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Research Article

Practical Asymptotic String Stability in Vehicular Platoons: Controller Design for Enhanced Stability

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Abstract: This paper introduces a new concept of *Global Practical Asymptotic String Stability* (GPAS) for vehicular platoons subjected to external disturbances. The proposed definition captures the ability of the system to maintain bounded inter-vehicle spacing errors, with convergence bounds that can be adjusted via a tunable control gain. Unlike classical string stability notions based on norms or transfer functions, the GPAS definition explicitly quantifies the trade-off between disturbance attenuation and control gain magnitude. To ensure GPAS, we design a distributed control law based on a backstepping structure, supported by a velocity observer for predecessor estimation. Lyapunov-based analysis establishes local exponential convergence, and sufficient conditions are provided for practical asymptotic string stability. Numerical simulations on a five-vehicle platoon illustrate the effectiveness of the proposed method under idealized conditions.

***Keywords*:** Stability analysis, String Stability, Platoon control, Networked systems, Distributed control, Vehicle safety.

1. Introduction

Autonomous vehicle platoon control is an emerging field, offering significant advantages for road safety and traffic efficiency—see [1–3]. One of the main drivers behind this research is the potential to improve road safety through precise vehicle coordination, eliminating human errors such as distraction, fatigue, or reckless driving. By optimizing inter-vehicle distances and speeds, platoons contribute to smoother traffic flow, reduced congestion, and fewer accidents. Furthermore, the use of autonomous vehicles in these formations can also lower emissions and shorten travel times.

However, despite these benefits, autonomous vehicle platooning still faces several technical challenges—see [4, 5]. These include the integration of vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communication, sensor fusion (RADAR, LiDAR), and the design of robust and scalable control algorithms—see [1, 6, 7]. Most existing approaches focus on achieving stability and safety under ideal conditions, but often fall short in capturing the impact of realistic constraints such as disturbances and partial state measurements.

In particular, external factors such as wind, rolling resistance, or road slope affect vehicle dynamics and must be properly accounted for—see [8]. Although several works investigate the modeling of these disturbances, many do not incorporate them into the controller design in a systematic way—see [9]. Robust control methods like H_∞ are often used to

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mitigate such effects—see [10, 11]—but they typically require full state measurements and entail high computational cost, especially in predictive control schemes—see [12]. Other approaches based on static output feedback, such as in [13], aim to reduce complexity while preserving a certain level of robustness, using only relative measurements with respect to the leader and predecessor.

Beyond these challenges, and as also noted in [14], there remains a need for quantitative and scalable analysis tools that can precisely characterize the impact of external disturbances on the overall platoon behavior. In this paper, we aim to address this by introducing a new concept which reveals that the effect of disturbances can be rendered arbitrarily small by tuning a scalar gain of the controller.

This analysis is closely tied to the classical notion of string stability—see [15]—which concerns the ability of a vehicle string to attenuate rather than amplify disturbances along the chain—see [16, 17]. Early definitions of string stability, such as in [18], require that disturbances or initial state variations do not lead to increasing fluctuations in the positions or velocities of following vehicles—see [19, 20]. The role of inter-vehicle spacing policies and information topologies in achieving string stability has been deeply explored in the literature—see [19, 21, 22].

In this paper, we introduce and highlight a new definition related to string stability, called Global Practical Asymptotic String (GPAS) Stability. Unlike classical definitions, this one explicitly incorporates a scalable gain in the stability inequality, allowing us to quantify and control the effect of disturbances in a principled way. Importantly, our definition should not be confused with the term "practical asymptotic string stability" as used in [23], [24], or [25], where it refers more generally to practical robustness under actuator and sensor limitations. In contrast, our GPAS definition is mathematically grounded in Lyapunov theory and emphasizes the gain-dependent decay of disturbance effects, forming the core conceptual contribution of the paper.

