division multiplexing (OFDM) are the latest advancements in radar applications according to Yadav et al. [99].

Additionally, as stated by Kumawat et al., radar-based target detection and classification offer several advantages over other surveillance methods like vision-based sensors, thermal cameras, acoustic sensors, and infrared sensors. These include no lighting requirements, robust weather noise immunity, day-and-night remote sensing capabilities, atmospheric propagation, and the ability to produce inconspicuous images with inherently false colour [100].

3.2 *Application of DoA in smart antenna system*

DoA estimation algorithms are generally incorporated in smart antenna technology for developing systems that provide precise information about geographical location for wireless functionalities. A smart antenna system is defined as a configuration that dynamically optimizes its radiation and reception patterns by combining numerous antenna components with advanced signal processing capabilities based on the prevailing signal environment. This technology serves a dual purpose i.e., it enhances the signal quality of radio-based systems by allowing focused transmission of radio signals, while simultaneously increasing system capacity through improved frequency reuse [101]. Smart antennas are primarily categorized into two systems: the Switched Beam system and the Adaptive Array system, the structures of which are illustrated in Figures 3 and 4.

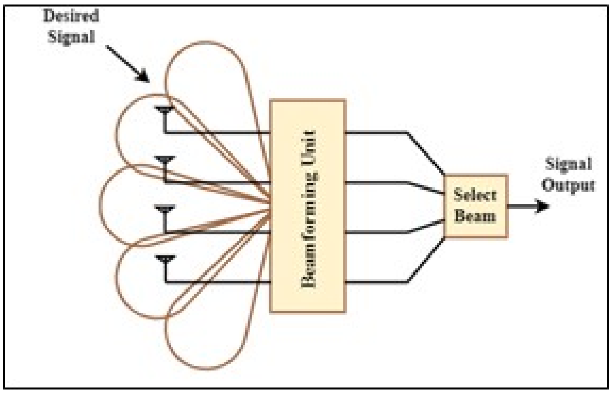

Figure 3. Switched beam antenna system

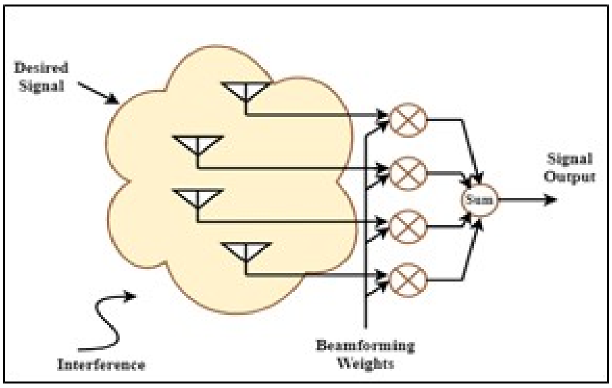

Figure 4. Adaptive array

The Switched beam utilizes a limited number of predetermined patterns or sectors and is simpler as compared to the adaptive array approach. In this system, when an incoming signal is detected, the base station determines the beam that is best aligned with the direction of the signal of interest and then switches to that beam to communicate with the user [102]. The Adaptive antenna system can create infinite patterns with real-time adjustment with respect to surrounding conditions and is more advanced than the switched beam antenna system. This technology forms nulls in the directions of interfering signals and steers the main beam towards the user to constantly track mobile users. The individually received signal is multiplied by weights which are complicated in nature and modulate phase and amplitude. The array output is generated by combining these signals. The complex weights are computed by an advanced adaptive algorithm, which is pre-programmed into the digital signal-processing unit managing the signal radiated by the base station [103].

3.2.1 *Architecture of smart antenna system*

The basic architecture of antenna consists of five main blocks namely Antenna arrays, RF module, Beam former unit, DoA estimator and Adaptive processor, as illustrated in Figure 5.

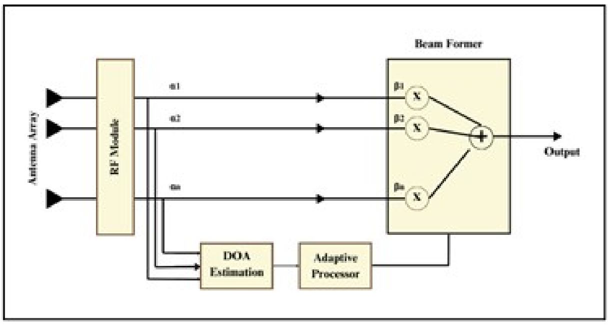

Figure 5. Architecture of smart antenna

The Antenna array consists of antenna elements (dipoles, patches, etc) which are arranged in specific configurations. The spacing between these elements is typically half of the signal wavelength. This is done to avoid spatial aliasing. The RF Module, or Radio Unit, primarily comprises three components: antenna arrays that receive RF signals from the environment, down-conversion chains that remove the carrier frequencies from these RF signals, and ADC that transform the resulting baseband signals into digital signals for further processing. The array should have relatively minimum number of elements to avoid unnecessary high complexity computation in the signal processing unit. These antenna arrays are arranged in one, two, or even three dimensions, depending on the spatial dimensions that need to be covered. The radiation pattern is influenced by the element type, excitation (amplitude and phase), and the relative positions of each element. The RF Module performs several key functions: converting digital signals from the baseband processing unit into analog signals for transmission and vice versa for reception, amplifying transmitted signals to ensure they reach the desired range while enhancing received signals to improve sensitivity and selectivity, performing frequency translation by mixing baseband signals with a carrier frequency for transmission and converting them back to baseband upon reception, filtering out unwanted noise and interference to meet specific bandwidth and spectral requirements, and managing the interfaces with antennas and other network components to ensure seamless connectivity and synchronization. The Beamforming Unit is responsible for combining signals from the individual antenna elements to form a desired beam pattern. It is responsible for forming the beam pattern and steering it in the desired direction. Let α be the data signals (i.e., the elements of the array) and let β be the set of weights. Thus, a set of beams directed at desired directions using pointing angles (leading to a signal peak at the output) can be formed by multiplying data signals with an appropriate set of weights can be denoted as:

$$o(\theta_i) = \sum_{0}^{n-1} \beta_i^k \alpha_i$$

The RF Module primarily functions as follows: it controls both the phase as well as amplitude of the signal emitted by each antenna element to form a directional beam, which can be dynamically adjusted to target specific users or areas. By focusing signals on desired directions, beamforming minimizes interference with other users and devices, thereby improving overall network performance. Additionally, beamforming increases the signal strength and quality at the receiver end, leading to better data rates and reduced error rates. The DoA estimator in smart antennas is essential for pinpointing the exact angle at which incoming signals arrive. Using sophisticated algorithms like MUSIC or ESPRIT, the estimator evaluates phase differences and time delays among signals received by the antenna array. The adaptive processing unit helps to determine the complex weights of the beam-former unit. The weights are optimized by maximization of data signals from the desired source and maximization of signal-to-interference ratio (SIR) suppressing the signal coming from the sources of interference.

The Space-time processor (smart antenna) is used to enhance the performance of mobile communication by increasing gain antenna, decreasing inter-symbol interference and decreasing co-channel interference. When compared to other antenna systems used for the estimation of DoA, smart antennas based on DoA of incident signals offer several advantages. These include superior Signal to Interference and Noise Ratio (SINR) performance, improved signal quality and range, increased capacity and efficiency, dynamic adaptability, enhanced security, and better utilization of the spectrum. Smart antenna systems can significantly increase capacity by allowing a greater number of users to share the same frequency, thus optimizing resource utilization. Improved signal range is achieved through focused gain directed towards communicating devices, which enhances coverage and reduces the number of necessary base stations. Smart antennas are also used to improve security by directing signals toward specific devices instead of radiating in all directions, increasing the complexity of interception for unauthorized users. Reduced interference is another key advantage, as the directionality of smart antennas minimizes the likelihood of signals interfering with one another. Furthermore, the ability to reuse frequencies effectively enhances bandwidth, enabling higher data rates and better overall network performance. Despite their advantages, smart antenna systems come with several limitations. One of the primary challenges is their complexity. Smart antennas require sophisticated technology and management which can complicate installations and maintenance. Smart antennas are generally more expensive than traditional antennas due to the advanced processing technology and multiple components they involve. Their physical size can be larger compared to conventional antennas, which lead to design concerns or space limitations in certain environments. Smart antennas are more susceptible to interference from other devices and environmental factors due to their reliance on precise signal processing. This leads to issues in multipath environments where multiple signals overlap [104]. The limitations of antennas include reliance on pre-defined beamforming algorithms, which lack the real-time adaptability of smart antennas. Additionally, while these systems focus on exploiting multipath propagation to increase capacity and reliability, their DoA estimation capabilities are typically secondary and not as refined or adaptable. Switched beam antennas, in particular, only switch between a set of predefined beams based on signal strength, lacking the continuous real-time adaptability that smart antennas offer. Other antenna systems that are used for DoA include Phased Array Antennas, MIMO systems, and Switched Beam Antennas.

Phased Arrays (PA) have garnered significant interest as a prominent multi-antenna technique, offering key advantages such as high directionality, effective spatial multiplexing, and strong anti-jamming capabilities. Despite these benefits, PA systems face challenges related to high hardware complexity and elevated power consumption. In contrast, Time-Modulated Arrays (TMA) have emerged as a low-complexity alternative with remarkable beamforming capabilities, making them an increasingly attractive option in recent years. The TMPA is divided into S subarrays, effectively multiplying the modulation frequency and the maximum bandwidth of the transmitted signal by S, which simplifies given filtering process. This method has three key advantages: (1) it significantly reduces the requirements for the band-pass filter (BPF) by expanding the frequency interval; (2) it enables the transmission of larger bandwidth signals using the same hardware; and (3) it improves the overall efficiency of the antenna [105]. However, smart antennas are generally considered superior to these alternatives due to their advanced adaptability and performance in dynamic environments.

4. DoA estimation methods

DoA estimation is a vital component of ASP utilized across diverse fields such as telecommunications, radar systems, and audio processing [106]. Diverse algorithms have been developed to accurately estimate DoA, with significant emphasis on subspace-based methods, which are categorized into two main types based on the utilized subspace. Orthogonal-Subspace Algorithms, which include the Pisarenko method, MUSIC (along with Root-MUSIC), and Min-Norm, that focus on separating the signal subspace from the noise subspace. In contrast, Signal-Subspace Algorithms, such as State-Space Realization, ESPRIT, and Matrix-Pencil, concentrate on the signal subspace itself. Furthermore, DoA estimation techniques can also be classified according to the numerical procedures they exploit. The Extrema-Searching Approach identifies peaks in the pseudo-spectrum and includes algorithms like MUSIC and Min-Norm. The Polynomial-Rooting Approach focuses on finding roots of polynomials derived from the data, encompassing methods like the Pisarenko method, Min-Norm, and Root-MUSIC. Lastly, the Matrix-Shifting Approach leverages shifts in matrices representing the signal model and features techniques such as State-Space Realization, ESPRIT, and Matrix-Pencil. Each of these techniques contributes unique strengths to the task of DoA estimation, offering solutions tailored to various applications and challenges encountered in practical scenarios [107]. Another widely popular method is Classical method of DoA estimation. These have long been fundamental in array signal processing, utilizing straightforward principles that emphasize computational efficiency. These methods primarily include techniques such as the Conventional Beamformer and Capon's Minimum Variance Distortionless Response (MVDR). The Conventional Beamformer operates by steering the array to a particular direction and measuring output power to determine the DoA based on the highest output. DoA estimation problems consist of two categories: spatial spectral-based problems and parametric problems.

4.1 *Sub-space based methods*

Subspace-based methods are advanced techniques used in signal processing for various applications, including parameter estimation, noise reduction, and data classification. Subspace-based methods, also known as high-resolution techniques, are forms of parametric spectral estimation that focus on estimating signal characteristics from noisy data [108]. They are particularly effective in scenarios where the signal is embedded within noise, allowing for better extraction of the signal's essential features. Subspace methods typically begin with eigenvalue decomposition of a sample covariance matrix, which helps in characterizing the signal environment [109]. By projecting signals into a signal subspace, they maintain crucial components while filtering out unwanted noise, which enables clearer and more reliable signal interpretation. These methods are utilized in tasks such as dimensionality reduction and noise reduction, attracting significant. interest in fields like speech enhancement, modelling, and classification research. By identifying and leveraging the signal subspace, these techniques can effectively reduce the impact of noise on signal integrity and clarity.

One notable advantage of subspace-based methods is their ability to yield sharper signal representations compared to traditional filtering techniques, which may incorporate gradual transitions between signal and noise signal components. This characteristic allows a more precise identification of signal features and improved performance in various applications.

4.1.1 *MUSIC*

MUSIC is one of the earliest DoA estimation methods that have been followed by several variants. It is a high-resolution sub-space technique used for estimating DoA. It works by using the method, which separates signal and noise subspaces. The DoA is then obtained by analysing the MUSIC pseudo-spectra over a grid in order to identify the respective peaks associated with angles [110]. This technique is based upon utilizing the eigen-structure of the input covariance matrix. In 1977 Schmidt proposed the use of geometric tools specifically, algebraic surfaces to look at signal parameter estimation from a different side. One step forward was made with the MUSIC algorithm being capable of dealing with arbitrary sensor arrays. Before the mid-70s, direction-finding techniques involved obtaining knowledge about the directional sensitivity pattern of an array in analytical form, which meant designing arrays having predetermined patterns of sensitivities for antenna designers. This constraint was relaxed by Schmidt who simplified the problem by stating that instead of an analytic calculation for complex array response, it is necessary to only measure and store this response. Though this did not reduce computational complexity in solving DoA estimation problems; using MUSIC, it expanded high-resolution estimation application to arbitrary sensor arrays [111]. Lincoln Laboratory at M.I.T. conducted an extensive examination that included thousands of simulations and found out that, MUSIC emerged as the most promising technique for further research and hardware applications. Computational costs associated with MUSIC are huge due to parameter space search and storage requirements for array calibration data although it has a huge performance advantage over other algorithms [112]. As this algorithm takes uncorrelated noise into account, the resulting covariance matrix is diagonal. In this process, the signal and noise subspaces are calculated using matrix algebra and are found to be orthogonal to each other. Therefore, this algorithm utilizes the orthogonality property to separate the signal and noise subspaces, as highlighted by Gunjal et al. [113].

The covariance matrix 'R_J' for the received data 'J' is the expectance of the matrix with its hermitian equivalent.

$$R_J = E[JJ^H]$$

The received data 'J' is denoted as:

$$J(t) = [J^1(t)J_2(t)\ldots J_D(t)]^T$$

Substitute the value of 'J':

$$R_J = E[(AS+N)(AS+N)^H]$$

$$R_J = AE\left[SS^H\right]A^H + E\left[NN^H\right]$$

$$R_J = AR_JA^H + R_N$$

The noise correlation matrix R_N can be formulated as:

$$R_N \ = \ \sigma^2 \ * \ I$$

I is the identity matrix for the antenna array with dimensions D*D. As the signals are correlated with the noise, the resulting correlation matrix, including the noise, can be expressed as follows:

$$R_J = \ Q_S \sum Q_S^H + \ Q_N * \sigma * \ Q_N^H$$

Since the MUSIC algorithm leverages the orthogonality between the signal and noise subspaces, the following equation holds true:

$$\beta^H(\theta)Q_N = 0$$

The angle of arrival can be expressed in terms of the incident signal sources and the noise subspaces, as given by the following equation:

$$\theta_{MUSIC} = arg.\min \beta^H(\theta)Q_NQ_N^H\beta(\theta)$$

The above equation can be inverted to obtain peaks in a spectral estimation plot:

$$P_{MUSIC} = \frac{1}{\beta^H(\theta)Q_NQ_N^H\beta(\theta)}$$

A pseudo-spectrum with high peaks is formed as a result of the equation above. Greater power is shown by the higher peaks, which also match the projected arrival angle. if the antenna array's element spacing is kept at half the wavelength of the received signal [113]. The SNR is set to less than 0 dB, and the number of snapshots is set at 300 dB. The signals are narrow-band and non-coherent. Ten antenna elements (D) are used in the system, and additive white Gaussian noise is assumed for noise. The Algorithm 1 given below illustrates the algorithm for MUSIC.

Algorithm 1 MUSIC Algorithm:

Start
Input: Sensor data matrix X, number of signals k, angle search grid θ (with resolution), steering vector $a(\theta)$
Compute covariance matrix $R = X * X^H$
Perform eigenvalue decomposition on R
Sort eigenvalues in descending order and partition:
    Signal subspace $(S) \leftarrow$ eigenvectors corresponding to k largest eigenvalues
    Noise subspace $(N) \leftarrow$ eigenvectors corresponding to smallest eigenvalues
Define the MUSIC pseudo-spectrum:
for each angle θ in search grid **do**
    $P(\theta) = \frac{1}{a(\theta)^H * N * N^H * a(\theta)}$
    where $a(\theta)$ is the steering vector based on array geometry
end for
Detect peaks in $P(\theta)$:
    Find top k peaks (or use threshold) corresponding to DoA estimates
Output: Estimated DoA angles
End

The schematic representation or flowchart of the explained MUSIC algorithm is provided in Figure 6. Figure 7 depicts the simulation result of the MUSIC algorithm. Signals at particular angles are confirmed by peaks in the spatial spectrum that match the calculated DOAs. If the input signals are at 10° and 30°, the MUSIC algorithm's output shows prominent peaks at these angles, demonstrating its effectiveness in resolving signals, even in the presence of noise. This verifies the algorithm's capability to separate closely spaced sources and accurately estimate their directions.

The MUSIC algorithm offers several advantages. It provides highly accurate DoA estimations and calculates DoA for multiple sources even in challenging conditions with small spacing between them by exploiting the subspace structures of the covariance matrix. MUSIC does not require calibration information about the array's directional bias and is versatile, capable of handling different geometries and useful for various signal processing tasks. The ideal number of snapshots for MUSIC is 700, at which the mean square error is effectively zero. Passive source localization using sensor arrays is a problem in ASP. One-dimensional MUSIC searches are less demanding but introduce finite-sample bias in environments with multiple sources. In low SNR conditions or with minor sensor array errors, the algorithm struggles to resolve closely spaced sources [114].

Narrow-band source localization has been a key focus for decades, leading to methods like MUSIC and Min-Norm, which offer high resolution with relatively low computational complexity by using the eigen-structure of the spatial correlation matrix. Despite extensive evaluations of eigenspace-based algorithms through simulations and experiments, interest in their statistical performance analysis has grown, particularly for scenarios like two sources with linear arrays. Analysis of the MUSIC algorithm's performance often involves Taylor series expansions of the estimated null spectrum to determine location estimates. Variations in handling second-order terms lead to significant differences in bias and variance expressions. While some studies have attempted to derive bias expressions for MUSIC estimates, many oversimplify by ignoring critical second-order terms or rely on restrictive assumptions [115]. Apart from this, the MUSIC algorithm has limitations in situations where background noise is correlated. It performs well for uncorrelated sources but struggles with coherent ones, and its accuracy is limited by snapshots and other factors. Theoretical models of the MUSIC spectral estimator are derived for uncorrelated noise fields, but practical environments often degrade performance as highlighted in the theoretical model [116].

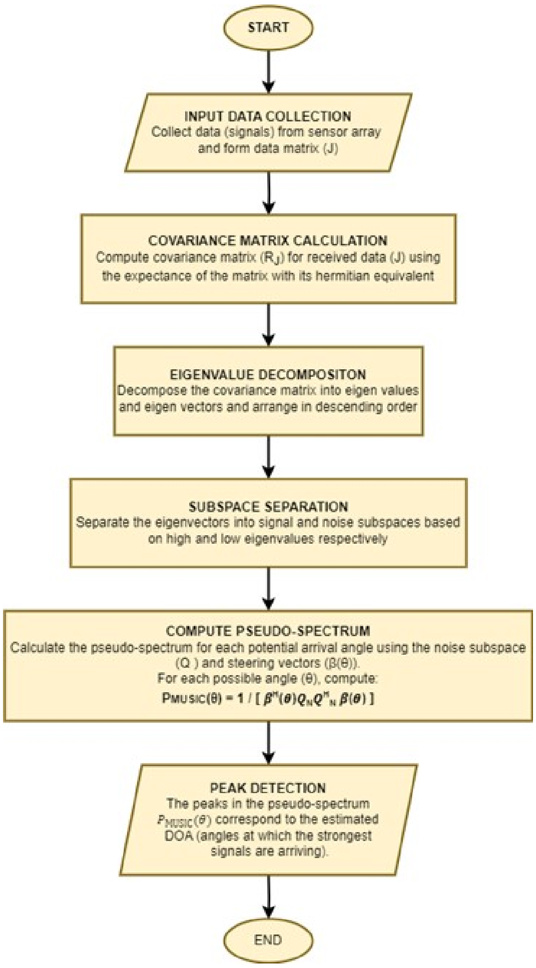

Figure 6. Schematic representation of the MUSIC algorithm

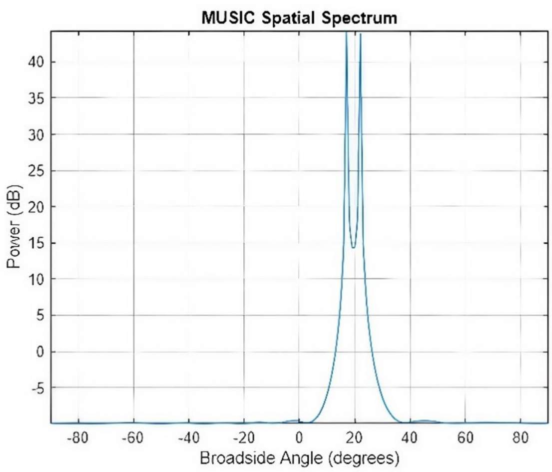

Figure 7. Simulation result of MUSIC

Despite its challenges, such as computational complexity and finite-sample bias, MUSIC is highly regarded for its accuracy and robustness. It leverages the eigenvalues and eigenvectors of the subspaces for estimating DoA and improves resolution as the number of antenna elements increases. MUSIC provides asymptotically unbiased estimates that approach the Cramer-Rao lower bound, demonstrating its high precision in DoA estimation. The MUSIC algorithm can be summarized as outlined in Table 2.

Table 2. Summary of MUSIC technique

ASPECT	DETAIL
Full form	Multiple signal classification
Purpose	High-resolution subspace algorithm for estimating DoA by separating signal and noise subspaces and analyzing MUSIC pseudo-spectrum to identify angle peaks.
Proposed by	Schmidt (1977), introduced algebraic tools for signal parameter estimation and simplified sensor array response analysis.
Advancements	-Relaxed constraints on analytical calculations for complex array responses. -Enabled arbitrary sensor arrays. - High-resolution applications.
Key principle	Utilizes orthogonality property between signal and noise subspaces in the covariance matrix.
Covariance matrix	$R_J = E[JJ^H]$, where J(t) is the received data matrix.
Pseudo-spectrum	Peaks in $P_{MUSIC} = \frac{1}{\beta^H(\theta)Q_N Q_N^H \beta(\theta)}$ represent DoA angles.
Challenges	High computational cost due to eigen-decomposition and exhaustive search over angle space.
Advantages	High-resolution estimation, effective for arbitrary arrays, and superior performance over conventional methods.

4.1.2 *Root-MUSIC*

Eigenspace-based techniques have been widely employed in recent years to estimate the DoA. The observed covariance matrix is usually broken down into two orthogonal components, or signal and noise subspaces, using these techniques. The subspace-based method was introduced by MUSIC, which also proved its benefits. One type of music is called Root-MUSIC. It is very similar to MUSIC, except the only difference is that it is suitable for linear equispaced sensor arrays [117]. Root-Music algorithm is an improvised version of the MUSIC algorithm. Using the MUSIC spectrum, the root of the polynomial is derived in order to estimate the angles of arrival. This algorithm is practically better, considering it gives results in a numerical pattern instead of spectrum plotting in MUSIC, from which we observe the peaks.

Root-MUSIC further simplifies the computational process by solving for the polynomial roots instead of searching for peaks in the spectrum. This method leverages the geometry of uniform linear arrays (ULA) to provide more accurate and efficient DoA estimation. The algorithm focuses on finding the roots lying closest to the unit circle in the complex plane. These roots correspond to the angles of arrival, making Root-MUSIC faster and more suitable for real-time applications. Its numerical approach also reduces the complexity involved in interpreting the results. The Algorithm 2 given below illustrates the algorithm for root-MUSIC.

4.1.3 *ESPRIT*

In practical signal processing problems, estimating the values of an assortment of fixed parameters upon which received signals are dependent is in general one of the main purposes. The problem parameterization estimation of quantities (parameters like frequency, DoA's of plane waves etc.) that the sensor output depends on are assumed to be time-invariant in these problems. The two most commonly used methods are Maximum Likelihood and Maximum Entropy methods. These are however discouraged because of the inappropriate measurement model. Schmidt used the MUSIC (Multiple Signal Classification) algorithm in chained form for arbitrary method sensor arrays. This was followed by studying a new algorithm ESPRIT which reduced the computation to a great extent. This method generally has mild computational requirements compared to algorithms such as MUSIC. Subspace-based ESPRIT algorithm is one of the most known algorithms because it exploits the subspaces for estimating the parameters of signals. It pays special attention to the invariance of rotation among signal subspaces for its goal, that is estimating those parameters. It exploits a computational advantage by employing sensor array displacement invariance. For instance, in DoA estimation, a sensor array is composed of two identical subarrays with a fixed displacement vector. By doing this, ESPRIT can estimate the parameters without actually finding individual sensors or using spectral searches as with MUSIC. Simulations indicate that ESPRIT is quite resilient to array imperfections, such as non-identical subarrays.

Algorithm 2 **Root-MUSIC algorithm**

1: **Start**
2: Input: Sensor data matrix X, number of sources d, Snapshot Factor (SF), Number of Elements Factor (EF), Signal Power Factor (PF)
3: Estimate Covariance Matrix: $R_{xx} = \sum \frac{1}{N} X_i X_i^H$
(Average over snapshots). Normalize R_{xx} if needed.
4: Identify Noise Subspace: Sort eigenvalues in ascending order
Noise subspace = eigenvectors corresponding to smallest eigenvalues
5: Generate MUSIC Spectrum Polynomial: Define steering vector $a(\mu)$ based on spatial frequency μ. Construct MUSIC spectrum polynomial using $a(\mu)$ and noise subspace.
6: Calculate Polynomial Roots: Determine the MUSIC polynomial's roots. Within the unit circle, roots are pairs.
7: Select Best Roots and Determine Angles: Select roots closest to the unit circle. Calculate angles of arrival θ using:

$$\theta = \left(\frac{\lambda_\mu}{2\pi\Delta} \right)$$

Detect peaks in the MUSIC spectrum for accurate angle estimation.
8: Optional Adjustments:
Increase number of snapshots: $\grave{S} = S * SF$
Increase number of array elements: $\grave{M} = M * EF$
Amplify signal: $\grave{x}(t) = x(t) * PF$
9: Output: Estimated angles of arrival (DoA).
10: **End**

With the sources placed at the far end of the array, we assume that the transmission medium is uniform, non-dispersive, and permits radiation to move in straight lines. As a result, a mixture of plane waves makes up the radiation that reaches the array. The location parameter space is reduced to a one-dimensional subset of R3 when we first assume a planar scenario, namely [114].

$$x_i(t) = \sum_{k=1}^{d} S_k(t) a_i(\theta) k + n_{xi}(t)$$

$$y_i(t) = \sum_{k=1}^{d} S_k(t)\, e^{\left(j\omega_0 \Delta \sin\left(\frac{\theta_k}{c}\right)\right)}\, a_i(\theta_k) + n_{yi}(t)$$

The subarray displacement vector A establishes the problem scale and the reference direction because sensor gain and phase patterns are arbitrary and ESPRIT does not need to know these sensitivities. Angles of arrival in relation to vector A's direction are the DoA estimates that are produced. As a result, a displacement vector is needed for each dimension where parameter estimations are requested. The received data vectors can be written as follows by merging the sensor outputs from the two subarrays:

$$x(t) = As(t) + n_x(t)$$

$$y(t) = A\varphi s(t) + n_y(t)$$

where, the vector $s(t)$ is the $d \times 1$ vector of incoming signals (wavefronts) observed at the reference sensor of subarray Zx. The signals can be correlated in the sense that $Es_i(t)s_j^*(t)$ is not equal to 0 for i not equal to j. The matrix ϕ is a diagonal $d \times d$ matrix representing the phase delays between the paired sensors for the d wavefronts and is defined as follows:

$$\varphi = diag\ \left(e^{j\gamma_1},\ \dots, e^{j\gamma_d}\right)$$

where,

$$\gamma(k) = \ \omega_0 \Delta \sin\left(\frac{\theta_k}{c}\right)$$

The measurements from subarray Zx are connected to those from subarray Zγ by the unitary operator matrix φ. Although φ is a scaling operator in the complex field, it is also known as a rotation operator since it is comparable to a real two-dimensional rotation operator. The narrow-band plane wave assumption, which produces unit-modulus sinusoidal signals in the spatial domain, is the cause of ϕ's unitary nature. Since the diagonal components of φ in time series analysis can be any complex number, ϕ can be either an expansive or contractive operator. The total array output vector z(t), which can be expressed as follows, is obtained by combining the outputs from the subarrays:

$$z(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = A' s(t) + n_z(t)$$

$$A' = \begin{bmatrix} A \\ A\varphi \end{bmatrix},\ n_z(t) = \begin{bmatrix} n_x(t) \\ n_y(t) \end{bmatrix}$$

For any non-singular diagonal matrix D, the structure is used to estimate the diagonal elements of ϕ without knowing A. It is also used to estimate the diagonal elements of s(t)—D − 1S(t) and A⁻A⁻D. Therefore, unless the gain pattern of one of the sensors is known, the estimations of the signals and related array manifold vectors are subject to an arbitrary scaling factor [118]. The Algorithm 3 given below illustrates the algorithm for ESPRIT.

Algorithm 3 ESPRIT algorithm

1: **Start**
2: Setup the Sensor Array: Divide the sensor array into two overlapping subarrays.
3: Calculate the Covariance Matrix: Collect incoming signal snapshots from the sensor array. Compute the covariance matrix R from the incoming signal array.
4: Perform Eigenvalue Decomposition: Perform eigenvalue decomposition on the covariance matrix R. Separate the eigenvectors corresponding to the individual signal and noise subspace.
5: Form the Signal Subspace: Select the eigenvectors corresponding to the largest eigenvalues to form the signal subspace (S).
6: Use Rotational Invariance: Take advantage of the rotational invariance property of the overlapping subarrays. Construct the rotation matrix using the subarrays' signals.
7: Solve the Shift-Invariance Equation: Use least squares or other appropriate methods to solve the shift-invariance equation.
8: Estimate Directions of Arrival (DoA): Extract the rotation matrix from the signal subspace. Compute the angles of arrival (DoA) from the rotation matrix using the equation:
9: Return DOA: Output the estimated DoA.
10: **End**

ESPRIT has several notable applications. In radar systems, it helps in finding targets by accurately determining the direction of incoming signals, which is crucial for locating and tracking targets. In sonar systems, ESPRIT is used to detect underwater objects or obstacles by analysing how sound waves reflect back. In wireless communications, ESPRIT improves signal quality by directing antenna beams toward the source of incoming signals, enhancing reliability and signal processing. In seismic exploration, ESPRIT helps locate and understand subsurface structures by analysing how seismic waves arrive at different points. In audio and acoustics, ESPRIT assists in locating sound sources and enhancing speech clarity by isolating it from background noise. In medical imaging, ESPRIT improves ultrasound images, enhancing the clarity and accuracy of diagnostic images [119].

ESPRIT's key strengths are its computational efficiency and accuracy compared to other DOA estimation techniques. Its core principle is exploiting the rotational invariance property of two identical, overlapping subarrays within the sensor array. This property allows ESPRIT to avoid the intensive spatial searching required by some other DOA estimation methods, like MUSIC. After dividing the overlapping subarrays, it extracts DOA estimates from this rotation matrix, providing an efficient method for determining signal directions without extensive spatial searching. ESPRIT provides high-resolution estimates of signal parameters, capable of resolving closely spaced signals with precision. It performs well in noisy environments and handles correlated signals effectively, offering reliable results under challenging conditions. Unlike MUSIC, ESPRIT does not require peak search for parameter estimation, simplifying processing and reducing computational demands. The algorithm can be applied to various array configurations, although typically used with uniform linear arrays. It delivers accurate DoA estimates and reduces the need for extensive calibration data, focusing on subspace decomposition. ESPRIT is adaptable to different numbers of sources and sensor configurations, offering scalability and flexibility in practical applications [120]. Despite its advantages, ESPRIT faces several challenges and limitations. It can only utilize one displacement invariance in the sensor array, leading to potential issues in cases where multiple subarray pairs might meet this condition without a clear rule for selecting the best one. Although ESPRIT can manage arrays with multiple displacement invariances through overlapping subarrays, it may benefit from a more tailored algorithm that fully leverages the array's physical layout. This challenge highlights the need for improved algorithms that can handle multiple invariances more effectively. Additionally, traditional models for electromagnetic vector sensors (EMVS) often encounter issues such as mutual coupling interference and significant computational demands. The ESPRIT algorithm using Geometric Algebra (GA-ESPRIT) models multidimensional signals in a holistic way, offering a promising approach for accurate DoA estimation with lower processing requirements [121, 122]. Thus, ESPRIT represents a sophisticated approach to DOA estimation that balances computational efficiency with high accuracy, making it a valuable tool in fields like radar, sonar, wireless communications, and other areas where determining signal directions is crucial. The ESPRIT algorithm can be summarized as outlined in Table 3.

Table 3. Summary of ESPRIT technique

ASPECT	DETAIL
Full form	Estimation of signal parameters via rotational invariance techniques
Purpose	Efficiently estimate the DoA of signals by utilizing rotational invariance between overlapping subarrays.
Key principle	Exploits rotational invariance of signal subspaces between two identical, overlapping subarrays, eliminating the need for spectral searches.
Covariance matrix	$R = E[ZZ^H]$, where $Z(t) = [X(t);Y(t)]$ combines outputs of two overlapping subarrays.
Rotation matrix (ϕ)	A diagonal matrix $\phi = diag\left(e^{j\gamma_1}, \ldots, e^{j\gamma_d}\right)$, where $\gamma(k) = \omega_0 \Delta \sin\left(\frac{\theta_k}{c}\right)$
Challenges	Assumes uniform, non-dispersive medium and planar wavefronts; sensitive to deviations from these assumptions.
Advantages	Computationally efficient, robust against array imperfections, and achieves high resolution with minimal bias via Total Least Squares (TLS).

4.1.4 *Total least squares ESPRIT*

An M-element sensor array is split into two identical subarrays with m elements each in previous ESPRIT models, which are identified by a fixed displacement vector A. Different measurement models are produced by overlapping subarrays, which allow M ≥ 2 m, and non-overlapping subarrays, which produce M = 2 m. An enhanced version of ESPRIT, TLS-ESPRIT (Total Least Squares Estimation of Signal Parameters via Rotational Invariance Techniques) is intended for estimating parameters of several intricate exponential signals that are impacted by noise. It improves ESPRIT by incorporating the TLS approach, thus enhancing its capability to handle real-world problems with noise. Signal modeling assumes that the received signal consists of multiple complex exponential components along with additive white Gaussian noise. The algorithm then organizes the modeled signal into a data matrix, where each column represents a snapshot of the signal at a specific time. Data preprocessing may include noise estimation and removal. Eigenvalue decomposition is performed to obtain eigenvectors and eigenvalues, focusing on the noise and signal-plus-noise subspaces. The rotational invariance property is used for forming the signal subspace, including eigenvectors corresponding to the signal-plus-noise component. The eigenvectors in the signal subspace are paired to estimate angles of the complex exponential signals using rotational invariance. TLS-ESPRIT minimizes the overall error between actual and estimated data matrices, providing accurate signal parameter estimates even in noisy conditions. The estimated parameters allow for signal reconstruction, making TLS-ESPRIT suitable for applications in radar, sonar, telecommunications, and array signal processing [119]. The Algorithm 4 given below illustrates the algorithm for TLS-ESPRIT.

Algorithm 4 TLS-ESPRIT algorithm

1: **Input:** Array of observed data X (Size $M \times N$). Number of signals d to estimate.
2: Compute Covariance matrix: $R = \frac{1}{N} X^H$
3: Perform Eigenvalue decomposition: Decompose R into eigenvalues and eigenvectors.
$R = U_s \Delta_s U_s^H + U_n \Delta_n U_n^H$; where U_s corresponds to the signal subspace and U_n corresponds to the noise subspace.
4: Form signal subspace: Extract d-dimensional signal subspace matrix U_s.
5: Partition signal subspace into two overlapping matrices: $U_1 = U_s(1 : M-1, :)$, $U_2 = U_s(2 : M, :)$
6: Solve for Rotation Matrix: Use Total Least Squares (TLS) to solve for the rotation matrix Φ that satisfies: $U_2 = \Phi U_1$
7: Extract eigenvalues of Φ: $\lambda_k = e^{j\omega_k}$; where ω_k represents the frequencies or DOAs.
8: Estimate Parameters: Estimate frequencies of angles from ω_k.

$$\theta_k = \frac{\arg(\lambda_k)}{2\pi}$$

9: **Output:** Estimated frequencies or DOAs $\{\theta_k\}$.

The comparative scrutiny of subspace-based DOA estimation algorithms reveals intricate performance variations across signal processing domains [123, 124]. MUSIC algorithm demonstrates superior spatial resolution and robust performance, especially in low SNR environments and circumstances with closely spaced signal sources. Empirical investigations substantiate MUSIC's enhanced capability to discriminate coherent sources with resolution thresholds approaching 0.5–1 degrees, significantly outperforming ESPRIT and Root-MUSIC methodologies [125]. The MUSIC algorithm, distinguished for its exceptional estimation accuracy, is characterized by an exponential growth in computational complexity as the number of sources increases. This phenomenon highlights a fundamental trade-off between the algorithm's precision in source localization and its computational efficiency, particularly in scenarios involving large source ensembles [126]. Root-MUSIC and ESPRIT algorithms offer complementary characteristics, with Root-MUSIC providing faster computational approaches at the expense of performance degradation under challenging signal conditions. The algorithmic paradigm's efficacy is particularly pronounced in specialized domains like wireless communications, radar systems and advanced sensor array processing, where high-precision angular estimation is paramount. Critically, the algorithmic performance is contingent upon several parametric constraints such as array geometry, source correlation, signal characteristics, and ambient noise statistical properties.

In sumary, the juxtaposition of DOA estimation algorithms reveals nuanced performance characteristics of MUSIC, ESPRIT, and Root-MUSIC, highlighting the critical importance of algorithm selection based on specific signal processing requirements. The research underscores the fundamental trade-offs between spatial resolution, computational complexity, and signal discrimination capabilities across varied signal environments. Future research should focus on developing adaptive algorithms that can dynamically optimize performance across diverse signal propagation scenarios, integrating advanced machine learning techniques to mitigate current algorithmic limitations [126].

4.2 *Beam-forming based methods*

Beamforming uses a group of antennas or sensors to direct signals toward a specific area while filtering out noise and interference from other directions. This makes it function like a spatial filter. Over the past decade, there has been renewed focus on beamforming, especially in wireless communications. Multi-antenna technology has become a key tool for managing the rapid increase in users and the growing demand for faster data services. The primary focus of beamforming is bearing estimation, with the goal of locating the source of the transmitted communication or radar signal [127, 128].

Antenna arrays are promising technology for enhancing coverage or boosting the capacity of terrestrial cellular systems. These arrays improve signal quality and increase overall network capacity, allowing efficient use of the available spectrum. This results in better performance, accommodating more users or extending coverage in a given area [129, 130]. A beamformer, typically considered a spatial filter, focuses on signals from a specific direction by processing inputs from multiple sensors arranged in an array. It operates in two stages: synchronization and weight-and-sum. Synchronization aligns the signals by adjusting their timing based on the TDoA, which is to be estimated if unknown. The weight-and-sum stage assigns weights to the aligned signals before summing them, fine-tuning the beam's characteristics. While both steps are essential, most attention is given to configuring the weights. These weights are sometimes set for a specific beam pattern but can be adaptively adjusted for better performance based on real signal and noise conditions [131]. In signal processing, three primary areas of research are (1) Determining signal numbers and detecting incoming signals (2) Finding the DoA of signals and (3) Enhancing signals of interest while suppressing interfering signals, directly related to beamforming [132]. In classical time-domain filtering, a signal is processed by linearly combining it with a set of weights for high-pass, low-pass, or band-pass filtering. Similarly, beamforming uses data from spatially distributed sensors to apply beamforming weights, achieving spatial filtering. This amplifies signals from the desired direction while suppressing unwanted signals, offering improved estimates of the transmitted signal [133].