The structure of the paper is as follows. In Section II, we introduce the problem formulation. Section III presents the observer and controller design. The main theoretical result is developed in Section IV. Section V provides numerical simulations, and Section VI concludes the paper.

2. Problem formulation & String stability

Consider a network of N vehicles with dynamics,

$$\dot{x}_i = v_i \tag{1a}$$

$$\dot{v}_i = u_i + J_i + F_i, \quad i \leq N \tag{1b}$$

where,

$$x_i := \begin{bmatrix} p_i^x \\ p_i^y \end{bmatrix}, v_i := \begin{bmatrix} v_i^x \\ v_i^y \end{bmatrix}, F_i := \begin{bmatrix} f_i^x/m \\ f_i^y/m \end{bmatrix}, J_i := \begin{bmatrix} d_i^x/m \\ d_i^y/m \end{bmatrix}, \quad \forall i \leq N,$$

such that f_i^x, f_i^y are the longitudinal and lateral resistances of the vehicle, d_i^x and d_i^y are external disturbances, m is the mass of the vehicle.

with f_i^x and f_i^y expressed as,

$$f_i^x = C_r mg\cos(\theta(t)) + mg\sin(\theta(t)) + \frac{1}{2}C_d\rho A(v_i^x)^2 \tag{2a}$$

$$f_i^y = C_f\alpha + \frac{1}{2}C_y\rho(v_i^x)^2 \tag{2b}$$

where C_f is the lateral stiffness, α_i the sideslip angle of the vehicle i , C_y the aerodynamic coefficient, ρ the air density, m the mass of the vehicle, C_d is the drag coefficient, ρ the air density, A the frontal area of the vehicle, C_r the rolling resistance coefficient and θ the slope angle. The graph corresponding to the platoon is presented in Fig 1, where each node $i \leq N$ corresponds to a vehicle.

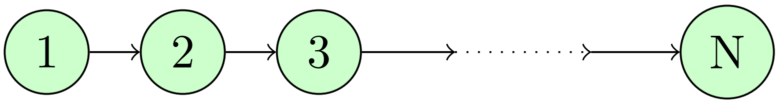

Figure 1. The graph corresponding to the platoon

In what follows, before producing a control input u_i for each vehicle $i \leq N$ that allows the platoon to satisfy String stability properties, we give more details about the latter. Essentially, this notion implies uniform boundedness of the state of all systems. In order to present the mathematical definitions of String Stability and Asymptotic String Stability, we consider $\chi_{i,i-1} = p_i - p_{i-1}$ to be the distance between two neighboring vehicles. To determine the equilibrium point of the platoon, we consider the nominal case without disturbances. Assuming that $p^* = \begin{bmatrix} p_x^* & p_y^* \end{bmatrix}^\top$ represents the desired intervehicular distance at steady-state, the equilibrium point for the pair $(i, i-1)$, is achieved when these vehicles have the same velocity and maintain the same distance,

$$\chi_{eq,i,i-1} = p^*.$$

Next, we recall these definitions.

Definition 1. ***(String Stability)*** *The equilibrium configuration* $\chi_{eq,i,i-1}$, *for* $i \leq N$, *is said to be* string stable *if for every* $\varepsilon > 0$, *there exists a* $\delta > 0$ *such that, whenever*

$$\max_{i \leq N} \left| \chi_{i,i-1}(0) - \chi_{eq,i,i-1} \right| < \delta,$$

it follows that for all $t \geq 0$,

$$\max_{i \leq N} \left| \chi_{i,i-1}(t) - \chi_{eq,i,i-1} \right| < \varepsilon.$$

A string of vehicles is stable if, for any set of bounded initial disturbances, the position fluctuations of all vehicles remain bounded as time goes to infinity.Key properties include boundedness and convergence of position fluctuations for any string length.