Beamforming techniques are classified as conventional or adaptive beamformers. Conventional beamformers use a predetermined weight vector based on the array's response for a specific DoA. These beamformers are data-independent, offering consistent responses regardless of signal or interference conditions. Adaptive beamformers, on the other hand, adjust weight vectors based on incoming data, optimizing performance and offering better resolution and interference rejection. However, traditional adaptive beamforming methods suffer in real-world scenarios if their assumptions, such as those about the environment, signal sources, or sensor array, are inaccurate. Adaptive beamforming techniques need to be robust against uncertainties and imperfections in these setups. Beamforming techniques include various subtypes. Static beamforming uses directional antennas to create a fixed radiation pattern, while dynamic beamforming adapts the pattern to optimize signal quality. Transmit beamforming adjusts phase-shifted signals for targeted transmission, either explicitly or implicitly. Analog beamforming tweaks individual antenna phases to shape the radiation pattern, while digital beamforming, or baseband beamforming, modifies signals before RF transmission, allowing multiple beams to target different users. Hybrid beamforming enhances flexibility and efficiency by combining analog and digital techniques, [134]. Advanced beamforming techniques using deep learning and AI have also emerged [135].

Conventional algorithms for beamforming scan a beam to measure the received power from various directions. The DoA's are identified as the directions with the highest received power. While these classical algorithms are simple, they often provide limited resolution and weaker performance [136].

4.2.1 *Conventional time domain beamformer*

To address the time delays between sensor data due to the array layout, these delays are corrected in the time domain. This involves applying precise time delays to the data from each sensor and then summing the corrected data. In a broadband time domain beamformer, interpolation filters are applied to each sensor to accurately adjust the time delay over a range of frequencies. The corrected outputs from all sensors are then combined to generate the final beamformer result.

A. Delay-and-Sum Technique: The DS beamformer showcases how using an array of sensors can improve the reception of desired signals while simultaneously reducing unwanted noise. By carefully delaying and then summing the signals from each sensor, the DS beamformer amplifies the signal coming from the target direction and diminishes interference from unwanted directions [132]. In a DS beamformer, the digitized sensor data is collected at a rate of F_i samples per second and stored before the beamforming calculation is performed. Once the final required data sample is available, the beam output can be computed. This method has several inherent limitations. First, it requires a sampling rate significantly higher than the Nyquist rate to accurately capture the time delays necessary for beam steering. Second, large arrays and high sampling rates often result in long delays, which demand substantial memory or storage capacity. Finally, when analog-to-digital (A/D) converters are located remotely from the beamformer, transmitting the high data rates becomes challenging due to the considerable bandwidth required for signal transmission [137].

B. Partial-Sum Beamformer: The partial-sum or sum-delay beamformer reduces the memory requirements compared to a traditional delay-sum beamformer. Instead of storing all the sensor data before performing beam steering, the partial-sum beamformer processes the sensor data immediately after sampling. This approach minimizes the amount of memory needed, as it avoids the necessity of buffering large scoops of data and performs beamforming in a more efficient manner. Even though the partial-sum concept cuts down on the amount of data that needs to be preserved by processing data right after sampling, it still generally requires a high input sampling rate for effective beam steering. (a) Accurate Beam Steering: To accurately steer the beam and capture the necessary time delays, the system needs a high sampling rate. This high rate ensures that the time delays can be approximated precisely, which is crucial for accurate beamforming. (b) Resolution of Time Delays: High sampling rates provide better resolution for time delays, which is essential for precise beam steering. Without a high sampling rate, the beamformer might not be able to differentiate between small delays, leading to less accurate beamforming. (c) Signal Integrity: A higher sampling rate helps preserve the integrity of the signal by capturing more detail, which is important for accurate processing and beam steering. In summary, while the partial-sum beamformer reduces memory requirements by processing data immediately, the need for high sampling rates remains to ensure accurate beam steering and signal quality [137].

C. Interpolation Beamforming for Bandpass Applications: The previously discussed techniques primarily focus on low-pass frequency applications; however, bandpass applications offer opportunities for additional optimization in areas such as analog-to-digital (A/D) converters, cable bandwidth, data storage, and computational throughput. These improvements are achieved by sampling the bandpass sensor data at a rate corresponding to the signal's bandwidth rather than its highest frequency. This method has significantly lead to the reduction in resource demands when the signal's bandwidth is considerably smaller than its central frequency. To prevent issues with frequency aliasing in bandpass applications, more complex sampling methods are necessary. Three commonly used methods are: (a) Analytic Signal Sampling: This technique involves sampling the signal in a way that helps to avoid aliasing by focusing on the analytic representation of the signal. (b) Second-order Sampling: This method samples the signal at twice the bandwidth, which helps to reduce aliasing effects and improve accuracy. (c) Quadrature Sampling: This approach samples the signal in two orthogonal components, allowing for more accurate representation and reducing aliasing. These advanced sampling procedures ensure that the bandpass signal is accurately captured and processed, maintaining signal integrity while optimizing resource usage.

D. Second-Order Sampling: Second-order sampling is an alternative approach that simplifies the signal processing workflow by eliminating the need for the Hilbert transform. This method involves sampling the signal in two interleaved sequences: one sampled at x(mT)x(mT) and the other at x(mT + α)x(mT + α), where TT is the sampling interval and α is a temporal offset. These sequences are uniformly spaced but staggered relative to each other. When α ≠ T/2 and the sampling rate exceeds the signal bandwidth, the original signal can be accurately reconstructed, ensuring the complete capture of its frequency components without aliasing. However, precise reconstruction requires the use of an ideal bandpass filter with complex frequency response characteristics to facilitate accurate interpolation. This filter is necessary to properly combine the two sequences and reconstruct the original signal. Overall, second-order sampling allows for effective bandpass signal processing by using interleaved sampling sequences, provided that the sampling rate is sufficient and appropriate filtering is applied.

E. Shifted Sideband Beamforming and Grating Lobes: Shifted sideband beamforming efficiently processes high-frequency signals by converting them to lower frequencies, which simplifies the processing and reduces the need for precise timing calculations. This approach leads to more straightforward and energy-efficient hardware. However, this method can lead to the formation of grating lobes in certain array configurations. Grating lobes appear when multiple sensors in the array are given the same delay during processing, which can create gaps in the array's coverage and result in additional, undesirable lobes in the output. While shifted sideband beamforming has benefits for hardware efficiency, it's important to manage and mitigate the risk of grating lobes to ensure effective performance. A shifted sideband beamformer can function without interpolation; however, incorporating interpolation with the shifted sideband technique offers the advantage of further reducing the input sampling rate. When applying complex sampling methods, the complex sensor data needs to be sampled only at approximately twice the highest signal frequency after frequency translation or at a rate equal to the signal's bandwidth. Interpolation is then employed to estimate the data at the precise time instances required for beam steering. Similar to other beamforming methods, interpolation can be implemented either before or after the beamforming process and can be combined with partial-sum or delay-sum techniques to achieve additional reductions in hardware costs [137].

F. Doppler Division Multiplexing (DDM): The Doppler Division Multiplexing (DDM) technique is applied at the transmitter station, while Doppler filtering at the receiver station enables the separation of echoes originating from multiple transmitters. The proposed beamforming framework operates in two stages. In the first stage, beamforming is independently performed on each channel, effectively addressing challenges related to near-field and wide-angle focusing. In the second stage, digital beamforming (DBF) and beampattern synthesis are executed to refine the beamforming process and enhance performance [138].

4.2.2 *Conventional frequency domain beamformer*

For narrowband signals, a conventional frequency domain beamformer approximates time delays as phase shifts. In the frequency domain, the data from the sensor array can be represented using the Fourier transform of the received signals. The system operates as follows: Initially, the delay between sensors is approximated as a phase shift, meaning that in the frequency domain, the sensor data vector can be represented with these phase modifications. The sensor array data is then transformed into the frequency domain to obtain X(ω)X(ω), which represents the array's data vector, with the array manifold vector (or steering vector) capturing the spatial characteristics of the array. For narrowband signals, phase shifts are applied to the sensor data to correct for differences in signal path lengths to each sensor, and the phase-adjusted data is summed to produce the beamformer output. For broadband signals, the signal is decomposed into frequency components using the Fast Fourier Transform (FFT), and each component is adjusted with its respective phase shift, determined by the steering vector. Finally, the phase-adjusted data from all frequency components is recombined to create the final output of the beamformer.

4.2.3 *Adaptive beamformer*

Adaptive beamforming aims to improve the clarity of a desired beacon meanwhile reducing noise and interference from the output of the sensor array. Adaptive Beamforming is an algorithm used for generating real-time beams, performing adaptive processing, accurately estimating angles, and suppressing interference from jammers as reviewed by De et al., in their work. It works by optimizing a set of weight vectors to better pinpoint a directional source. Various methods can be employed to solve this optimization problem. In situations where the strength of the signal is not known and is constantly present, applying linear constraints to the weight vectors allows for precise control over how the beamformer adapts its performance [139]. The detailed structure of the adaptive beamformer, which dynamically adjusts its weights to optimize signal reception by minimizing interference and noise while enhancing the desired signal, is illustrated in Figure 8. This figure provides a comprehensive representation of the system's components and their interconnections.

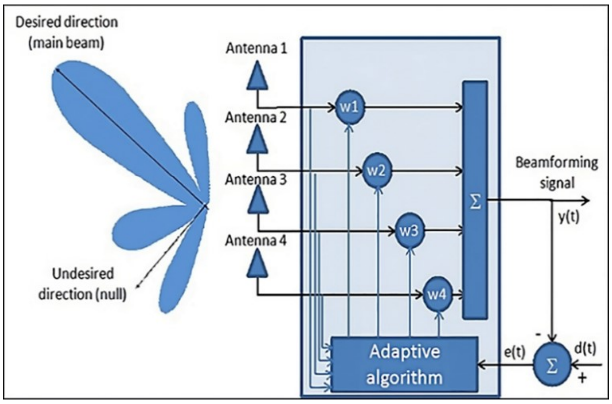

Figure 8. Adaptive beamformer

A. Linearly Constrained Minimum Variance Beamforming: In many cases, the traditional methods may not be adequate. For instance, it is possible that the desired signal is either weak or constant, which can prevent precise estimation of the signal and noise covariance matrices in maximum SNR processors and necessitate the use of signal cancellation techniques such as Minimum Squared Cancellation (MSC). Furthermore, using reference signals becomes infeasible when the desired signal is unknown. LCMV beamforming applies linear constraints to the weight vector in order to overcome these drawbacks. This method provides a great deal of control over the beamformer's response. LCMV beamforming employs several techniques to optimize its performance. First, it uses linear constraints to ensure that signals from the direction of interest are transmitted with the desired strength and phase, allowing for precise control over the beamformer's response to inputs. Additionally, LCMV beamforming aims to minimize the output variance or power by selecting weights that adhere to the specified response constraints, effectively reducing noise and interference while enhancing the desired signal. To further refine the beamformer's response and suppress unwanted sidelobes, the generalized sidelobe canceler is often integrated with LCMV beamforming [140]. Figure 9 shows the simulation result of LCMV.

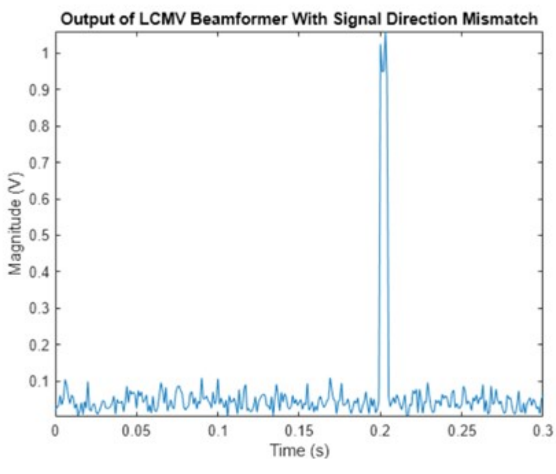

Figure 9. Simulation result of LCMV

To prevent signal self-nulling in beamforming, the LCMV (Linearly Constrained Minimum Variance) beamformer is used. It allows multiple constraints to be applied along the target direction, reducing the chance of suppressing the target signal when it arrives slightly off the expected angle. First, the LCMV beamformer is created, enabling the output of the beamformer weights. Constraints are defined by specifying steering vectors for the expected direction and nearby angles to avoid signal loss. The steering vectors are calculated for the desired directions, and the corresponding constraint matrix and desired response are set. When the beamformer is applied to the receiving signal, output shows that the targeted signal can still be detected despite a mismatch between the true and expected signal direction.

B. Maximum SNR Filter: Fixed beamforming techniques make the most of the array's geometry and the known location of the signal source to create an optimal beam pattern. However, their ability to suppress noise and interference from other sources is often constrained by factors like the array's aperture. To overcome these limitations and achieve a greater SNR when the array's geometry is fixed, adaptive beamforming techniques are used. These techniques leverage the characteristics of source as well as noise, ensuing in a range of advanced array processing algorithms that adapt to the signal environment for better performance.

C. Diagonal Loading: Because of its ease of use and efficiency in handling a variety of error kinds, including finite sample errors and steering vector inaccuracies, diagonal loading is one of the most widely used robust adaptive beamforming techniques. It is especially resistant to errors in finite samples. The absence of a dependable technique for choosing the diagonal loading factor, which can have an immediate effect on the technique's performance, is a major disadvantage of diagonal loading.

D. Eigenspace-Based Technique: The eigenspace-based beamformer is a widely used method for robust adaptive beamforming. It utilizes the eigen decomposition of the sample covariance matrix to identify the estimated signal-plus-interference subspace and projects the signal steering vector onto this subspace. This approach offers excellent resistance to steering vector errors, particularly when the number of interference directions is known and the signal-plus-interference subspace is small. However, the performance of the eigenspace-based beamformer can significantly degrade if the low-rank interference-plus-signal model assumption is violated or if the subspace dimension is either unknown or incorrectly specified. For instance, if the interferers are spatially distributed and incoherent, or if there are shifting interferers or wavefronts, the low-rank assumption may no longer hold, which can reduce the method's effectiveness. In such cases, the eigenspace-based beamformer may not be the optimal choice. Additionally, even when the low-rank model is valid, this technique tends to perform best in environments where the signal-to-noise ratio (SNR) is sufficiently high.

E. LMCV Beamformer: To make the LCMV beamformer more robust against errors in DoA, we can add additional constraints. These constraints help in creating a wider main beam, which covers all potential directions of the desired signal. This approach is also useful when dealing with rapidly moving interference sources. As these sources shift, they might move out of the sharply designed null areas of the beamformer's pattern, leading to a drop in the SINR. A common solution is to artificially widen the nulls toward the interference directions by applying derivative constraints. This adjustment helps maintain good performance in challenging conditions. For more complex situations, including both point sources and scattered sources or fluctuating wavefronts, a robust beamformer has been developed. This method models uncertainties in the signal and data covariance and focuses on optimizing performance under worst-case scenarios. It also provides efficient, closed-form solutions and practical online implementations.

F. Capon Beamformer: The Robust Capon beamformer improves on the standard Capon method by adding a restriction that ensures the beamformer's response remains above a specified threshold for all directions within an ellipsoid (or sphere) centred on the expected direction of interest. This adjustment helps to handle variations in the signal's direction. This method uses a quadratic inequality constraint and is implemented using a gradient descent algorithm. While previous methods have focused on narrowband signals, many applications require handling wideband signals. A common approach to achieve this is to decompose the broadband signals into narrower frequency bands (sub-bands) and apply narrowband beamformers to each sub-band individually. The Algorithm 5 given below illustrates the algorithm for MVDR.

G. Tapped Delay Line Beamformer: Using tapped delay lines (TDLs) is an additional method for creating wideband beamformers. By producing a frequency-dependent response for every broadband signal that is received, this technique aids in adjusting for phase differences between different frequency components. A strong algorithm utilizing worst-case optimization is presented for broadband arrays. In order to address mismatches in the desired signal, this method applies a set of constraints at particular frequency points within the range of interest. The method's high computational complexity and limited capacity to maintain a consistent response when handling mismatched desired signals are two of its drawbacks. These new techniques combine worst-case performance optimization with constraints on frequency invariance. This method is demonstrated as a convex optimization issue, which is to be resolved by applying well-known convex optimization techniques [126].

Algorithm 5 MVDR

1: **Input:** Steering vector $a(\theta)$ for the desired direction. Covariance matrix of received signals R. Desired signal direction θ.
2: Compute Steering vector: Define the steering vector $a(\theta)$, which models the array response to a signal coming from direction θ.
3: Calculate inverse covariance matrix: R^{-1}.
4: Compute MVDR weights: $w = \frac{R^{-1}a(\theta)}{a^H(\theta)R^{-1}a(\theta)}$
5: Apply the beamforming weights to process the received signals: $y(t) = w^H x(t)$
6: **Output:** Beamformed output $y(t)$.

Figure 10 observes the simulation result of the MVDR algorithm. The signal coming from the intended direction of 45 degrees in azimuth is maintained by the MVDR beamformerwhile suppressing signals from other directions. It creates two deep nulls at the interference directions of 30 and 50 degrees. With a gain of 0 dB along the target direction of 45 degrees, the beamformer effectively preserves the target signal and suppresses interference signals.

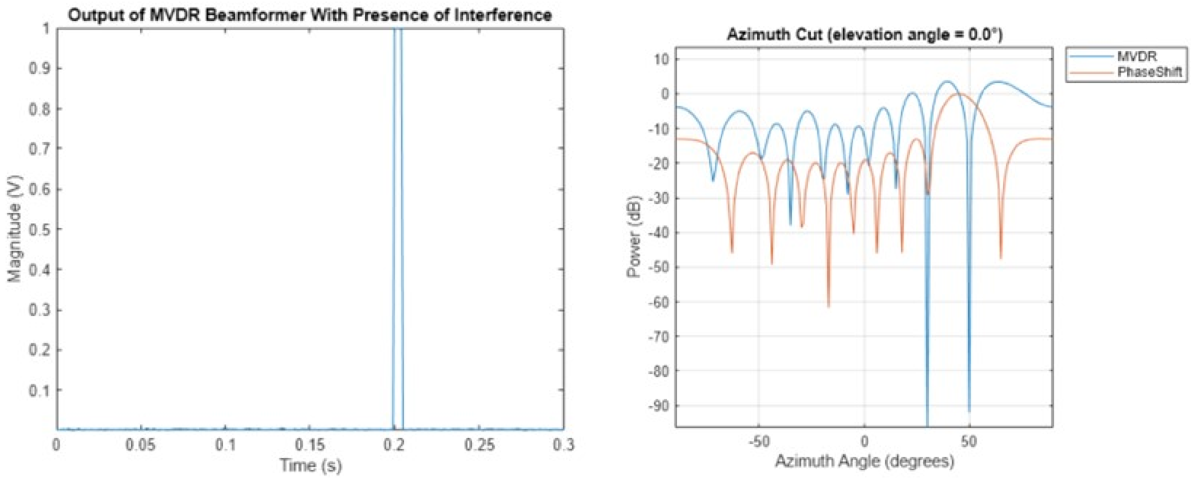

Figure 10. Simulation result of MVDR (Adaptive beamformer)

Figure 11 observes the comparison of simulation results of MUSIC and MVDR methods. The MVDR algorithm correctly estimates the DOAs offering enhanced resolution. However, this increased resolution comes at a cost. MVDR is more sensitive to sensor position errors, and inaccuracies in sensor placement can result in a distorted spatial spectrum. In situations where sensor positions are inaccurate, MVDR may produce worse results than Beamscan. Furthermore, if the difference between the two signal directions is reduced to a level smaller than the beamwidth of an MVDR beam, the algorithm will fail to distinguish between the sources.

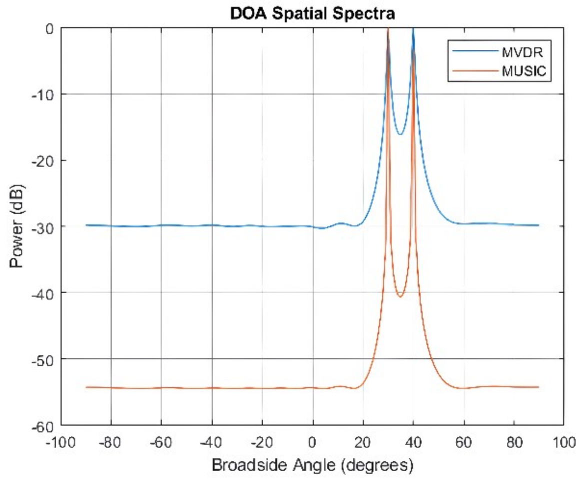

Figure 11. Comparison of simulation result of MUSIC and MVDR (Adaptive beamformer)

The MUSIC algorithm can also resolve closely spaced signals. It correctly estimates the DOAs and provides better spatial resolution as compared to MVDR. Although, like MVDR, MUSIC is sensitive to sensor position errors, which can affect the accuracy of its spatial spectrum. Additionally, the number of sources must be known or accurately estimated. When the number of sources is incorrect, MVDR and Beam-scan may return insignificant peaks from the correct spatial spectrum, while MUSIC may produce an inaccurate spatial spectrum. Moreover, the amplitudes of MUSIC's spectral peaks cannot be interpreted as the power of the sources.

Beamforming, while beneficial, has several limitations. Managing interference and noise can be difficult, particularly in noisy environments or those with multiple sources of interference. Advanced beamforming techniques require complex algorithms and significant computational resources, leading to higher costs. In multipath environments, signal processing and communication systems face considerable challenges. Signals that reflect off various surfaces and arrive at the receiver from different paths can degrade the signal. The signals combine in and out of phase at each antenna, causing signal fading, which may vary with the antenna elements' location. This results in a weakened or distorted signal, leading to fluctuations in signal strength and quality. For GPS or radar applications, such reflections can introduce errors, affecting the accuracy of location or target detection. Managing these multipath effects requires sophisticated signal processing techniques, which can increase computational demands and delays, ultimately limiting the effective capacity of wireless channels, reducing data throughput, and increasing error rates. Multipath environments thus present significant challenges that can impair signal clarity and the performance of communication and radar systems. The difference between SNRs becomes more noticeable unless the interference is really near the look direction, in which case it occurs within the main lobe. When interference is outside the main lobe, the difference between SNRs is less pronounced. As length of the filter increases, the contrast between SNRs decreases, and this effect is more significant in high-bandwidth scenarios. Conversely, filter length has less of an effect in situations with lower bandwidth. When interference is outside the main lobe, adding more elements to the array can decrease the difference between SNRs; but, when interference is inside the main lobe, the larger array can raise the difference between SNRs. This suggests that although a wider array is beneficial in some situations, it creates additional difficulties when interference occurs inside the main lobe. In massive MIMO systems, each terminal uses a unique pilot sequence for uplink communication. However, due to a limited number of unique pilot sequences, they are reused, leading to pilot contamination. When pilot sequences are reused across cells, the base station's antenna array receives mixed signals from multiple terminals, making it difficult to separate them. This contamination results in interference during downlink beamforming, reducing its effectiveness by limiting the ability to enhance the desired signal while suppressing unwanted noise. As the antenna number increases, the pilot contamination effect worsens, further complicating the beamforming process and potentially degrading overall system performance [141].

Future 5G systems will need to use sophisticated antenna systems with beamforming and DoA estimation capabilities in order to achieve low latency. The methods covered have proven successful in bringing down the price of beamforming in big sonar systems that are utilized for applications for underwater environments. Because digital, microprocessor-based configurations are so versatile, these techniques have also shown to be relevant in a number of other fields. These days, they are used in seismic monitoring to find earthquakes, in aeroacoustics to pinpoint aircraft noise sources and in ultrasonography to enhance image quality [132]. Beamforming is an imaging technique widely used in aeroacoustics and is continually advancing to address new challenges. While it shares similarities with methods like near acoustic holography, beamforming excels in handling distributed, broadband, and incoherent sources located at varying distances from the array. This versatility makes it particularly effective in capturing and analysing complex acoustic environments [131]. Recent advancements in Very High-Speed Integrated Circuit (VHSIC) and Very Large-Scale Integration (VLSI) technologies have further expanded the potential of these techniques, making them viable for use in radar systems to enhance detection and tracking capabilities. Gupta et al. have studied various applications and designs using a unique Field Programmable Gate Array (FPGA) based pulse detection and characterization approach [142]. Rathor et al. studied the Digital Radar Receiver and Signal Processing Algorithm Implementation on an FPGA-Based Board [143]. Rajkumar et al. designed and developed FMCW based Radar Altimeter DSP Interfaces and Algorithm using FPGA based board [144, 145]. Hybrid beamforming in 5G is a key technology that balances performance and complexity by combining benefits of Digital and Analog Beamforming, making it well-suited for the demands of 5G networks. Analog beamforming adjusts signal phases at the RF level, helping to direct signals and improve coverage with fewer components. Digital beamforming, on the other hand, allows for precise control over signal amplitude and phase at the baseband level, enabling the formation of multiple beams to serve different users simultaneously. The hybrid approach optimizes both power and cost by integrating these two methods. It allows 5G systems to manage high-frequency signals and massive MIMO antenna arrays efficiently. This results in better signal quality, higher data rates, and more reliable connections, which are essential for the fast and flexible services 5G promises to deliver. Future 5G systems will require sophisticated antenna systems with beamforming and DoA estimation capabilities in order to achieve low latency and large throughputs [146]. In the UMTS (Universal Mobile Telecommunications System) framework, beamforming significantly boosts network performance by enhancing signal quality through precise direction towards users, resulting in clearer calls and faster data speeds. This technique also improves coverage, especially in challenging environments like urban areas or rough terrains, where traditional methods might struggle. By focusing signals more efficiently, beamforming increases network capacity, allowing more users to connect simultaneously without causing interference. Overall, it plays a crucial role in delivering better, more reliable mobile connections within the UMTS network [128]. In underwater sonar systems, beamforming offers a cost-effective way to boost the detection and tracking of underwater objects. By focusing sound waves more precisely, it enhances the clarity of sonar images, making it easier to identify and follow objects beneath the surface. In seismic monitoring, beamforming improves the detection and analysis of earthquakes by processing seismic waves more accurately. This results in a better understanding and quicker response to seismic events, which is crucial for assessing and mitigating natural disasters. Aero-acoustic analysis benefits from beamforming by helping to identify sources of noise in aircraft design. By analyzing the acoustic signatures of different components, it supports the development of quieter aircraft, contributing to reduced noise pollution and improved passenger comfort. In medical ultrasound imaging, beamforming enhances diagnostic imaging by providing clearer and more precise images. This improvement helps doctors make better-informed decisions, leading to more accurate diagnoses and effective treatments. The rapidly developing technology known as Impulse Ultra-Wide Band (UWB) communication has a number of unique advantages over narrowband and broadband signals, such as superior interception and detection capabilities, exceptionally high data rates, remarkable robustness in dense multipath environments, and the capacity to function concurrently with traditional systems [147]. Beamforming techniques improve target identification and tracking in radar systems by more efficiently directing radar signals. This enhances the ability to detect and monitor objects, which is crucial for applications ranging from military surveillance to air traffic management.

5. Deployment of DoA

5.1 *Pre-processing and Post-processing techniques used for DoA estimation*

In environments characterized by complex multipath effects such as urban settings, indoor spaces, or densely populated signal environments—achieving reliable DoA estimation is notably challenging. The presence of multiple path propagation can lead to signals arriving via multiple indirect paths, which often results in overlapping or closely spaced signals. Traditional DoA estimation methods frequently struggle to separate these signals effectively. To tackle these challenges, advanced preprocessing and postprocessing techniques have emerged as essential strategies to enhance both the accuracy and robustness of estimation of DoA.

Effectiveness of preprocessing and postprocessing techniques in enhancing Direction of Arrival (DoA) estimation accuracy critically depends on sophisticated signal conditioning strategies designed to mitigate multipath environment complexities [148, 149, 150, 151]. In complex electromagnetic environments characterized by urban settings, indoor spaces, and densely populated signal scenarios, advanced preprocessing methodologies emerge as essential techniques for addressing signal interference and estimation uncertainties. The Adaptive Directional Time-Frequency Distributions (ADTFD) approach represents a particularly innovative preprocessing technique, dynamically adjusting kernel directions within the time-frequency domain to facilitate clearer signal component separation and minimize signal overlap. Advanced preprocessing methodologies, including blind source separation, subspace clustering, and beam-space processing, demonstrate remarkable potential in addressing spatial correlation and signal decomposition challenges [152]. Table 4 depicts the comparison of these preprocessing methodologies in terms of signal separation efficiency, Noise reduction, Computational complexity, and multipath scenario adaptability.

Complementary postprocessing techniques, such as the QRD-RLS algorithm and spatial filtering techniques, further enhance DoA estimation by providing temporal consistency and adaptive refinement of initial estimation results. Emerging research highlights the transformative impact of machine learning-based preprocessing techniques, particularly deep neural networks and compressive sensing approaches, which adaptively extract salient signal features and suppress noise-induced distortions with unprecedented precision. The synergistic integration of preprocessing and postprocessing strategies creates a comprehensive approach to signal refinement, particularly effective in scenarios featuring overlapping signals, reflected paths, and multiple interference sources. Innovative techniques such as Kalman filtering and adaptive spatial filtering enable continuous estimate refinement, addressing the fundamental challenges encountered in complex multipath propagation scenarios. These sophisticated approaches collectively address critical problems like reduced SNR, coherent signal interference, spatial resolution constraints, ultimately pushing the boundaries of DoA estimation capabilities in next-generation wireless communication and sensing systems [153]. Table 5 depicts the comparison of postprocessing techniques in terms of estimation accuracy, temporal consistency, adaptive refinement capability and interference suppression.

The integration of preprocessing and postprocessing strategies creates a comprehensive approach to signal refinement, particularly in complex multipath scenarios. Techniques such as ADTFD in the preprocessing stage ensure that only the most pertinent signal components are relayed to the DoA estimation algorithms. In turn, postprocessing techniques, like QRD-RLS, enhance these estimates by adapting to the temporal and spatial dynamics of the environment. This synergistic combination significantly elevates DoA estimation accuracy, leading to more reliable performance in scenarios featuring overlapping signals, reflected paths, or multiple sources of interference.

Table 4. Comparison of preprocessing techniques

Technique	Signal separation efficiency	Noise reduction (dB)	Computational complexity	Multipath scenario adaptability
Adaptive directional time-frequency distributions (ADTFD)	92.3%	15.7	High	Excellent
Blind source separation	85.6%	12.4	Very high	Good
Subspace clustering	88.2%	13.9	Moderate	Very Good
Beam-space processing	79.5%	10.6	Low	Moderate
Deep neural network preprocessing	94.1%	17.3	Extremely high	Outstanding

Table 5. Comparison of postprocessing techniques

Technique	Estimation accuracy (%)	Temporal consistency	Adaptive refinement capability	Interference suppression
QRD-RLS algorithm	89.7%	High	Excellent	Very Good
Kalman filtering	86.5%	Very High	Outstanding	Good
Spatial filtering	82.3%	Moderate	Good	Moderate
Compressive sensing	91.2%	Moderate	Moderate	Excellent
Machine learning refinement	93.6%94.1%	High	Outstanding	Excellent

5.2 *Hardware implementation*

The layout and configuration of antenna arrays are fundamental for the success of DoA estimation systems. Array types, such as Uniform Linear Arrays (ULAs) or Coprime Arrays, can significantly impact both the resolution of DoA estimates and the cost of deployment. For example, ULAs are compatible with simpler algorithms, such as Root-MUSIC, due to their regular spacing, which allows for efficient computation. In contrast, arrays with complex configurations (e.g., nested or sparse arrays) may require advanced algorithms that can leverage non-uniform spacing, providing greater flexibility and enhanced resolution but demanding more complex processing [154].

Practical hardware deployment of advanced Direction of Arrival (DoA) estimation systems demands sophisticated computational infrastructure, typically requiring multi-core processors with clock speeds between 2.5–3.5 GHz. Advanced implementations require significant GPU acceleration, with a minimum of 1024 CUDA cores and 8–16 GB dedicated VRAM, enabling complex preprocessing and machine learning-based signal processing techniques. The computational intensity varies across different DoA estimation methods, ranging from 10^9 to 10^{14} floating-point operations, with machine learning preprocessing representing the most computationally demanding approach. Memory requirements are critical, with a minimum of 16 GB DDR4/DDR5 RAM recommended, scaling to 32–64 GB for intricate multipath environments. High-speed NVMe SSD storage (minimum 256 GB) ensures rapid data processing and algorithm execution. For specialized implementations, FPGA or ASIC configurations with 100,000–500,000 logic elements provide additional customization potential, allowing for optimized signal processing architectures tailored to specific electromagnetic sensing requirements [155, 156, 157].

Power and thermal management are pivotal considerations in Direction of Arrival (DoA) estimation systems, particularly in mobile and remote sensing applications with stringent resource constraints. Advanced implementations of DoA leverage low-power FPGAs and energy-efficient antenna arrays to facilitate sophisticated signal processing while maintaining energy consumption between 1–5 watts. Thermal management strategies in these systems include dynamic power scaling, adaptive clock gating, intelligent thermal throttling, and distributed thermal sensing, enabling efficient heat regulation. Intended to function throughout a broad temperature range of −40 °C to +85 °C, it emphasizes localized heat dissipation, adaptive cooling mechanisms, and predictive thermal stress mitigation. Emerging approaches incorporate machine learning algorithms for proactive thermal performance optimization, shifting from reactive to predictive paradigms [158].

Scalability is an important consideration for DoA estimation systems that need to operate in diverse environments, from small-scale to large-scale deployments. For instance, the system's capacity to predict the direction of arrival, with the increase in number of antenna elements in an array, improves with higher precision, but this also increases the complexity of the hardware. For large systems, scalable hardware solutions like modular antenna arrays and distributed computing systems become essential. Such solutions allow for the addition of more antennas or processing units without a significant overhaul of the entire system, ensuring the hardware can handle the increased data flow and processing requirements [159].

Accurate synchronization of multiple antenna elements is crucial for DoA estimation. Errors in synchronization can lead to inaccuracies in the directionality estimates. Typically, time synchronization is achieved using GPS clocks or other precision timing sources, but in many cases, the cost and complexity of such solutions may not be feasible. Therefore, ensuring that the hardware design includes robust synchronization mechanisms is key to obtaining reliable results. Additionally, antenna array calibration is a significant aspect of deployment, as the system performance depends on the proper alignment as well as calibration of the antennas, which must be done regularly to avoid errors due to environmental factors like temperature fluctuations or physical obstructions.

The deployment environment can dramatically influence hardware performance. Factors like temperature, humidity, and electromagnetic interference (EMI) can diminish the calibre of the received signals, causing inaccurate DoA estimates. In outdoor deployments, for example, antennas and hardware components must be designed to withstand environmental stresses, such as high winds, rain, or temperature variations. The system must also be shielded or protected from EMI to ensure signal integrity. This might require additional hardware such as environmental enclosures, heat sinks, or EMI shields to ensure consistent performance under a variety of conditions.

In many applications, especially in mobile or remote systems (e.g., UAVs, automotive radar), the physical constraints of the hardware, such as size, weight, and form factor, become crucial factors in deployment. Antenna arrays must be compact yet capable of providing the required resolution, while the associated processing hardware must be small and light enough to be deployed without significantly affecting the system's mobility or energy consumption. Miniaturization techniques, such as the use of microstrip antennas and smaller, integrated circuit designs, are often employed to meet these constraints.

Hardware costs are always a major factor in real-world deployment. High-performance systems, particularly those using sophisticated algorithms and high-end components (like GPUs or FPGAs), can be costly to manufacture and deploy. For large-scale systems, the total cost of ownership (TCO) becomes an important consideration. Efficient, cost-effective design strategies, such as using commercial off-the-shelf (COTS) components or designing hardware for mass production, can help reduce costs while maintaining system performance. In addition, balancing between high performance and affordability is a key challenge in developing systems for wide-scale commercial deployment.

DoA estimation systems are often part of larger systems, such as communication networks, radar systems, or autonomous vehicles. Hence, the hardware must be compatible with existing infrastructure and be able to communicate with other system components (e.g., sensors, data storage, or decision-making systems). Hardware interfacing, such as proper communication protocols (Ethernet, PCIe, etc.), I/O support, and integration with cloud computing systems, plays an important role in the overall effectiveness of the deployment. This integration should not only ensure data exchange but also ensure that the system operates in a coordinated manner with other subsystems, such as navigation or control.

In critical applications, such as autonomous vehicles or aerospace systems, the reliability and fault tolerance of the DoA estimation hardware are of paramount importance. Systems must be designed with redundancy in critical components to ensure that the failure of an individual part cannot compromise the entire system. For example, redundant antenna elements, backup power supplies, and error-correction mechanisms can provide higher levels of reliability. Additionally, the system must include diagnostics to detect faults or performance degradation, alerting operators to potential issues before they become critical [160, 161].

5.3 *Hybrid analog and digital architecture*

In order to resolve the DOA estimation problem, hybrid architectures—which integrate both analog and digital components—have grown in popularity, particularly in high-frequency systems like mm-Wave MIMO and huge antenna arrays. These architectures seek to strike a balance between the hardware efficiency of analog systems and the performance advantages of completely digital systems. Hybrid architectures can provide tremendous benefits in terms of computational complexity, power consumption, and cost-effectiveness by employing analog beamforming techniques at the front end to minimize the number of necessary radio frequency (RF) chains, followed by digital signal processing for finer resolution and accuracy.

The application of hybrid architectures to DOA estimation is examined by Zhang et al. Large antenna arrays, which usually need a lot of RF chains in a fully digital design, present a DOA estimation difficulty that makes the system costly and power-hungry. Only a small number of RF chains are utilized in a hybrid design, and analog beamforming techniques are used to minimize the number of chains needed while preserving the capability of high-resolution DOA estimation. The authors suggest an effective hybrid DOA estimation technique that uses digital processing for accurate angle estimate after utilizing analog beamforming to lower the problem's complexity. By combining these techniques, the system achieves both reduced hardware requirements and high DOA estimation accuracy, making it suitable for large-scale antenna arrays used in modern communication systems [162].

Zhang et al. focuses on the application of hybrid architectures in millimeter-wave (mm-Wave) MIMO systems, which are integral to next-generation wireless communication technologies. Mm-Wave systems have a very high bandwidth but are also highly susceptible to sparse channel behaviors, which require efficient methods for channel estimation. In a hybrid architecture, analog beamforming is used at the front end to capture the spatial diversity of the mm-Wave channels, and then a digital processing unit performs sparse recovery to estimate the wideband channel information. A block sparse recovery method is proposed that exploits the sparsity of mm-Wave channels in both domains. By doing so, the system significantly reduces the amount of data that needs to be processed, which lowers computational requirements while achieving accurate channel estimation. This approach is particularly beneficial for wideband mm-Wave MIMO systems, where channel estimation can be computationally expensive [163]. Integrating MIMO with mm-Wave technology delivers exceptional performance improvements for future wireless communications. Accurate channel state information is difficult to obtain in wideband mm-Wave massive MIMO systems with hybrid transceiver architectures, especially in high-mobility situations when Doppler effects are noticeable. At THz bands, massive multiple-input multiple-output (m-MIMO) communication and integrated sensing provide a wealth of bandwidth resources and a great deal of geographical freedom [164, 165].

Tensor decomposition is a mathematical technique used in hybrid mm-Wave massive MIMO to extract latent structures from multi-dimensional data, which can be particularly useful in various signal processing and communication applications. It generalizes matrix decomposition by extending the concept to higher-order arrays, called tensors. In the context of wireless communication, tensor decomposition can help estimate multi-path channel parameters, such as angles of arrival (DoA) and gains, by leveraging multi-dimensional data collected from multiple antennas and receiver positions [166].