Definition 2. ***(Asymptotic String Stability)*** *The equilibrium configuration* $\chi_{eq,i,i-1}$, *for* $i \leq N$, *is said to be* asymptotically string stable *if it is string stable and, in addition, satisfies*

$$\lim_{t \to \infty} \left| \chi_{i,i-1}(t) - \chi_{eq,i,i-1} \right| = 0, \quad \textit{for all } i \leq N.$$

The equilibrium of the system is *asymptotically string stable* if it is *string stable* and the state vector norms approach zero asymptotically. Asymptotic string stability extends string stability to include asymptotic convergence and requires construction of Lyapunov functions. These two notions are widely used in the literature for stability analysis of vehicular platoons–see [15, 26–28].

Moreover, several other definitions related to *string stability* can be found in the literature, such as *Frequency-Domain String Stability*, where for linear platoon systems, the system is Frequency-Domain String Stable if the transfer function of outputs between the leading vehicle and any other vehicle has an H_∞ norm less than or equal to 1. This definition is more flexible regarding information flow topologies, but assumes linear systems and specific disturbance types.

There is also the notion of L_p *String Stability*, where the equilibrium point of the system is L_p string stable if the L_p norm of the output remains bounded by a function of the initial disturbance and the control input for any platoon length. This notion handles some types of disturbances but does not regulate convergence. If in addition of being L_p String Stable, the L_p norm of the output of each vehicle is less than or equal to that of its predecessor, the system is said to be *Strictly* L_p *String Stable*.

Finally, *Input-to-State String Stability* and *Input-to-Output String Stability* are both crucial concepts in the analysis of vehicular platoon control, yet they differ significantly in their focus and application. *Input-to-State String Stability* ensures that the state vector norms are bounded by a function of the initial disturbances and the L_∞ norm of the disturbances over time, making it suitable for all types of disturbances. This definition generalizes other stability definitions and requires the construction of $\mathscr{K}\mathscr{L}$ and $\mathscr{K}$ functions, which provide a rigorous foundation for analyzing the internal dynamics of the system. On the other hand, *Input-to-Output String Stability* focuses on the mapping from inputs to outputs, ensuring that the output norms are bounded by the input norms for any platoon length. While *Input-to-Output String Stability* is effective in regulating the relationship between inputs and outputs, it does not account for the internal state descriptions of the system. Therefore, *Input-to-State String Stability* offers a more comprehensive approach by considering both the internal and external influences on the system's stability, whereas *Input-to-Output String Stability* provides a more straightforward method focused solely on input-output relationships.

These definitions provide a comprehensive framework for understanding and analyzing string stability in vehicular platoons, each with its own strengths and limitations depending on the specific application and system characteristics. But none of them presents an explicit relation between the amplitude of the effect of the perturbation and some control gain, that is why we introduce the following definition.

Definition 3. ***(Global Practical Asymptotic String Stability)****: Let $\beta_1, \beta_2 \in \mathscr{K}\mathscr{L}$ be continuous functions that are strictly decreasing in their second argument and satisfy $\beta_j(r,t) \to 0$ as $t \to \infty$ for each fixed $r \geq 0$, for $j = 1,2$. We say that the system is* globally practically asymptotically string stable *if, for each $\delta > 0$, there exists $\gamma^*(\delta) > 0$ such that for all $\gamma \geq \gamma^*$, and for all initial conditions $\chi_{i,i-1}(0) \in \mathbb{R}^n$, the solutions $\chi_{i,i-1}(t)$ to the relative dynamics $\dot{\chi}_{i,i-1} = v_i - v_{i-1}$ satisfy:*

$$|\chi_{i,i-1}(t) - \chi_{\mathrm{eq},i,i-1}|^2 \leq \delta + \beta_1\left(|\chi_{i,i-1}(0)|,t\right) + \frac{1}{\gamma}\beta_2\left(|\chi_{i,i-1}(0)|,t\right), \quad \forall t \geq 0.$$