A tensor decomposition-based method is proposed to estimate the parameters of multi-path channel components, allowing the reconstruction of the wireless channel between any pair of Tx and Rx MA positions within the respective Tx and Rx regions, while achieving high channel estimation accuracy with minimal overhead of pilot training. For pilot training, a two-stage Tx-Rx sequential antenna movement pattern is presented initially. The angle and gain parameters needed to reconstruct the channel between any Tx/Rx MA sites can then be estimated by applying canonical polyadic decomposition to obtain the factor matrices of the tensor. Furthermore, examination of the tensor decomposition's uniqueness condition is conducted to make sure that channel data from the whole Tx and Rx regions may be used to fully rebuild the channel from a finite set of Tx/Rx MA positions [167].

6. Applications of DoA

Estimation of DoA has widespread applications in a wide spectrum of domains, such as radar, smart antennas, communication, acoustics, hearing aids, sonar, seismology, oceanography, and surveillance. The range of applications for DoA estimate has grown, and with it have come new implementation hurdles. These difficulties include the need for more memory and longer computation times. Various environments require particular adjustments to current practices. Computational time and cost are important considerations in real-time settings, especially in military applications where the quickest technique is needed for DoA estimate. Determining the DoA in an under-determined case—where there are fewer sensors than sources—is one of the major issues in DoA calculation. Determining the DoA in an under-determined instance, or when there are fewer sensors than sources, is one of the major issues in DoA calculation [168]. By optimizing signal reception and enhancing service quality, DoA estimation improves wireless and mobile communication in communication systems. It supports beamforming techniques and spatial multiplexing, boosting network efficiency and capacity. In smart antenna systems, DoA estimation is crucial for adaptive beamforming and spatial diversity. It allows smart antennas to adjust their radiation patterns dynamically, improving signal quality and minimizing interference. In seismology, this technology aids in pinpointing the epicenter of earthquakes and tracking seismic wave directions. Accurate DoA estimation is essential for understanding earthquake mechanics and refining early warning systems. In oceanography, DoA estimation is used in sonar systems for underwater exploration and monitoring. It helps locate underwater objects, map ocean floors, and track marine species, contributing to marine research and navigation. In acoustics, DoA estimation improves sound source localization and environmental acoustics analysis. It is applied in designing concert halls, controlling noise, and conducting audio surveillance [169]. For surveillance and security systems, DoA estimation enhances the ability to locate and track sound sources or signals, which is crucial for detecting and monitoring potential threats. In hearing aids, DoA estimation boosts performance by distinguishing the direction of different sound sources, helping users focus on conversations and important sounds amid background noise. In teleconferencing systems, DoA estimation improves audio source localization, ensuring clear sound capture from participants, which enhances the quality of virtual meetings and discussions. In radar systems, DoA estimation is vital for detecting and tracking targets. It supports air traffic control, weather monitoring, and military defence by determining the direction of reflected signals. When it comes to estimation models, several common assumptions are often made [170]. Sensor amplitudes are often considered to be identical, and a Taylor series expansion is used for approximating the phase difference. In far-field settings, where only the first-order elements of the Taylor series are taken into consideration, the direction cosine is linearly connected to the phase differences in the array. However, the Fresnel model preserves the second-order elements of the Taylor series in near-field conditions [171]. Multiple targets are recognized based on their relative velocity and/or range in many real-world automotive radar applications, guaranteeing that each processing cell has a maximum of one target. The simple computational beamformer (BF) spectrum can be used to produce optimal DoA estimations for a single target. Nevertheless, when more than one target reflection overlaps in a processing cell, especially with horizontal multipath near a close guardrail, the BF spectrum may not resolve these paths. This can lead to false localization, where the observed vehicle appears to be positioned closer to the guardrail. To accurately identify and resolve such multipath situations and accurately localize both observed vehicle as well as ghost targets, high-resolution DoA estimation is necessary [172].

As the use of DoA estimation grows, so do the challenges related to computation time and memory requirement. In real-time scenarios such as military applications, the need for fast and efficient algorithms is critical. A significant challenge arises in the under-determined case, where there are fewer sensors than sources, complicating the direction determination process. Addressing these challenges involves ongoing development and optimization of DoA estimation algorithms to meet the demands of various applications.

7. Future scope

7.1 *Deep learning approach in DoA estimation*

Over the last decade, Compressed Sensing (CS) techniques have been utilized to tackle Direction of Arrival (DoA) estimation problems by exploiting the sparsity of signal sources in the spatial domain, particularly with respect to angles. These approaches are typically categorized into three types: grid-less, off-grid, and on-grid. Both on-grid and off-grid methods offer a reasonable trade-off, requiring less computational power but experiencing some performance degradation due to grid mismatch issues. In contrast, grid-less methods provide superior performance at the cost of significantly higher computational complexity. DoA estimation is generally accomplished by solving sparse minimization problems, which are often approached using two main strategies: greedy methods based on the l_g pseudo-norm and convex relaxation techniques utilizing the 1-norm. A more recent development in DoA estimation involves Deep Learning (DL) techniques, which offer several advantages over traditional optimization-based methods. Once a network is trained, no further optimization is required, and solutions can be quickly derived using simple operations such as multiplications and additions. Moreover, DL techniques do not necessitate fine-tuning of parameters, unlike optimization-based approaches, where accuracy is highly sensitive to parameter adjustments. Additionally, DL methods exhibit robustness to data imperfections, handling fewer snapshots effectively and maintaining strong performance even under low Signal-to-Noise Ratio (SNR) conditions [173]. For instance, a Deep Neural Network (DNN) with fully connected layers has been employed for DoA classification of two targets using the signal covariance matrix. However, it is noteworthy that these methods tend to underperform in high SNR scenarios [174].

The landscape of DoA estimation is undergoing a transformative evolution, driven by convergent advancements in machine learning, signal processing hardware, and antenna technologies, presenting unprecedented opportunities for innovative research and technological breakthroughs [175, 176, 177, 178, 179, 180]. Deep learning models, particularly

DNNs and CNNs, are emerging as revolutionary approaches for enhancing DoA estimation in dynamic and interference-rich environments, demonstrating exceptional capabilities in automatically learning complex signal patterns and overcoming traditional algorithmic limitations. Emerging trends demonstrate a paradigm shift towards hybrid intelligent systems that seamlessly integrate physics-informed neural networks, advanced signal processing architectures, and adaptive hardware configurations. Reconfigurable antenna arrays represent a critical technological advancement, offering dynamic configuration capabilities that significantly improve spatial resolution and system flexibility across diverse scenarios. The exploration of hybrid architectures combining traditional DoA estimation algorithms with machine learning-based methods presents a particularly promising research direction, aiming to leverage computational efficiency and adaptive performance simultaneously [181]. Advanced signal processing hardware, including FPGAs, Application-Specific Integrated Circuits (ASICs), and Graphics Processing Units (GPUs), are enabling the execution of increasingly complex computational models without compromising system performance or power efficiency. Quantum machine learning and neuromorphic computing architectures represent cutting-edge research frontiers, offering potential computational paradigms that could dramatically enhance DoA estimation's computational efficiency and adaptive learning capabilities [182]. The integration of edge computing and distributed machine learning frameworks presents novel opportunities for real-time, low-latency DoA estimation, particularly in scenarios requiring instantaneous spatial signal characterization such as autonomous systems, wireless communications, and radar technologies [183]. The convergence of advanced machine learning algorithms, specialized signal processing hardware, and innovative antenna technologies heralds a new era of intelligent, adaptive DoA estimation methodologies with transformative potential across multiple technological domains.

7.2 *Convolutional neural networks*

The future scope of using CNNs in DoA estimation includes several promising advancements. CNNs can process large datasets with complex spatial features, leading to more accurate DoA estimation in challenging environments like dense multipath or low SNR scenarios. With the continuous development of deep learning hardware accelerators, such as GPUs and TPUs, CNNs are expected to enable real-time estimation of DoA, which is critical for applications like radar, sonar, and wireless communication systems. Moreover, CNNs can be designed to adapt to varying environmental conditions, enhancing performance in dynamic settings like mobile networks, smart antennas, and 5G/6G systems. They also automate the feature extraction process, reducing the need for manual tuning of parameters, and simplifying the development of scalable systems for large antenna arrays. Furthermore, CNN-based DoA systems are anticipated to improve robustness against interference, noise, and hardware impairments, making them suitable for military, automotive, and IoT applications. Future work may involve integrating CNNs with other AI/ML techniques, such as reinforcement learning or transfer learning, to further optimize DoA estimation under various conditions [184, 185, 186].

7.3 *MIMO*

The classical MUSIC algorithm's performance, when applied to a UCA, is significantly limited by the array aperture. Additionally, most DoA estimation algorithms require that the number of array elements exceeds the number of incident signals. Wave-forming is a recently developed technique that uses virtual antennas spread throughout the environment to harness spatial diversity, leading to greater flexibility and improved performance. As bandwidth expands, it uncovers more independent multipath components, which in turn, enhances the degree of freedom, allowing faster data rates and dependable communication. Wave-forming can be engineered to tackle the interference issues that arise as wireless traffic surges and the number of users in 5G networks grows, which can become a performance hindrance [187]. The flexibility of MIMO is particularly noteworthy, as it allows for the selection and adaptation of waveforms to suit the specific requirements and objectives of different applications. The resolution improves, and the sensitivity increases, particularly when detecting slow-moving targets. Additionally, the ability to identify parameters improves significantly, and adaptive techniques can also be applied to the array for enhanced performance, as noted by Deepshikha et al. [188]. In essence, MIMO beamforming and wideband wave-forming are comparable. MIMO was developed to simulate multipath communication utilizing multiple transmit (TX) and/or receive (RX) antennas when bandwidth is insufficient to identify distinct multipaths. This makes use of spatial diversity and spatial multiplexing benefits to enable high data rate transmission. Radar systems are heavily influenced by the positions of the antennas. On the one hand, a large array aperture and great DoA precision can be achieved by placing antennas far apart. However, they must be positioned close enough to avoid spatial aliasing, or grating lobes. Massive MIMO is a novel MIMO paradigm that has attracted a lot of interest from both industry and academics. By improving spectrum efficiency through aggressive spatial multiplexing, this strategy promises to satisfy future capacity demands [189]. A new multiple DoA estimate approach based on a rotating array has been suggested lately [190] to address this problem. It offers reduced complexity and lower energy loss. Using a sparse array, sometimes referred to as a thinned array, is one way to strike a balance between these criteria. This technique ensures that at least one pair of antennas meets the anti-aliasing criteria (such as half-wavelength separation) while fracturing the uniform spacing of antennas to produce a wider aperture with the same number of array elements. Larger sidelobes are the price paid for this improvement in DoA estimation accuracy, albeit [191]. In the middle of the 20th century, certain antenna placement techniques were investigated in an effort to balance sidelobe height and main beam width. An additional consideration is the incorporation of MIMO systems. A 2-element MIMO antenna system used in modern wireless terminals can be represented as a ULA (Uniform Linear Array) in the case of (M = 2). MIMO antennas on bigger devices, like as laptops and tablets, or at access points can be represented by higher-order situations, like (M = 4) or (M = 8) [192]. High-capacity communication networks are obviously needed given the sharp rise in the number of devices connecting to the Internet. By adjusting parts of the antenna beam to the locations of several Mobile Stations (MSs), Beam Division Multiple Access (BDMA) increases capacity and facilitates more effective data transfer. For achieving optimal performance, the Base Station (BS) must accurately determine the locations of the MS's. Using the DoA approach for locating MS's is a unique approach. This involves utilizing an array of antennas with two common configurations: Uniform Linear Array (ULA) and Uniform Circular Array (UCA). UCA is preferred due to its advantage of providing 360-degree coverage [193]. Multiple carrier frequencies assist prevent grating lobes and increase the accuracy of DoA estimation by enabling arbitrarily tiny sensor spacing while preserving a large aperture. In addition to joint range-DoA estimation, a simplified DoA estimator that performs similarly to the joint estimator after range elimination is also observed. Multi-Carrier MIMO (MC-MIMO) is a wireless communication technique where multiple base stations cooperate to enhance data transmission and reception by jointly utilizing multiple antennas across cells, improving spectral efficiency and reducing interference. This multi-carrier technique, however, produces larger sidelobes than single-carrier systems, which raises the estimate threshold in a manner akin to sparse array designs. Compared to single-carrier systems, the primary benefit of the MC-MIMO system is its much larger aperture, which results in a lower Cramér-Rao Bound (CRB) when enough SNR is available. DoA-based systems offer several advantages over conventional temporal reference-based solutions. For example, because of their accurate directional information, they are more suited for downlink applications.

Estimation of DoA in massive MIMO systems represents a complex technological challenge characterized by multifaceted limitations in spatial signal processing. Traditional algorithmic approaches encounter significant performance bottlenecks arising from intricate spatial correlations, mutual coupling effects, and exponential computational complexity, which fundamentally compromise estimation accuracy and real-time processing capabilities [194, 195]. Classical subspace-based techniques like MUSIC and ESPRIT experience substantial degradation in high-dimensional array configurations, struggling with signal interference, angular resolution ambiguity, and increased spatial spectrum sidelobes [196]. Emerging interdisciplinary research demonstrates promising solutions through advanced methodological paradigms, particularly deep learning and sparse array techniques, which offer transformative strategies for mitigating these critical constraints [197, 198].

Physics-informed neural networks and convolutional architectures provide innovative mechanisms for adaptive feature extraction, enabling robust signal separation, interference suppression, and enhanced spatial resolution beyond conventional signal processing methodologies [199, 200]. Sparse array configurations further contribute to performance improvements by utilizing non-uniform element spacing, minimizing mutual coupling, improving computational efficiency, and achieving superior angular resolution with reduced array elements [201]. The convergence of machine learning algorithms, compressed sensing techniques, and advanced signal processing frameworks represents a comprehensive approach to addressing fundamental DoA estimation challenges, potentially revolutionizing wireless communication, radar systems, and sensor array processing technologies [202]. By integrating sophisticated neural network architectures with domain-specific signal propagation knowledge, researchers are developing increasingly robust estimation methodologies capable of handling ultra-dense wireless network scenarios with unprecedented accuracy, computational efficiency, and adaptive learning capabilities.

Deep learning also offers the potential for real-time adaptability in dynamic environments, enabling algorithms to adjust quickly to changing conditions, which is crucial in massive MIMO where environmental factors can vary broadly and rapidly. Additionally, as neural networks are suited for parallel processing, they can efficiently utilize hardware like GPUs and TPUs, making them practical for real-time applications even when dealing with large-scale antenna arrays. This integration of advanced techniques and technologies presents a promising avenue for enhancing the operational effectiveness of DoA estimation in massive MIMO systems [203, 204, 205, 206, 207, 208].

A key technology for upcoming sixth-generation (6G) wireless networks is integrated sensing and communication, or ISAC. This is especially true for new applications that need both high-performance sensing and communication capabilities. ISAC can optimize the utilization of available spectrum and drastically lower hardware costs by allowing the simultaneous operation of sensing and communication operations on a single hardware platform. This dual functionality allows for the sharing of the same waveform for both sensing and communication tasks, leading to more efficient spectrum utilization and improved resource management. As 6G networks continue to develop, Integrated Sensing and Communication (ISAC) is expected to play a pivotal role in enabling advanced applications such as autonomous systems, smart cities, and IoT networks. In these contexts, the integration of sensing and communication capabilities will be crucial for ensuring seamless connectivity and real-time data processing. By leveraging the extensive spatial degrees of freedom, the combination of ISAC with massive MIMO is expected to improve both spectral and energy efficiencies while enhancing sensing performance. However, the deployment of a large number of antennas in massive MIMO-ISAC systems presents significant challenges in accurately obtaining both channel state information (CSI) and target parameters. To address these issues within a unified framework, the system models are initially analyzed, followed by the introduction of a novel tensor-based method that simultaneously handles both channel estimation and target sensing. Specifically, by parameterizing the high-dimensional communication channel using a limited set of physical parameters, the channel state information is linked to the target sensing parameters, such as angular, delay, and Doppler dimensions [209, 210].

Furthermore, when handling tiny angular spreads, DoA-based beamforming typically performs better than alternative techniques. DoA estimation takes a lot of time, though. Accelerating the DoA estimating process is essential for the successful implementation of a DoA-based system. The dearth of affordable digital processing equipment that can manage the significant computational load has made this difficult in real-world systems. The concomitant requirements for speed, compactness, and low power consumption necessary for such devices are usually beyond the capabilities of general von Neumann architecture CPUs. FPGA-based digital signal processing is therefore one of the most recent methods to address this problem [211, 212].

8. Conclusions

Recent advancements in DoA estimation algorithms, particularly MUSIC and Root-MUSIC, and their applications in smart antenna systems and radar systems are analysed. These algorithms have superior performance in resolution and accuracy, even in challenging scenarios involving closely spaced signal sources and low SNR environments. The introduction of amplification ratios for criterions such as the number of elements, number of snapshots, and signal power has significantly improved the resolution of DoA estimation, allowing for the detection of angular separations as small as 1 degree. This enhancement represents a substantial improvement over the traditional methods. Additionally, the modified MUSIC algorithm presented in this study effectively addresses the limitations of the standard MUSIC algorithm when dealing with coherent sources, ensuring accurate DoA estimations in more complex signal environments.

Root-MUSIC, derived from the traditional MUSIC algorithm, offers improved performance, particularly in low SNR conditions. Its simplified root-finding process enhances both stability and computational efficiency. The integration of beamforming techniques with these algorithms has proven highly effective, allowing for precise focusing of signal energy and thereby improving overall system performance.

To mitigate grating lobes, it is crucial to maintain an inter-element spacing of 0.5λ or less in antenna arrays, highlighting the importance of hardware design in conjunction with algorithm development. The advancements discussed have far-reaching implications for various fields, including 5G and future communication networks, autonomous systems, and advanced radar technologies.

The potential integration of deep neural networks with MUSIC and Root-MUSIC algorithms presents a promising avenue for future research. This combination could lead to even more accurate and efficient DoA estimation in increasingly complex and dynamic signal environments. The adaptation of these algorithms to more intricate signal scenarios and their application to emerging technologies represent exciting areas for future research and development.

In conclusion, while MUSIC and Root-MUSIC algorithms have significantly advanced the futuristic approaches in DoA estimation, there remains ample opportunity for further innovation. These ongoing advancements will undoubtedly contribute to more efficient, intelligent, and capable signal processing systems, meeting the evolving demands of modern communication networks and beyond.

Conflict of interest

There is no Conflict of interest in this study.

References

[1] Z. Chen, G. Gokeda, and Y. Yu, *Introduction to Direction of-arrival Estimation*. Norwood, MA, USA: Artech House, 2010.

[2] T. Dhope, D. Simunic, and M. Djurek, "Application of DoA estimation algorithms in smart antenna systems," *Stud. Inform. Control*, vol. 19, no. 4, pp. 445–452, 2010.

[3] R. Shari and S. Jacob, "Comparative study of DoA estimation algorithms," in *Proc. 2022 IEEE 19th India Council Int. Conf. (INDICON)*, Kochi, India, Nov. 24–26, 2022.

[4] J. Foutz, A. Spanias, and M. Banavar, *Narrowband DoA Estimation for Antenna Arrays*. New York, NY, USA: Springer Nature, 2022.

[5] G. K. Papageorgiou, M. Sellathurai, and Y. C. Eldar, "Deep Networks for DoA Estimation in Low SNR," *IEEE Trans. Signal Process.*, vol. 69, pp. 3714–3729, 2021. https://doi.org/10.1109/TSP.2021.3089927.

[6] C. Rajkumar and A. A. Bazil Raj, "Design and Development of DSP Interfaces and Algorithm for FMCW Radar Altimeter," in *Proc. 2019 4th Int. Conf. Recent Trends Electron., Inf., Commun. & Technol. (RTEICT)*, Bangalore, India, May 17–18, 2019, pp. 720–725. https://doi.org/10.1109/RTEICT46194.2019.9016960.

[7] A. Pandey, M. Hasan, and A. A. B. Raj, "Space Time Adaptive Processing for High Signal to Clutter Ratio Target's Detection Radar System," in *Proc. 2022 Int. Conf. Augmented Intell. Sustainable Syst. (ICAISS)*, Trichy, India, Nov. 24–26, 2022, pp. 1298–1302. https://doi.org/10.1109/ICAISS55157.2022.10010908.

[8] Z. Yang, J. Li, P. Stoica, L. Xie, "Sparse methods for direction-of-arrival estimation," in *Academic Press Library in Signal Processing*, Burlington, MA, USA: Academic Press, 2018, pp. 509–581, Volume 7.

[9] N. T. N. Tho, S. Zhao, and D. L. Jones, "Robust DoA estimation of multiple speech sources," in *Proc. 2014 IEEE Int. Conf. Acoust., Speech Signal Process. (ICASSP)*, Florence, Italy, May 4–9, 2014, pp. 2287–2291. https://doi.org/10.1109/ICASSP.2014.6854007.

[10] A. Dey, A. Nandi, and B. Basu, "Gold-MUSIC based DoA estimation using ULA antenna of DS-CDMA sources with propagation delay diversity," *AEU-Int. J. Electron. Commun.*, vol. 84, pp. 162–170, 2018.

[11] Z. Shi, C. Zhou, Y. Gu, N. A. Goodman, F. Qu, "Source estimation using coprime array: A sparse reconstruction perspective," *IEEE Sens. J.*, vol. 17, no. 3, pp. 755–765, 2016.

[12] C. Zhou, Y. Gu, Y. D. Zhang, Z. Shi, T. Jin, X. Wu, "Compressive sensing-based coprime array direction-of-arrival estimation," *IET Commun.*, vol. 11, no. 11, pp. 1719–1724, 2017.

[13] H. Zheng, Z. Shi, C. Zhou, M. Haardt, J. Chen, "Coupled coarray tensor CPD for DoA estimation with coprime L-shaped array," *IEEE Signal Process. Lett.*, vol. 28, pp. 1545–1549, 2021.

[14] K. Hameed, W. Khan, Y. S. Abdalla, F. F. Al-Harbi, A. Armghan, M. Asif, M. Salman Qamar, et al., "Far-field DoA estimation of uncorrelated RADAR signals through coprime arrays in low SNR regime by implementing Cuckoo search algorithm," *Electronics*, vol. 11, no. 4, pp. 558, 2022.

[15] Y. Hioka and N. Hamada, "DoA estimation of speech signal using equilateral-triangular microphone array," in *Proc. INTERSPEECH*, 2003.

[16] T. Basikolo, K. Ichige, and H. Arai, "Direction of arrival estimation for quasi-stationary signals using nested circular array," in *Proc. 2016 4th Int. Works. Compressed Sens. Theory Appl. Radar, Sonar Remote Sens. (CoSeRa)*, Aachen, Germany, Sept. 19–22, 2016, pp. 193–196. https://doi.org/10.1109/CoSeRa.2016.7745727.

[17] L. Xu and F. Wen, "Fast noncircular 2D-DoA estimation for rectangular planar array," *Sensors*, vol. 17, no. 4, p. 840, 2017.

[18] A. Feng, Z. Zhao, and Q. Yin, "Wideband direction-of-arrival estimation using fast chirplet-based adaptive signal decomposition algorithm," in *Proc. IEEE 54th Vehicular Technol. Conf. VTC Fall 2001. Proc. (Cat. No.01CH37211)*, Atlantic City, NJ, USA, Oct. 7–11, 2001, pp. 1432–1436. https://doi.org/10.1109/VTC.2001.956433.

[19] A. Gupta and A. A. Bazil Rai, "Feature Extraction of Intra-Pulse Modulated LPI Waveforms Using STFT," in *Proc. 2019 4th Int. Conf. Recent Trends Electron., Inf., Commun. & Technol. (RTEICT)*, Bangalore, India, May 17–18, 2019, pp. 742–746. https://doi.org/10.1109/RTEICT46194.2019.9016799.

[20] A. B. Gershman and M. G. Amin, "Wideband DoA estimation of multiple chirp signals using spatial time-frequency distributions," *IEEE Signal Process. Lett.*, vol. 7, no. 6, pp. 152–155, 2000.

[21] M. Bouri, "Source Detection and Localization in Array Signal Processing," in *Proc. 2006 1st Int. Symp. Environ. Identities Mediterr. Area*, Corte-Ajaccio, France, Jul. 9–12, 2006, pp. 12–17. https://doi.org/10.1109/ISEIMA.2006.344928.

[22] S. Mohan, M. E. Lockwood, M. L. Kramer, D. L. Jones, "Localization of multiple acoustic sources with small arrays using a coherence test," *J. Acoust. Soc. Am.*, vol. 123, no. 4, pp. 2136–2147, 2008.

[23] T. Nishiura, S. Nakamura, and K. Shikano, "Talker localization in a real acoustic environment based on DOA estimation and statistical sound source identification," in *Proc. 2002 IEEE Int. Conf. Acoust., Speech Signal Process.*, Orlando, FL, USA, May 13–17, 2002, pp. I-893–I-896. https://doi.org/10.1109/ICASSP.2002.5743883.

[24] H. Sawada, R. Mukai, and S. Makino, "Direction of arrival estimation for multiple source signals using independent component analysis," in *Proc. 7th Int. Symp. Signal Process. Its Appl.*, Paris, France, Jul. 4, 2003, pp. 411–414. https://doi.org/10.1109/ISSPA.2003.1224901.

[25] M. Matsuo, Y. Hioka, N. Hamada, and Y. K. Hiyoshi, "Estimating DOA of multiple speech signals by improved histogram mapping method," in *Proc. IWAENC 2005*, Eindhoven, The Netherlands, Sept. 12–15, 2005, pp. 129–132.

[26] M. Swartling, B. Sallberg, and N. Grbic, "Direction of arrival estimation for speech sources using fourth order cross cumulants," in *2008 IEEE Int. Symp. Circuits Syst. (ISCAS)*, Seattle, WA, USA, May 18–21, 2008, pp. 1696–1699. https://doi.org/10.1109/ISCAS.2008.4541763.

[27] W. Wang and W. Zhang, "Joint Beam Training and Positioning for Intelligent Reflecting Surfaces Assisted Millimeter Wave Communications," *IEEE Trans. Wireless Commun.*, vol. 20, no. 10, pp. 6282–6297, Oct. 2021. https://doi.org/10.1109/TWC.2021.3073140.

[28] Y. Liu, H. Chen, and B. Wang, "DoA estimation based on CNN for underwater acoustic array," *Appl. Acoust.*, vol. 172, pp. 107594, 2021.

[29] F. Asano, S. Hayamizu, T. Yamada, S. Nakamura, "Speech enhancement based on the subspace method," *IEEE Trans. Speech Audio Process.*, vol. 8, no. 5, pp. 497–507, 2000.

[30] E. Visser, T. Lee, and M. Otsuka, "Speech enhancement in a noisy car environment," in *Proc. 3rd Int. Conf. Independent Component Anal. Source Separ.*, Dec. 2001, pp. 272–277.

[31] N. Mitianoudis and M. E. Davies, "Audio source separation: Solutions and problems," *Int. J. Adapt. Control Signal Process.*, vol. 18, no. 3, pp. 299–314, 2004.

[32] L. Lin, C. She, Y. Chen, Z. Guo, X. Zeng, "TB-NET: A Two-Branch Neural Network for DoA Estimation under Model Imperfections," *Electronics*, vol. 11, no. 2, p. 220, 2022.

[33] K. Lee, "Deep learning-aided coherent DoA estimation with the FTMR algorithm," *IEEE Trans. Signal Process.*, vol. 70, pp. 1118–1130, Jan. 2022.

[34] Y. Kase, T. Nishimura, T. Ohgane, Y. Ogawa, D. Kitayama, and Y. Kishiyama, "DOA Estimation of Two Targets with Deep Learning," in *Proc. 15th Works. Pos. Navig. Commun.*, Bremen, Germany, Oct. 25–26, 2018, pp. 1–5, https://doi.org/10.1109/WPNC.2018.8555814.

[35] N. A. Khan, S. Ali, and M. Jansson, "DoA estimation using adaptive directional time-frequency distributions," *Multidimensional Syst. Signal Process.*, vol. 29, no. 2, pp. 503–521, 2018.

[36] E. A. R. Habets and S. Gannot, "Dual-Microphone Speech Dereverberation using a Reference Signal," in *Proc. 2007 IEEE Int. Conf. Acoust., Speech Signal Process.—ICASSP'07*, Honolulu, HI, USA, Apr. 15–20, 2007, pp. IV-901–IV-904. https://doi.org/10.1109/ICASSP.2007.367216.

[37] J. F. Gu, S. C. Chan, W. P. Zhu, M. N. S. Swamy, "Joint DoA estimation and source signal tracking with Kalman filtering and regularized QRD RLS algorithm," *IEEE Trans. Circ. Syst. II: Express Briefs*, vol. 60, no. 1, pp. 46–50, 2013.

[38] V. Bamane, J. Sapkale, A. Pawar, P. G. Chilveri, N. Akhter, A. A. B. Raj, "A Review on AI Based Target Classification Advanced Techniques," *Int. J. Eng. Res. Rev.*, vol. 10, no. 4, pp. 88–99, Dec. 2022. https://doi.org/10.5281/zenodo.7491584.

[39] Z. Yang, J. Li, P. Stoica, L. Xie, "Sparse methods for direction-of-arrival estimation," in *Academic Press Library in Signal Processing*, Burlington, MA, USA: Academic Press, 2018, vol. 7, pp. 509–581.

[40] G. Sharma and A. Arockia Bazil Raj, "Low complexity interference mitigation technique in IRCI-free SAR imaging algorithms," *IEEE Geosci. Remote Sens. Lett.*, vol. 19, pp. 1–5, 2022. https://doi.org/10.1109/LGRS.2022.3148994.

[41] N. Bhavsar, A. Tayade, H. Ghodmare, N. M. Wagdharikar, N. Akhter, A. A. B. Raj, "A comprehensive review on foliage penetration radar systems," *Int. J. Eng. Res. Rev.*, vol. 10, no. 4, pp. 28–40, 2022.

[42] J. Mazumder and A. B. Raj, "Detection and Classification of UAV Using Propeller Doppler Profiles for Counter UAV Systems," in *Proc. 2020 5th Int. Conf. Commun. Electron. Syst. (ICCES)*, Coimbatore, India, Jun. 10–12, 2020, pp. 221–227. https://doi.org/10.1109/ICCES48766.2020.9138077.

[43] H. C. Kumawat and P. Samczyński, "Spectrum localization and Hough transform-based β tuning for LSS targets' accurate micro-Doppler imaging system," *IEEE Trans. Instrum. Meas.*, vol. 71, pp. 1–11, 2022.

[44] S. Kumar and A. A. B. Raj, "Design of X-Band FMCW Radar Using Digital Doppler Processor," in *Proc. 2021 Int. Conf. Syst., Comput., Autom. Netw. (ICSCAN)*, Puducherry, India, Jul. 30–31, 2021, pp. 1–5. https://doi.org/10.1109/ICSCAN53069.2021.9526461.

[45] M. Kameshwara Rao and A. Bazil Raj, "Reduced Radar Cross-Section Target Imaging System," in *Proc. 2021 Int. Conf. Syst., Comput., Autom. Netw. (ICSCAN)*, Puducherry, India, Jul. 30–31, 2021, pp. 1–6. https://doi.org/10.1109/ICSCAN53069.2021.9526517.

[46] H. C. Kumawat and A. A. Bazil Raj, "Extraction of Doppler signature of micro-to-macro rotations/motions using continuous wave radar-assisted measurement system," *IET Sci. Meas. Technol.*, vol. 14, no. 7, pp. 772–785, 2020.

[47] H. C. Kumawat and A. A. Bazil Raj, "Data Acquisition and Signal Processing System for CW Radar," in *Proc. 2019 5th Int. Conf. Comput., Commun., Control Autom. (ICCUBEA)*, Pune, India, Sept. 19–21, 2019, pp. 1–5. https://doi.org/10.1109/ICCUBEA47591.2019.9128921.

[48] H. C. Kumawat and A. A. Bazil Raj, "Extraction of Doppler signature of micro-to-macro rotations/motions using continuous wave radar-assisted measurement system," *IET Sci. Meas. Technol.*, vol. 14, no. 7, pp. 772–785, 2020.

[49] A. P. Lakshminarayanan, H. C. Kumawat, and A. B. Raj, "Moving Target Detection in Foliage Environment using FMCW Radar," in *Proc. 2020 5th Int. Conf. Commun. Electron. Syst. (ICCES)*, Coimbatore, India, Jun. 10–12, 2020, pp. 418–421. https://doi.org/10.1109/ICCES48766.2020.9138095.

[50] M. Upadhyay, S. K. Murthy, and A. A. B. Raj, "Intelligent System for Real time detection and classification of Aerial Targets using CNN," in *Proc. 2021 5th Int. Conf. Intell. Comput. Control Syst. (ICICCS)*, Madurai, India, May 6–8, 2021, pp. 1676–1681. https://doi.org/10.1109/ICICCS51141.2021.9432136.

[51] A. Rana, H. C. Kumawat, and A. A. B. Raj, "Wall See Through Low-Velocity Moving Target Detection," in *Proc. 2023 IEEE 3rd Int. Conf. Technol., Eng., Manag. Societal Impact Using Mark., Entrep. Talent (TEMSMET)*, Mysuru, India, Feb. 10–11, 2023, pp. 1–6. https://doi.org/10.1109/TEMSMET56707.2023.10149977.

[52] J. Mazumder and A. B. Raj, "Detection and Classification of UAV Using Propeller Doppler Profiles for Counter UAV Systems," in *Proc. 2020 5th Int. Conf. Commun. Electron. Syst. (ICCES)*, Coimbatore, India, Jun. 10–12, 2020, pp. 221–227. https://doi.org/10.1109/ICCES48766.2020.9138077.

[53] U. Garg, A. A. Bazil Raj, and K. P. Ray, "Cognitive Radar Assisted Target Tracking: A Study," in *Proc. 2018 3rd Int. Conf. Commun. Electron. Syst. (ICCES)*, Coimbatore, India, Oct. 15–16, 2018, pp. 427–430. https://doi.org/10.1109/CESYS.2018.8723894.

[54] P. Z. Peebles, *Radar Principles*. Hoboken, NJ, USA: John Wiley and Sons, 2007.

[55] H. C. Kumawat and A. Arockia Bazil Raj, “SP-WVD with adaptive-filter-bank-supported RF sensor for low RCS targets’ nonlinear micro-Doppler signature/pattern imaging system,” *Sensors*, vol. 22, no. 3, p. 1186, 2022.
[56] R. Vaishnavi, G. Unnikrishnan, and A. A. Bazil Raj, “Implementation of algorithms for Point target detection and tracking in Infrared image sequences,” in *Proc. 2019 4th Int. Conf. Recent Trends Electron., Inf., Commun. & Technol. (RTEICT)*, Bangalore, India, May 17–18, 2019, pp. 904–909. https://doi.org/10.1109/RTEICT46194.2019.9016871.
[57] J. Scheer and W. A. Holm, *Principles of Modern Radar*. Raleigh, NC, USA: SciTech Publishing, 2010, pp. 3–4.
[58] J. Eaves and E. Reedy, *Principles of Modern Radar*. New York, NY, USA: Springer Science and Business Media, 2012.
[59] H. Rahman, *Fundamental Principles of Radar*. Boca Raton, FL, USA: CRC Press, 2019.
[60] J. C. Toomay and P. J. Hannen, *Radar Principles for the Nonspecialist*, Surrey, UK: SciTech Publishing, 2004, vol. 2.
[61] S. De and A. A. Bazil Raj, “A survey on photonics technologies for radar applications,” *J. Opt.*, vol. 52, no. 1, pp. 90–119, 2023.
[62] H. C. Kumawat and A. B. Raj, “Approaching/Receding Target Detection using CW Radar,” in *Proc. 2020 5th Int. Conf. Commun. Electron. Syst. (ICCES)*, Coimbatore, India, Jun. 10–12, 2020, pp. 136–141. https://doi.org/10.1109/ICCES48766.2020.9137863.
[63] A. S. Akella and A. A. B. Raj, “DCNN Based Activity Classification of Ornithopter using Radar micro-Doppler Images,” in *Proc. 2021 5th Int. Conf. Intell. Comput. Control Syst. (ICICCS)*, Madurai, India, May 6–8, 2021, pp. 780–784. https://doi.org/10.1109/ICICCS51141.2021.9432228.
[64] A. Rana and A. A. Bazil Raj, “Non-Linear Orbital Path Tracking of Ornithopters,” in *Proc. 2021 Int. Conf. Syst., Comput., Autom. Netw. (ICSCAN)*, Puducherry, India, Jul. 30–31, 2021, pp. 1–6. https://doi.org/10.1109/ICSCAN53069.2021.9526513.
[65] U. Singh and A. A. B. Raj, “A Comparative Study of Multiple Radar Waveform Design Techniques,” in *Proc. 2020 2nd Int. Conf. Inventive Res. Comput. Appl. (ICIRCA)*, Coimbatore, India, Jul. 15–17, 2020, pp. 1177–1182. https://doi.org/10.1109/ICIRCA48905.2020.9182810.
[66] Y. Zhang, Y. Wang, W. Li, Y. Huang, J. Yang, “Data-aided signal-to-noise-ratio estimation for scanning radar angular superresolution based on iterative adaptive approach,” in *Proc. 2015 IEEE Int. Geoscience Remote Sens. Symp. (IGARSS)*, Milan, Italy, Jul. 26–31, 2015, pp. 4400–4403. https://doi.org/10.1109/IGARSS.2015.7326802.
[67] Y. Ju, L. Yu, Y. Wang, X. Chu, “Inverse Synthetic Aperture Radar Imaging at Low Signal-to-noise Ratio,” in *Proc. 2010 Int. Conf. Artif. Intell. Comput. Intell.*, Sanya, China, Oct. 23–24, 2010, pp. 106–110. https://doi.org/10.1109/AICI.2010.145.
[68] Y. Wang, Q. Liu, and A. E. Fathy, “CW and pulse–Doppler radar processing based on for human sensing applications,” *IEEE Trans. Geosci. Remote Sens.*, vol. 51, no. 5, pp. 3097–3107, 2012.
[69] P. Anju, A. A. Bazil Raj, and C. Shekhar, “Pulse Doppler Processing—A Novel Digital Technique,” in *Proc. 2020 4th Int. Conf. Intell. Comput. Control Syst. (ICICCS)*, Madurai, India, May 13–15, 2020, pp. 1089–1095. https://doi.org/10.1109/ICICCS48265.2020.9120950.
[70] S. De, S. Elayaperumal, and A. A. Bazil Raj, “Angle Estimation Using Modified Subarray Level Monopulse Ratio Algorithm and S-Curve in Digital Phased Array Radar,” in *Proc. 2020 2nd Int. Conf. Inventive Res. Comput. Appl. (ICIRCA)*, Coimbatore, India, Jul. 15–17, 2020, pp. 936–941. https://doi.org/10.1109/ICIRCA48905.2020.9182859.
[71] H. C. Kumawat and A. A. Bazil Raj, “Extraction of Doppler signature of micro-to-macro rotations/motions using continuous wave radar-assisted measurement system,” *IET Sci. Meas. Technol.*, vol. 14, no. 7, pp. 772–785, 2020.
[72] K. Tirumalesh and A. A. Bazil Raj, “Laboratory Based Automotive Radar for Mobile Targets Ranging,” in *Proc. 2021 Int. Conf. Syst., Comput., Autom. Netw. (ICSCAN)*, Puducherry, India, Jul. 30–31, 2021, pp. 1–6. https://doi.org/10.1109/ICSCAN53069.2021.9526346.
[73] A. Wankhede, S. De, and G. N. Surname, “Development of L-Band FMCW Radar on SDR using GNU RADIO,” in *Proc. 2024 2nd Int. Conf. Emerging Trends Inf. Technol. Eng. (ICETITE)*, Vellore, India, Feb. 22–23, 2024, pp. 1–6. https://doi.org/10.1109/ic-ETITE58242.2024.10493476.
[74] T. Y. Gite, P. G. Pradeep, and A. A. Bazil Raj, “Design and Evaluation of C-Band FMCW Radar System,” in *Proc. 2018 2nd Int. Conf. Trends Electron. Inform. (ICOEI)*, Tirunelveli, India, May 11–12, 2018, pp. 1274–1276. https://doi.org/10.1109/ICOEI.2018.8553838.
[75] R. J. Burkholder, L. J. Gupta, and J. T. Johnson, “Comparison of monostatic and bistatic radar images,” *IEEE Antennas Propag. Mag.*, vol. 45, no. 3, pp. 41–50, Jun. 2003. https://doi.org/10.1109/MAP.2003.1232162.

[76] H. D. Griffiths, “Bistatic and multistatic radar,” in *Proc. IEE Military Radar Sem.*, 2004.