Remark 1. *Although the proposed definition does not explicitly follow classical string stability formulations such as $\mathscr{L}_\infty$, H_∞, or ISS-based notions, it is closely related. In particular, the inclusion of two $\mathscr{K}\mathscr{L}$ functions allows us to capture both transient behavior and control gain-dependent effects. As shown below, the definition implies norm-based boundedness properties under mild conditions on the decay rates of β_1 and β_2.*

Assume the $\mathscr{K}\mathscr{L}$ functions $\beta_1(r,t) \leq a_1 r e^{-c_1 t}$ and $\beta_2(r,t) \leq a_2 r e^{-c_2 t}$ for some constants $a_1, a_2, c_1, c_2 > 0$. Then, for any $\delta > 0$ and $\gamma \geq \gamma^*(\delta)$, the solution $\chi_{i,i-1}(t)$ satisfies

$$\|\chi_{i,i-1}(t) - \chi_{\mathrm{eq},i,i-1}\| \leq \sqrt{\delta + Ce^{-ct}},$$

for some constants $C, c > 0$, uniformly in time. Hence, the system is string stable in the uniform norm sense.

Remark 2. *The control strategy is implemented per vehicle using a backstepping-based design and guarantees local exponential convergence of the tracking error for each vehicle. However, the current analysis does not extend to the entire vehicle string as a coupled system. In particular, we do not provide formal guarantees on disturbance propagation or scalability with respect to the number of vehicles. Addressing string-wise robustness and amplification effects remains an important direction for future research.*

Remark 3. *The current analysis focuses on Lyapunov-based design and stability verification at the level of individual vehicle pairs. However, no inter-agent coupling tools such as the small-gain theorem, passivity theory, or input-output* *stability are used. While this approach suffices to ensure local convergence per vehicle, it does not provide global guarantees on the stability of the interconnected platoon. Incorporating such system-theoretic tools could offer stronger guarantees on scalability, robustness to disturbances, and interconnection effects, and is a promising direction for future extensions.*

Remark 4. *The proposed notion of practical string stability relies on increasing the control gain γ to reduce the steady-state bound and improve transient performance. However, this gain-dependent design introduces trade-offs not explicitly addressed in the current analysis. In particular, increasing γ may lead to larger control effort, higher sensitivity to measurement noise, and risks of actuator saturation, especially in real-world implementations. While our theoretical framework confirms convergence with large γ, practical constraints necessitate careful tuning and robustness considerations. Investigating these trade-offs and establishing performance bounds under actuation and sensing limitations remains a crucial direction for future research.*

In what follows, the external disturbances acting on vehicle i J_i in equation (1) are assumed to be unknown. Furthermore, the preceding vehicle's velocities provided by the sensors are not necessarily the exact values of the velocity, errors can occur, or the information exchange between two neighboring vehicles can be distorted by noise or errors. Consequently, we add an observer to let each vehicle evaluate the velocity of its corresponding leading vehicle, in order to follow a distributed control approach. Distributed control of vehicular platoons involves coordinating multiple vehicles such that adjusts its speed and spacing based on local information and communication with neighboring vehicles —see [29–31]. The decentralized control system allows each vehicle to make real-time decisions, ensuring a stable and adaptive platoon formation.

3. Observer and control design

The control design consists of designing a distributed control law u_i for each $i \leq N$ to stabilize $p_i - p_{i-1}$ for all $i \leq N$. The control approach, on one hand, is based on Backstepping, and, on the other hand, uses a preceding vehicle velocity observer. Then, we implement a certainty-equivalence output feedback controller. The aim of our control is to ensure that the distance between two neighboring vehicles

$$p_i(t) - p_{i-1}(t) \to p^* \qquad \text{for} \quad t \to \infty.$$

Therefore, we define, $\delta p_{i,i-1} = p_i - p_{i-1} - p^*$.