[77] A. Raut and A. A. Bazil Raj, “Signal Processing for Digital Beamforming on Transmit in MIMO Radar,” in *Proc. 2020 2nd Int. Conf. Inventive Res. Comput. Appl. (ICIRCA)*, Coimbatore, India, Jul. 15–17, 2020, pp. 1106–1111. https://doi.org/10.1109/ICIRCA48905.2020.9183290.

[78] I. Bradaric, G. T. Capraro, D. D. Weiner, M. C. Wicks, “Multistatic radar systems signal processing,” in *Proc. 2006 IEEE Conf. Radar*, Verona, NY, USA, Apr. 24–27, 2006, p. 8. https://doi.org/10.1109/RADAR.2006.1631783.

[79] A. Raut and A. A. Bazil Raj, “Signal Processing for Digital Beamforming on Transmit in MIMO Radar,” in *Proc. 2020 2nd Int. Conf. Inventive Res. Comput. Appl. (ICIRCA)*, Coimbatore, India, Jul. 15–17, 2020, pp. 1106–1111. https://doi.org/10.1109/ICIRCA48905.2020.9183290.

[80] R. Vaishnavi, G. Unnikrishnan, and A. A. Bazil Raj, “Implementation of algorithms for Point target detection and tracking in Infrared image sequences,” in *Proc. 2019 4th Int. Conf. Recent Trends Electron., Inf., Commun. & Technol. (RTEICT)*, Bangalore, India, May 17–18, 2019, pp. 904–909. https://doi.org/10.1109/RTEICT46194.2019.9016871.

[81] M. Chakraborty, H. C. Kumawat, S. V. Dhavale, A. A. B. Raj, “DIAT-μ RadHAR (micro-doppler signature dataset) & μ RadNet (a lightweight DCNN)—For human suspicious activity recognition,” *IEEE Sens. J.*, vol. 22, no. 7, pp. 6851–6858, 2022.

[82] M. Chakraborty, H. C. Kumawat, and S. V. Dhavale, “Application of DNN for radar micro-doppler signature-based human suspicious activity recognition,” *Pattern Recognit. Lett.*, vol. 162, pp. 1–6, 2022.

[83] G. Sharma and A. B. Raj, “Spotlight SAR Imaging with Sufficient Cyclic Prefix-OFDM Waveform,” in *Proc. 2020 Int. Conf. Smart Electron. Commun. (ICOSEC)*, Trichy, India, Sep. 10–12, 2020, pp. 625–630, https://doi.org/10.1109/ICOSEC49089.2020.9215293.

[84] K. Hemanth Sai and A. A. Bazil Raj, “Deep CNN Supported Recognition of Ship Using SAR Images in Maritime Environment,” in *Proc. 2021 Int. Conf. Syst. Comput. Autom. Netw. (ICSCAN)*, Puducherry, India, Jul. 30–31, 2021, pp. 1–5, https://doi.org/10.1109/ICSCAN53069.2021.9526447.

[85] G. Sharma and A. B. Raj, “Effects of Antenna Radiation Pattern on Airborne SAR Imaging with Sufficient Cyclic Prefix-OFDM,” in *Proc. 2020 IEEE 4th Conf. Inf. Commun. Technol. (CICT)*, Chennai, India, Dec. 3–5, 2020, pp. 1–6, https://doi.org/10.1109/CICT51604.2020.9312080.

[86] G. Yadav, K. S. Rajpoot, and A. A. Bazil Raj, “Synthetic Aperture Radar (SAR) Image of Small Unmanned Aerial Vehicle (sUAV),” in *Proc. 2022 Int. Conf. Augmented Intell. Sustain. Syst. (ICAISS)*, Trichy, India, Nov. 24–26, 2022, pp. 1012–1018, https://doi.org/10.1109/ICAISS55157.2022.10011085.

[87] P. Shakya and A. A. Bazil Raj, “Inverse Synthetic Aperture Radar Imaging Using Fourier Transform Technique,” in *Proc. 2019 1st Int. Conf. Innov. Inf. Commun. Technol. (ICIICT)*, Chennai, India, Apr. 25–26, 2019, pp. 1–4, https://doi.org/10.1109/ICIICT1.2019.8741461.

[88] S. Latthe, S. Mohite, L. Tagde, P. G. Chilveri, N. Akhter, and A. A. B. Raj, “A review on ISAR imaging techniques for low RCS targets,” *Int. J. Eng. Res. Rev.*, vol. 10, no. 4, pp. 15–27, 2022.

[89] G. Xu, F. Xu, X. G. Xia, H. Yu, H. Zhou, J. Kang, et al., “A Robust Nonlocal Tensor Decomposition Method for InSAR Phase Denoising,” *IEEE Trans. Geosci. Remote Sens.*, vol. 62, p. 5216511, 2024, https://doi.org/10.1109/TGRS.2024.3423384.

[90] G. Xu, B. Zhang, H. Yu, J. Chen, M. Xing, and W. Hong, “Sparse Synthetic Aperture Radar Imaging from Compressed Sensing and Machine Learning: Theories, applications, and trends,” *IEEE Geosci. Remote Sens. Mag.*, vol. 10, no. 4, pp. 32–69, Dec. 2022, https://doi.org/10.1109/MGRS.2022.3218801.

[91] H. C. Kumawat and A. Bazil Raj, “Data Acquisition and Signal Processing System for CW Radar,” in *Proc. 2019 5th Int. Conf. Computing, Commun. Control Autom. (ICCUBEA)*, Pune, India, Sep. 19–21, 2019, pp. 1–5, https://doi.org/10.1109/ICCUBEA47591.2019.9128921.

[92] U. Garg, A. A. Bazil Raj, and K. P. Ray, “Cognitive Radar Assisted Target Tracking: A Study,” in *Proc. 2018 3rd Int. Conf. Commun. Electron. Syst. (ICCES)*, Coimbatore, India, Oct. 15–16, 2018, pp. 427–430, https://doi.org/10.1109/CESYS.2018.8723894.

[93] K. S. Rajpoot and A. A. B. Raj, “Cognitive Radar for Target Tracking,” in *Proc. 2022 Int. Conf. Trends Quantum Comput. Emerging Bus. Technol. (TQCEBT)*, Pune, India, Jun. 29–Jul. 2, 2014, pp. 1–5, https://doi.org/10.1109/TQCEBT54229.2022.10041644.

[94] T. Qin, H. Zhang, and X. Zhang, “Criterion for narrowband beamforming,” *Electron. Lett.*, vol. 40, no. 14, p. 1, 2004.

[95] H. C. Kumawat, M. Chakraborty, and A. A. Bazil Raj, “DIATRadSATNet—A novel lightweight DCNN architecture for micro-Doppler-based small unmanned aerial vehicle (SUAV) targets’ detection and classification,” *IEEE Trans. Instrum. Meas.*, vol. 71, pp. 1–11, 2022.

[96] D. Ünalir, S. Sezgın, C. S. Yuva, B. Y. Gökdoğan, and E. Aydin, “Low Radar Cross Section UAV Design in X-Band,” in *Proc. 2022 30th Signal Process. Commun. Appli. Conf. (SIU)*, Safranbolu, Turkey, May 15–18, 2022, pp. 1–4, https://doi.org/10.1109/SIU55565.2022.9864894.

[97] G. Kouemou, ed., *Radar Technology*. Geesthacht, Germany: BoD–Books on Demand, 2010.

[98] S. Pisa, E. Pittella, and E. Piuzzi, “A survey of radar systems for medical applications,” *IEEE Aerosp. Electron. Syst. Mag.*, vol. 31, no. 11, pp. 64–81, 2016.

[99] N. Yadav and A. Rockia Bazil Raj, “Orthogonal Frequency Division Multiplexing Waveform Based Radar System,” in *Proc. 2021 Int. Conf. Syst. Comput. Autom. Netw. (ICSCAN)*, Puducherry, India, Jul. 30–31, 2021, pp. 1–4, https://doi.org/10.1109/ICSCAN53069.2021.9526501.

[100] H. C. Kumawat, M. Chakraborty, A. A. B. Raj, S. V. Dhavale, “DIAT-μSAT: Small aerial targets’ micro-Doppler signatures and their classification using CNN,” *IEEE Geosci. Remote Sens. Lett.*, vol. 19, pp. 1–5, 2021.

[101] S. Bellofiore, J. Foutz, C. A. Balanis, and A. Spanias, “Smart antennas for wireless communications,” in *Proc. IEEE Antennas Propag. Soc. Int. Symp. 2001 Digest*, Boston, MA, USA, Jul. 8–13, 2001, pp. 26–29, vol. 4, https://doi.org/10.1109/APS.2001.959392.

[102] S. Katariya, “A survey on smart antenna system,” *Int. J. Electron. Commun. Technol.*, vol. 2, no. 3, pp. 123–126, 2011.

[103] S. Vashisht, S. Jain, and G. S. Aujla, “MAC protocols for unmanned aerial vehicle ecosystems: Review and challenges,” *Computer Commun.*, vol. 160, pp. 443–463, 2020.

[104] J. Vikstedt, “Test challenges of smart antenna systems,” in *Proc. 2017 IEEE Int. Symp. Electromagn. Compat. Signal/Power Integrity (EMCSI)*, Washington, DC, USA, Aug. 7–11, 2017, pp. 1–47, https://doi.org/10.1109/ISEMC.2017.8078068.

[105] Y. Ma, R. Ma, Z. Lin, R. Zhang, Y. Cai, W. Wu, et al., “Improving Age of Information for Covert Communication with Time-Modulated Arrays,” *IEEE Internet Things J.*, https://doi.org/10.1109/JIOT.2024.3466855.

[106] F. Shu, Y. Qin, T. Liu, L. Gui, Y. Zhang, J. Li, et al., “Low-complexity and high-resolution DOA estimation for hybrid analog and digital massive MIMO receive array,” *IEEE Trans. Commun.*, vol. 66, no. 6, pp. 2487–2501, 2018.

[107] F. Li and R. J. Vaccaro, “Unified analysis for DOA estimation algorithms in array signal processing,” *Signal Process.*, vol. 25, no. 2, pp. 147–169, 1991.

[108] S. Motamedi-Fakhr, M. Moshrefi-Torbati, M. Hill, C. M. Hill, P. R. White, “Signal processing techniques applied to human sleep EEG signals—A review,” *Biomed. Signal Process. Control*, vol. 10, pp. 21–33, 2014.

[109] R. J. Vaccaro, “The Role of Subspace Estimation in Array Signal Processing,” in *Proc. 2019 53rd Asilomar Conf. Signals, Syst., Comput.*, Pacific Grove, CA, USA, Nov. 3–6, 2019, pp. 1566–1572, https://doi.org/10.1109/IEEECONF44664.2019.9048994.

[110] G. K. Papageorgiou, M. Sellathurai, and Y. C. Eldar, “Deep networks for DoA estimation in low SNR,” *IEEE Trans. Signal Process.*, vol. 69, pp. 3714–3729, 2021.

[111] C. R. Dongarsane and A. N. Jadhav, “Simulation study on DOA estimation using MUSIC algorithm,” *Int. J. Technol. Eng. Syst. (IJTES)*, vol. 2, no. 1, pp. 54–57, 2011.

[112] R. Roy and T. Kailath, “ESPRIT-estimation of signal parameters via rotational invariance techniques,” *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 37, no. 7, pp. 984–995, Jul. 1989.

[113] M. M. Gunjal and A. A. B. Raj, “Improved Direction of Arrival Estimation Using Modified MUSIC Algorithm,” in *Proc. 2020 5th Int. Conf. Commun. Electron. Syst. (ICCES)*, Coimbatore, India, Jun. 10–12, 2020, pp. 249–254, https://doi.org/10.1109/ICCES48766.2020.9137982.

[114] T. S. Dhope, *Application of MUSIC, ESPRIT and Root MUSIC in DOA Estimation*. Faculty of Electrical Engineering and Computing, University of Zagreb, Zagreb, Croatia, 2010.

[115] K. Adhikari and B. Drozdenko, “Symmetry-Imposed Rectangular Coprime and Nested Arrays for Direction of Arrival Estimation With Multiple Signal Classification,” *IEEE Access*, vol. 7, pp. 153217–153229, 2019, https://doi.org/10.1109/ACCESS.2019.2948503.

[116] M. Job and R. S. Yadav, “High Resolution DOA Estimation of Narrowband Signal for MUSIC, MVDR and Beamscan Algorithm,” in *Proc. 2023 11th Int. Symp. Electron. Syst. Dev. Comput. (ESDC)*, Sri City, India, May 4–6, 2023, pp. 1–5, https://doi.org/10.1109/ESDC56251.2023.10149863.
[117] R. L. Haupt, *Antenna Arrays: A Computational Approach*. Hoboken, NJ, USA: John Wiley and Sons, 2010.
[118] R. Schmidt, “Multiple emitter location and signal parameter estimation,” *IEEE Trans. Antennas Propag.*, vol. 34, no. 3, pp. 276–280, 1986.
[119] R. Roy and T. Kailath, “ESPRIT-estimation of signal parameters via rotational invariance techniques,” *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 37, no. 7, pp. 984–995, Jul. 1989, https://doi.org/10.1109/29.32276.
[120] G. K. Papageorgiou, M. Sellathurai, and Y. C. Eldar, “Deep Networks for DoA Estimation in Low SNR,” *IEEE Trans. Signal Process.*, vol. 69, pp. 3714–3729, 2021.
[121] A. L. Swindlehurst, B. Ottersten, R. Roy, T. Kailath, “Multiple invariance ESPRIT,” *IEEE Trans. Signal Process.*, vol. 40, no. 4, pp. 867–881, Apr. 1992.
[122] R. Wang, Y. Wang, Y. Li, W. Cao, Y. Yan, “Geometric algebra-based ESPRIT algorithm for DOA estimation,” *Sensors*, vol. 21, no. 17, p. 5933, 2021.
[123] F. Li, H. Liu, and R. J. Vaccaro, “Performance analysis for DOA estimation algorithms: unification, simplification, and observations,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 29, no. 4, pp. 1170–1184, 1993.
[124] S. U. Pillai, *Array Signal Processing*. New York, NY, USA: Springer Science & Business Media, 2012.
[125] P. Stoica and A. Nehorai, “MUSIC, maximum likelihood, and Cramer-Rao bound,” *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 37, no. 5, pp. 720–741, May 1989.
[126] I. Ziskind and M. Wax, “Maximum likelihood localization of multiple sources by alternating projection,” *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 36, no. 10, pp. 1553–1560, Oct. 1988.
[127] V. Krishnaveni, T. Kesavamurthy, and B. Aparna, “Beamforming for DOA estimation—A survey,” *Int. J. Comput. Appl.*, vol. 61, no. 11, pp. 4-11, 2013.
[128] P.-J. Chung, M. Viberg, and J. Yu, “DOA estimation methods and algorithms,” *Acad. Press Libr. Signal Process.*, vol. 3, pp. 599–650, 2014.
[129] K. I. Pedersen, P. E. Mogensen, and J. R. Moreno, “Application and performance of downlink beamforming techniques in UMTS,” *IEEE Commun. Mag.*, vol. 41, no. 10, pp. 134–143, Oct. 2003.
[130] O. Shouffi, I. Shaglil, K. Stayanova, R. Benotsmane, “Design and Implementation of DOA Algorithms for Smart Antenna Systems,” in *Proc. 2024 25th Int. Carpathian Control Conf. (ICCC)*, Krynica Zdrój, Poland, May 22–24, 2024, pp. 1–6, https://doi.org/10.1109/ICCC62069.2024.10569403.
[131] J. Benesty, J. Chen, and Y. Huang, “Conventional beamforming techniques,” *Microphone Array Signal Process.*, vol. 1, p. 3965, 2008.
[132] E. Ali, M. Ismail, R. Nordin, N. F. Abdulah, “Beamforming techniques for massive MIMO systems in 5G: overview, classification, and trends for future research,” *Front. Inf. Technol. Electron. Eng.*, vol. 18, pp. 753–772, 2017.
[133] J. C. Chen, K. Yao, and R. E. Hudson, “Source localization and beamforming,” *IEEE Signal Process. Mag.*, vol. 19, no. 2, pp. 30–39, Mar. 2002.
[134] A. Bindle, T. Gulati, and N. Kumar, “A detailed introduction of different beamforming techniques used in 5G,” *Int. J. Commun. Syst.*, vol. 34, no. 5, p. e4718, 2021.
[135] H. Al Kassir, Z. D. Zaharis, P. I. Lazaridis, N. V. Kantartzis, T. V. Yioultsis, T. D. Xenos, “A review of the state of the art and future challenges of deep learning-based beamforming,” *IEEE Access*, vol. 10, pp. 80869–80882, 2022.
[136] Z. Jaafer, S. Goli, and A. S. Elameer, “Best Performance Analysis of DOA Estimation Algorithms,” in *Proc. 2018 1st Annu. Int. Conf. Inf. Sci. (AiCIS)*, Fallujah, Iraq, Nov. 20–21, 2018, pp. 235–239, https://doi.org/10.1109/AiCIS.2018.00050.
[137] B. D. Van Veen and K. M. Buckley, “Beamforming: A versatile approach to spatial filtering,” *IEEE ASSP Mag.*, vol. 5, no. 2, pp. 4–24, Apr. 1988.
[138] B. Zhang, G. Xu, R. Zhou, H. Zhang, W. Hong, “Multi-Channel Back-Projection Algorithm for mmWave Automotive MIMO SAR Imaging With Doppler-Division Multiplexing,” *IEEE J. Sel. Topics Signal Process.*, vol. 17, no. 2, pp. 445–457, Mar. 2023, https://doi.org/10.1109/JSTSP.2022.3207902.
[139] S. De, S. Elayaperumal, and A. A. Bazil Raj, “Angle Estimation Using Modified Subarray Level Monopulse Ratio Algorithm and S-Curve in Digital Phased Array Radar,” in *Proc. 2020 Sec. Int. Conf. Inventive Res. Comput. Appl. (ICIRCA)*, Coimbatore, India, Jul. 15–17, 2020, pp. 936–941, https://doi.org/10.1109/ICIRCA48905.2020.9182859.

[140] F. Anderson, W. Christensen, L. Fullerton, B. Kortegaard, "Ultra-wideband beamforming in sparse arrays," *IEE Proc. H (Microw. Antennas Propag.)*, vol. 138, no. 4, pp. 342–346, Aug. 1991.

[141] H. Bölcskei, D. Gesbert, C. B. Papadias, A. J. Van der Veen, *Space-Time Wireless Systems: From Array Processing to MIMO Communications*. Cambridge, UK: Cambridge University Press, 2006.

[142] D. Gupta, A. A. Bazil Raj, and A. Kulkarni, "Multi-Bit Digital Receiver Design for Radar Signature Estimation," in *Proc. 2018 3rd IEEE Int. Conf. Recent Trends Electron. Inf. & Commun. Technol. (RTEICT)*, Bangalore, India, May 18–19, 2018, pp. 1072–1075, doi: 10.1109/RTEICT42901.2018.9012283.

[143] S. Bhan Rathor, S. Dayalan, and A. A. Bazil Raj, "Digital Implementation of Radar Receiver and Signal Processing Algorithms," in *Proc. 2021 Int. Conf. Syst. Comput. Autom. Netw. (ICSCAN)*, Puducherry, India, Jul. 30–31, 2021, pp. 1–6, doi: 10.1109/ICSCAN53069.2021.9526463.

[144] J. Helton, C.-I. H. Chen, D. M. Lin, and J. B. Y. Tsui, "FPGA-Based 1.2 GHz Bandwidth Digital Instantaneous Frequency Measurement Receiver," in *Proc. 9th Int. Symp. Q. Electron. Des. (ISQED)*, San Jose, CA, USA, Mar. 17–19, 2008, pp. 568–571, doi: 10.1109/ISQED.2008.4479798.

[145] V. D. Kanzarkar, K. S. Kayat, A. B. Jadhav, P. kumar Tingare, S. De, C. Joseph, et al., "FPGA Board Ramp Generation for 2 GHz RF Bandwidth in X-Band Application." *Int. J. Elec. Electron. Res.*, vol. 12, no. 2, pp. 10–15, 2024.

[146] M. Rzymowski, K. Trzebiatowski, K. Nyka, and L. Kulas, "DoA Estimation Using Reconfigurable Antennas in Millimeter-Wave Frequency 5G Systems," in *Proc. 2019 17th IEEE Int. New Circ. Syst. Conf. (NEWCAS)*, Munich, Germany, Jun. 23–26, 2019, pp. 1–4, doi: 10.1109/NEWCAS44328.2019.8961285.

[147] B. B. Bhagat and A. A. Bazil Raj, "Detection of Human Presence Using UWB Radar," in *Proc. 2021 Int. Conf. Syst. Comput. Autom. Netw. (ICSCAN)*, Puducherry, India, Jul. 30–31, 2021.

[148] A. Ramírez-Arroyo, A. Alex-Amor, P. Padilla, and J. F. Valenzuela-Valdés, "Wideband Elliptical Arrays for DoA and ToA Estimation in Multipath Environments," in *Proc. 2023 17th Europ. Conf. Antennas Propag. (EuCAP)*, Florence, Italy, Mar. 26–31, 2023, pp. 1–5, doi: 10.23919/EuCAP57121.2023.10133110.

[149] K. Kaneko and A. Sano, "Music-Like Iterative DoA Estimation in Multipath Environments," in *Proc. 2008 5th IEEE Sens. Array Multich. Signal Process. Works.*, Darmstadt, Germany, Jul. 21–23, 2008, pp. 212–215, doi: 10.1109/SAM.2008.4606857.

[150] A. Cidronali, G. Collodi, S. Maddio, M. Passafiume, G. Pelosi, S. Selleri, "Improving Phaseless DoA Estimation in Multipath-Impaired Scenarios by Exploiting Dual-Band Operations," in *Proc. 2016 IEEE MTT-S Int. Microw. Symp. (IMS)*, San Francisco, CA, USA, May 22–27, 2016, pp. 1–4, doi: 10.1109/MWSYM.2016.7540326.

[151] Y. Harkouss, "Direction of Arrival Estimation in Multipath Environments Using Deep Learning," *Int. J. Commun. Syst.*, vol. 34, no. 11, p. e4882, 2021.

[152] M. Hasib, S. Kandeepan, W. S. T. Rowe, A. Al-Hourani, "Direction of Arrival (DoA) Estimation Performance in a Multipath Environment with Envelope Fading and Spatial Correlation," in *Proc. 2022 13th Int. Conf. Inf. Commun. Technol. Convergence (ICTC)*, Jeju Island, Korea, Oct. 19–21, 2022, pp. 48–53, doi: 10.1109/ICTC55196.2022.9952516.

[153] V. Bhatia, M. R. Tripathy, and P. Ranjan, "Deep Learning for Massive MIMO: Challenges and Future Prospects," in *Proc. 2020 IEEE 9th Int. Conf. Commun. Syst. Netw. Techn. (CSNT)*, Gwalior, India, Apr. 10-12, 2020, pp. 26–31, doi: 10.1109/CSNT48778.2020.9115783.

[154] N. Tayem, S. A. Raza, M. Omer, M. El-Lakki, J. F. Nayfeh, "Hardware Implementation of a Proposed Qr-Tls DOA Estimation Method and MUSIC, ESPRIT Algorithms on Ni-PXI Platform," *Prog. Electromagn. Res. C*, vol. 45, pp. 203–221, 2013.

[155] S. Lahti, T. Aaltonen, E. Rastorgueva-Foi, J. Talvitie, B. Tan, T. D. Hämäläinen, "An Efficient High-Level Synthesis Implementation of the MUSIC DoA Algorithm for FPGA," in *Proc. 2024 27th Int. Symp. Des. Diagn. Electron. Circ. Syst. (DDECS)*, Kielce, Poland, Apr. 3–5, 2024, pp. 142–147, doi: 10.1109/DDECS60919.2024.10508912.

[156] Y. Xie, C. Peng, X. Jiang, S. Ouyang, "Hardware Design and Implementation of DOA Estimation Algorithms for Spherical Array Antennas," in *Proc. 2014 IEEE Int. Conf. Signal Proc. Commun. Comput. (ICSPCC)*, Guilin, China, Aug. 5–8, 2014, pp. 219–223, doi: 10.1109/ICSPCC.2014.6986186.

[157] A. A. Hussain, N. Tayem, M. O. Butt, A. H. Soliman, "FPGA Hardware Implementation of DOA Estimation Algorithm Employing LU Decomposition," *IEEE Access*, vol. 6, pp. 17666–17680, 2018.

[158] H. Park, H. Chung, and S. Kim, "Semi-Supervised Learning-Based Approach for DOA Estimation Under Hardware Impairments," in *Proc. 2023 IEEE 33rd Int. Works. Mach. Learn. Signal Process. (MLSP)*, Rome, Italy, Sept. 17–20, 2023, pp. 1–6, doi: 10.1109/MLSP55844.2023.10286004.

[159] A. A. Hussain, N. Tayem, and A.-H. Soliman, "Matrix Decomposition Methods for Efficient Hardware Implementation of DOA Estimation Algorithms: A Performance Comparison," in *Proc. 2019 4th Int. Conf. Recent Adv. Innov. Eng. (ICRAIE)*, Kedah, Malaysia, Nov. 27–29, 2019, pp. 1–7, doi: 10.1109/ICRAIE47735.2019.9037778.

[160] A. A. Hussain, N. Tayem, and A.-H. Soliman, "FPGA Hardware Implementation of Computationally Efficient DOA Estimation of Coherent Signals," in *Proc. 2021 Int. Conf. Radar Antenna Microw. Electron. Telecommun. (ICRAMET)*, Bandung, Indonesia, Nov. 23–24, 2021, pp. 103–108, doi: 10.1109/ICRAMET53537.2021.9650476.

[161] E. M. Al-Ardi, R. M. Shubair, and M. E. Al-Mualla, "Investigation of high-resolution DOA estimation algorithms for optimal performance of smart antenna systems," in *Proc. 4th Int. Conf. 3G Mobile Commun. Technol.*, London, UK, Jun. 25–27, 2003, pp. 460–464.

[162] R. Zhang, B. Shim, and W. Wu, "Direction-of-arrival estimation for large antenna arrays with hybrid analog and digital architectures," *IEEE Trans. Signal Process.*, vol. 70, pp. 72–88, 2022, doi: 10.1109/TSP.2021.3119768.

[163] R. Zhang, J. Zhang, T. Zhao, and H. Zhao, "Block sparse recovery for wideband channel estimation in hybrid mmWave MIMO systems," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Abu Dhabi, United Arab Emirates, Dec. 9–13, 2018, pp. 1–6, doi: 10.1109/GLOCOM.2018.8647482.

[164] R. Zhang, X. Wu, Y. Lou, F. G. Yan, Z. Zhou, W. Wu, et al., "Channel training-aided target sensing for terahertz integrated sensing and massive MIMO communications," *IEEE Internet Things J.*, pp. 1–1, 2024, doi: 10.1109/JIOT.2024.3447584.

[165] R. Zhang, L. Cheng, S. Wang, Y. Lou, W. Wu, and D. W. K. Ng, "Tensor decomposition-based channel estimation for hybrid mmWave massive MIMO in high-mobility scenarios," *IEEE Trans. Commun.*, vol. 70, no. 9, pp. 6325–6340, Sep. 2022, doi: 10.1109/TCOMM.2022.3187780.

[166] R. Zhang, L. Cheng, W. Zhang, X. Guan, Y. Cai, W. Wu, et al., "Channel estimation for movable-antenna MIMO systems via tensor decomposition," *IEEE Wireless Commun. Lett.*, vol. 13, no. 11, pp. 3089–3093, Nov. 2024, doi: 10.1109/LWC.2024.3450592.

[167] R. Zhang, L. Cheng, S. Wang, Y. Lou, Y. Gao, W. Wu, et al., "Integrated sensing and communication with massive MIMO: A unified tensor approach for channel and target parameter estimation," *IEEE Trans. Wireless Commun.*, vol. 23, no. 8, pp. 8571–8587, Aug. 2024, doi: 10.1109/TWC.2024.3351857.

[168] P. Gupta and S. P. Kar, "MUSIC and improved MUSIC algorithm to estimate direction of arrival," in *Proc. Int. Conf. Commun. Signal Process. (ICCSP)*, Melmaruvathur, India, Apr. 2–4, 2015, pp. 757–761, doi: 10.1109/ICCSP.2015.7322593.

[169] P. K. Eranti and B. D. Barkana, "An overview of direction-of-arrival estimation methods using adaptive directional time-frequency distributions," *Electronics*, vol. 11, no. 9, p. 1321, 2022.

[170] H.-A. Nguyen, K. Mahler, M. Peter, W. Keusgen, T. Eichler, and H. Mellein, "Estimation of DoA based on large-scale virtual array data," in *Proc. 10th Eur. Conf. Antennas Propag. (EuCAP)*, Davos, Switzerland, Apr. 10–15, 2016, pp. 1–4, doi: 10.1109/EuCAP.2016.7481756.

[171] N. Ruan, H. Wang, F. Wen, J. Shi, "DOA estimation in B5G/6G: Trends and challenges," *Sensors*, vol. 22, no. 14, p. 5125, 2022.

[172] P. Heidenreich and A. M. Zoubir, "Fast maximum likelihood DOA estimation in the two-target case with applications to automotive radar," *Signal Process.*, vol. 93, no. 12, pp. 3400–3409, 2013.

[173] G. K. Papageorgiou, M. Sellathurai, and Y. C. Eldar, "Deep networks for DoA estimation in low SNR," *IEEE Trans. Signal Process.*, vol. 69, pp. 3714–3729, 2021, doi: 10.1109/TSP.2021.3089927.

[174] S. Ge, K. Li, and S. N. B. M. Rum, "Deep learning approach in DOA estimation: A systematic literature review," *Mobile Inf. Syst.*, vol. 2021, p. 6392875, 2021.

[175] M. Weiß, M. Kohler, A. Saam, and J. Worms, "Single snapshot DoA estimation from a Rotman lens using machine learning techniques," in *Proc. 21st Int. Radar Symp. (IRS)*, Warsaw, Poland, Oct. 5–8, 2020, pp. 35–39, doi: 10.23919/IRS48640.2020.9253832.

[176] H. Al Kassir, Z. D. Zaharis, P. I. Lazaridis, N. V. Kantartzis, T. V. Yioultsis, T. D. Xenos, "A review of the state of the art and future challenges of deep learning-based beamforming," *IEEE Access*, vol. 10, pp. 80869–80882, 2022.

[177] C. Vieira, V. Lücken, A. R. Diewald, and H.-P. Beise, "Steering-layer: A hybrid DSP-machine learning layer for sensor DoA estimation," in *Proc. 15th German Microw. Conf. (GeMiC)*, Duisburg, Germany, Mar. 11–13, 2024, pp. 245–248, doi: 10.23919/GeMiC59120.2024.10485352.

[178] M. Huang, B. Zheng, T. Cai, X. Li, J. Liu, C. Qian, et al., "Machine-learning-enabled metasurface for direction of arrival estimation," *Nanophotonics*, vol. 11, no. 9, pp. 2001–2010, 2022.

[179] Y. Chai, "Advanced techniques in adaptive beamforming for enhanced DOA estimation," in *Proc. Int. Wireless Commun. Mobile Comput. (IWCMC)*, Ayia Napa, Cyprus, May 27–31, 2024, pp. 269–273, doi: 10.1109/IWCMC61514.2024.10592555.

[180] O. Lange and B. Yang, "Array geometry optimization for direction-of-arrival estimation including subarrays and tapering," in *Proc. Int. ITG Works. Smart Antennas (WSA)*, Bremen, Germany, May 23–24, 2024, pp. 135–142, doi: 10.1109/WSA.2010.5456461.

[181] S. Theodoridis and R. Chellappa, *Academic Press Library in Signal Processing: Signal Processing Theory and Machine Learning*. London, UK: Academic Press, 2013. vol. 1.

[182] J. Biamonte, P. Wittek, N. Pancotti, P. Rebentrost, N. Wiebe, S. Lloyd, "Quantum machine learning," *Nature*, vol. 549, no. 7671, pp. 195–202, 2017.

[183] W. Xu, Z. Yang, D. W. K. Ng, M. Levorato, Y. C. Eldar, M. Debbah, "Edge learning for B5G networks with distributed signal processing: Semantic communication, edge computing, and wireless sensing," *IEEE J. Sel. Topics Signal Process.*, vol. 17, no. 1, pp. 9–39, 2023.

[184] M. Chakraborty, H. C. Kumawat, and S. V. Dhavale, "DIAT-RadHARNet: A lightweight DCNN for radar-based classification of human suspicious activities," *IEEE Trans. Instrum. Meas.*, vol. 71, pp. 1–10, 2022.

[185] P. Madhuri, N. Akhter, and A. A. Bazil Raj, "Digital implementation of depthwise separable convolution network for AI applications," in *Proc. IEEE Pune Sect. Int. Conf. (PuneCon)*, Pune, India, Dec. 14–16, 2023, pp. 1–5, doi: 10.1109/PuneCon58714.2023.10450113.

[186] S. Soni, M. Chakraborty, and A. A. B. Raj, "AI-based small unmanned aerial vehicle (SUAV) targets detection and tracking techniques," in *Proc. Int. Conf. Augmented Intell. Sustain. Syst. (ICAISS)*, Trichy, India, Nov. 24–26, 2022, pp. 43–49, doi: 10.1109/ICAISS55157.2022.10010718.

[187] Q. Xu, C. Jiang, Y. Han, B. Wang, and K. J. R. Liu, "Waveforming: An overview with beamforming," *IEEE Commun. Surveys Tuts.*, vol. 20, no. 1, pp. 132–149, 2018, doi: 10.1109/COMST.2017.2750201.

[188] Deepshikha, A. Raut, and A. A. Bazil Raj, "Signal processing for digital beamforming on transmit in MIMO radar," in *Proc. Sec. Int. Conf. Inventive Res. Comput. Appl. (ICIRCA)*, Coimbatore, India, Jul. 15–17, 2020, pp. 1106–1111, doi: 10.1109/ICIRCA48905.2020.9183290.

[189] R. Shafin, L. Liu, J. Zhang, Y. C. Wu, "DoA estimation and capacity analysis for 3-D millimeter-wave massive-MIMO/FD-MIMO OFDM systems," *IEEE Trans. Wireless Commun.*, vol. 15, no. 10, pp. 6963–6978, 2016.

[190] X. Lan, L. Wan, G. Han, J. J. P. C. Rodrigues, "A novel DoA estimation algorithm using array rotation technique," *Future Internet*, vol. 6, no. 1, pp. 155–170, 2014.

[191] M. Ulrich and B. Yang, "Multi-carrier MIMO radar: A concept of sparse array for improved DoA estimation," in *Proc. IEEE Radar Conf. (RadarConf)*, Philadelphia, PA, USA, May 2–6, 2016, pp. 1–5, doi: 10.1109/RADAR.2016.7485156.

[192] S. A. Alawsh, A. H. Muqaibel, and M. S. Sharawi, "DoA estimation in MIMO systems with compressive sensing for future handsets," in *Proc. IEEE Jordan Conf. Appl. Electr. Eng. Comput. Technol. (AEECT)*, Amman, Jordan, Nov. 3–5, 2015, pp. 1–6, doi: 10.1109/AEECT.2015.7360532.

[193] N. Labbaf, H. R. D. Oskouei, and M. R. Abedi, "Robust DoA estimation in a uniform circular array antenna with errors and unknown parameters using deep learning," *IEEE Trans. Green Commun. Netw.*, vol. 7, no. 4, pp. 2143–2152, 2023.

[194] L. Lu, G. Y. Li, A. L. Swindlehurst, A. Ashikhmin, R. Zhang, "An overview of massive MIMO: Benefits and challenges," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 742–758, 2014.

[195] A. L. Swindlehurst, B. D. Jeffs, G. Seco-Granados, J. Li, "Applications of array signal processing," in *Academic Press Library in Signal Processing*. Amsterdam, The Netherlands: Elsevier, 2014, vol. 3, pp. 859–953.

[196] A. Patwari, "Sparse linear antenna arrays: A review," *Antenna Syst.*, vol. 10, pp. 97–123, 2021.

[197] R. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propag.*, vol. 34, no. 3, pp. 276–280, 1986.

[198] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, et al., "Scaling up MIMO: Opportunities and challenges with very large arrays," *IEEE Signal Process. Mag.*, vol. 30, no. 1, pp. 40–60, 2012.

[199] X. Zhao, R. Jiang, K. S. Chen, Y. Yang, "A physics-based neural estimation of the direction of arrival over sea surfaces," *IEEE Trans. Geosci. Remote Sens.*, vol. 60, pp. 1–14, 2022.

[200] M. P. Singman and R. M. Narayanan, "Applying machine learning to adaptive array signal processing weight generation," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 60, pp. 4952–4962, 2024.

[201] P. Stoica and A. Nehorai, “MUSIC, maximum likelihood, and Cramer-Rao bound,” *IEEE Trans. Acoust. Speech Signal Process.*, vol. 37, no. 5, pp. 720–741, 1989.
[202] P. Wu, Y. Liu, Z. Zhao, Q. H. Liu, “Sparse antenna array design methodologies—a review,” *J. Electron. Sci. Technol.*, p. 100276, 2024. https://doi.org/10.1016/j.jnlest.2024.100276.
[203] I. Ziskind and M. Wax, “Maximum likelihood localization of multiple sources by alternating projection,” *IEEE Trans. Acoust. Speech Signal Process.*, vol. 36, no. 10, pp. 1553–1560, 1988.
[204] A. S. Assoa, A. Bhat, S. Ryu, and A. Raychowdhury, “A scalable platform for single-snapshot direction of arrival (DOA) estimation in massive MIMO systems,” in *Proc. Great Lakes Symp. VLSI 2023*, Knoxville, TN, USA, Jun. 5–7, 2023, pp. 631–637.
[205] B. Shi, Q. Jie, F. Shu, Z. Zhuanga, S. Feng, Y. Zhanga, et al., “DOA estimation using massive receive MIMO: Basic principles, key techniques, performance analysis, and applications,” *arXiv*, 2021, arXiv:2109.00154.
[206] Y. Chen, L. Yan, C. Han, M. Tao, “Millidegree-level direction-of-arrival estimation and tracking for terahertz ultra-massive MIMO systems,” *IEEE Trans. Wirel. Commun.*, vol. 21, no. 2, pp. 869–883, 2021.
[207] F. Shu, B. Shi, Y. Chen, J. Bai, Y. Li, T. Liu, et al., “A new heterogeneous hybrid massive MIMO receiver with an intrinsic ability of removing phase ambiguity of DOA estimation via machine learning,” *IEEE Trans. Mach. Learn. Commun. Netw.*, vol. 3, pp. 17–29, 2024.
[208] S. Ravindran and R. Jose, “Direction of arrival and channel estimation using machine learning for multiple input multiple output system,” in *Proc. 2019 Int. Conf. Commun. Electron. Syst. (ICCES)*, Coimbatore, India, Jul. 17–19, 2019, pp. 1327–1330, doi: 10.1109/ICCES45898.2019.9002097.
[209] F. Wang, X. Zhu, Z. Chen, Y. Zhang, G. Yu, Y. Liu, “Low-complexity DOA estimation of wideband signals based on intelligent optimization algorithms in massive MIMO systems,” in *Proc. 2023 8th Int. Conf. Signal Image Process. (ICSIP)*, Wuxi, China, Jul. 8–10, 2023, pp. 542–548, doi: 10.1109/ICSIP57908.2023.10270941.
[210] R. Zhang, L. Cheng, S. Wang, Y. Lou, Y. Gao, W. Wu, et al., “Integrated sensing and communication with massive MIMO: A unified tensor approach for channel and target parameter estimation,” *IEEE Trans. Wirel. Commun.*, vol. 23, no. 8, pp. 8571–8587, Aug. 2024, doi: 10.1109/TWC.2024.3351856.
[211] W. Zhou, R. Zhang, G. Chen, and W. Wu, “Integrated sensing and communication waveform design: A survey,” *IEEE Open J. Commun. Soc.*, vol. 3, pp. 1930–1949, 2022, doi: 10.1109/OJCOMS.2022.3215683.
[212] N. Anughna, M. Ramesha, “Antenna reconfiguration based DOA estimation for AWGN channel in MIMO applications,” *Prog. Electromagn. Res. C*, vol. 128, pp. 73–84, 2023.