Assume v_{i-1} known and let v_i be a virtual control input. Clearly, **if**

$$v_i = v_{i-1} - \lambda_1 \delta p_{i,i-1}, \quad \lambda_1 > 0,$$

we obtain the closed-loop equation

$$\dot{\delta} p_{i,i-1} = -\lambda_1 \delta p_{i,i-1}$$

for which the origin $\delta p_{i,i-1} = 0$ is exponentially stable, so $p_i(t) - p_{i-1}(t) \to p^*$ as $t \to \infty$.

Finally, we use the second control input, u_i to steer v_i to $v_i^* = v_{i-1} - \lambda_1 \delta p_{i,i-1}$. Consequently, we set

$$u_i = \dot{v}_i^* - F_i - \lambda_2 \delta v_i, \quad \delta v_i = v_i - v_i^*, \quad \lambda_2 > 0.$$

The previous reasoning leads to the closed-loop system,

$$\dot{\delta} p_{i,i-1} = -\lambda_1 \delta p_{i,i-1} + \delta v_i$$

$$\dot{\delta} v_i = -\lambda_2 \delta v_i + J_i.$$

Furthermore, since the velocity of the preceding vehicle v_{i-1} for all $i \leq N$, is assumed to be unknown, we consider $\hat{v}_{i-1}$ to be the observed velocity of the vehicle $i \leq N$. We use a fixed-gain linear observer to obtain estimation of the preceding vehicle's velocity from information of the follower's onboard sensors.

$$\dot{\hat{v}}_{i-1} = -\lambda_3 \hat{v}_{i-1} - \delta p_{i,i-1}, \quad \lambda_3 > 0.$$

Now that the observer is taken into account, we redefine

$$v_i^* := \hat{v}_{i-1} - \lambda_1 \delta p_{i,i-1},$$

and

$$u_i = \dot{v}_i^* - F_i - \lambda_2 \delta v_i - \delta p_{i,i-1}, \tag{3}$$

which turns the closed-loop dynamics of $\delta p_{i,i-1}$ into,

$$\dot{\delta} p_{i,i-1} = -\lambda_1 \delta p_{i,i-1} + \delta v_i + \bar{v}_{i-1}$$

where

$$\bar{v}_{i-1} = \hat{v}_{i-1} - v_{i-1}, \quad \text{for all} \quad i \in \{1, 2, \ldots, N\},$$

is the observation error.

As a result, for each pair of leader follower vehicles $(i-1, i)$, we obtain the closed-loop system

$$\dot{\delta} p_{i,i-1} = -\lambda_1 \delta p_{i,i-1} + \delta v_i + \bar{v}_{i-1}$$

$$\dot{\delta} v_i = -\lambda_2 \delta v_i + J_i - \delta p_{i,i-1}$$

$$\dot{\bar{v}}_{i-1} = -\lambda_3 \hat{v}_{i-1} - \delta p_{i,i-1} - \dot{v}_{i-1}$$

In the next part, we introduce our statement related to the *Practical asymptotic string stability* of the system.

4. Main result

Proposition 1. *Consider a platoon of N vehicles with dynamics* (1)-(3)*. Then, if there exists* $a > 0$ *such that, for each* $i \leq N$*, the norm of velocities* $|v_{i-1}| \leq a_1$*,* $|\dot{v}_{i-1}| \leq a_2$*, the solutions to* (1)-(3) *are globally bounded. Moreover, the system* (1)-(3) *is Globally Practically String Stable, according to Definition 3.*

Proof. To begin with, we recall that, for $(\delta p_{i,i-1}, \delta v_i, \bar{v}_{i-1}) = (0,0,0)$, the control objective is achieved. Therefore, we consider the Lyapunov function

$$V(\delta p_{i,i-1}, \delta v_i, \bar{v}_{i-1}) = \frac{1}{2}(||\delta p_{i,i-1}||^2 + ||\delta v_i||^2 + ||\bar{v}_{i-1}||^2).$$

Its derivative along the trajectories of (4) yields

$$\dot{V}(\delta p_{i,i-1}, \delta v_i, \bar{v}_{i-1}) = -\lambda_1 ||\delta p_{i,i-1}||^2 + \delta p_{i,i-1}^\top \delta v_i - \delta v_i^\top \delta p_{i,i-1} + \delta p_{i,i-1}^\top \bar{v}_{i-1} - \lambda_2 ||\delta v_i||^2 + J_i^\top \delta v_i$$

$$- \lambda_3 \hat{v}_{i-1}^\top \bar{v}_{i-1} - \delta p_{i,i-1}^\top \bar{v}_{i-1} - \bar{v}_{i-1}^\top \dot{v}_{i-1}.$$

Next, using the triangular inequality,

$$xy \leq \beta^2 x^2 + \frac{y^2}{\beta^2}, \qquad \forall \beta > 0,$$

we obtain

$$\dot{V}(\delta p_{i,i-1}, \delta v_i, \bar{v}_{i-1}) \leq -\alpha_1 ||\delta p_{i,i-1}||^2 - \alpha_2 ||\delta v_i||^2 - \alpha_3 ||\bar{v}_{i-1}||^2 + b_i,$$

with

$$\alpha_1 = \lambda_1, \qquad \alpha_2 = \frac{\lambda_2}{2}, \qquad \alpha_3 = \frac{\lambda_3}{2}, \qquad b_i = \frac{2}{\lambda_2}||J_i|| + \lambda_3 \frac{a_1 + \frac{a_2}{\lambda_3}}{2}.$$

As a result,

$$\dot{V}(\delta p_{i,i-1}, \delta v_i, \bar{v}_{i-1}) \leq -\Psi(\delta p_{i,i-1}, \delta v_i, \bar{v}_{i-1}) \leq 0, \qquad \forall (\delta p_{i,i-1}, \delta v_i, \bar{v}_{i-1}) \in \mathbb{R}^6 \setminus \mathscr{B}_r,$$

where $\mathscr{B}_r$ is a ball of radius

$$r := \sqrt{\frac{b_i}{\min\{\alpha_1, \alpha_2, \alpha_3\}}}$$

and,

$$\Psi(\delta p_{i,i-1}, \delta v_i, \bar{v}_{i-1}) = \alpha_1 ||\delta p_{i,i-1}||^2 + \alpha_2 ||\delta v_i||^2 + \alpha_3 ||\bar{v}_{i-1}||^2 - b_i.$$

Hence, the trajectories of (4) are globally bounded, in other words,

$$\forall t \geq 0, \forall (\delta p_{i,i-1}(0), \delta v_i(0), \bar{v}_{i-1}(0)) \in \mathbb{R}^6, \qquad \text{such that} \|(\delta p_{i,i-1}(0), \delta v_i(0), \bar{v}_{i-1}(0))\| \leq r_o,$$

there exists $r > 0$ such that

$$\|(\delta p_{i,i-1}(t), \delta v_i(t), \bar{v}_{i-1}(t))\| \leq r \quad \forall t > 0.$$

Now, we focus on the global *Practical asymptotic string stability*, considering $W(\delta p_{i,i-1}) = \frac{1}{2}||\delta p_{i,i-1}||^2$, we have

$$\dot{W}(\delta p_{i,i-1}) \leq -\lambda_1 ||\delta p_{i,i-1}||^2 + \delta p_{i,i-1}^{\top} \delta v_i + \delta p_{i,i-1}^{\top} \bar{v}_{i-1}.$$

Since the trajectories are proven to be globally bounded, we have

$$||\delta p_{i,i-1}^{\top}(\delta v_i + \bar{v}_{i-1})|| \leq 2r^2,$$

and,

$$\dot{W}(\delta p_{i,i-1}) \leq -2\lambda_1 W(\delta p_{i,i-1}) + 2r^2.$$

It follows that

$$W(\delta p_{i,i-1}(t)) \leq \delta p_{i,i-1}(0) \exp(-2\lambda_1 t) + 2r^2.$$

Moreover,

$$||\delta p_{i,i-1}(t)||^2 \leq \beta_1(|\delta p_{i,i-1}(0)|,t) + \frac{\beta_2(|\delta p_{i,i-1}(0)|,t) + \delta}{\lambda_1},$$

with

$$\beta_1(|\delta p_{i,i-1}(0)|,t) = \frac{2r^2}{2\lambda_1} \exp(-2\lambda_1 t),$$

$$\beta_2(|\delta p_{i,i-1}(0)|,t) = \delta p_{i,i-1}(0) \exp(-2\lambda_1 t),$$

$$\delta = r^2.$$

The *Practical asymptotic string stability* of the system follows. □

5. Simulations

We proceed now to numerical simulations to validate our results. We consider a network of 5 vehicles with dynamics (1) in closed loop with (3) for each $i \in \{1,2,3,4,5\}$. The systems are interconnected in leader-follower scheme, *i.e.,* over a graph $\mathcal{G}$ presented in Fig 2.

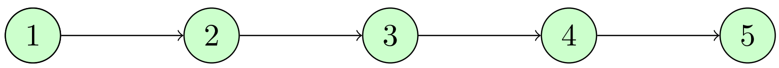

Figure 2. Interconnection graph $\mathcal{G}$ of the network

For $J(t)$, we generate a table following a standard normal distribution with random values. The noise generated affecting velocity in (1) represents the disturbances that vehicles can face, they can be due to road conditions . The vehicles are asked to follow a linear trajectory generated by a virtual leader, which is virtually interconnected with node 1. We set the velocities of the virtual leader at $v_0^x = 2$m/s and $v_0^y = 0$m/s. Concerning the physical and mechanical parameters of the vehicles, the mass of the vehicles is taken $m = 1000$ kg, which represents a typical value for a mid-sized car, gravity is taken $g = 9.81$ m/s², corresponding to the standard gravitational acceleration on Earth, the rolling resistance coefficient $Cr = 0.01$, the aerodynamic drag coefficient $Cd = 0.3$ is chosen to represent air resistance, taking into account the air density $\rho = 1.225$ kg/m³ and the frontal area of the vehicle $A = 2.5$ m². Finally, the lateral stiffness coefficient $Cf = 50000$ N/m and the lateral drag coefficient $Cy = 50000$ N/m are used to model the forces acting on the vehicle during lateral movements. These values allow for realistic simulation of the dynamic behavior of vehicles under various driving conditions.

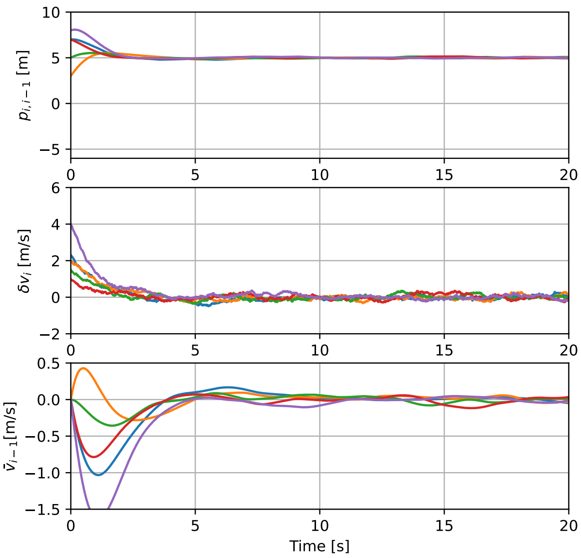

Figure 3. Trajectories of the intervehicular distance $p_{i,i-1}$, the velocity error δv_i and the estimation error $\bar{v}_{i-1}$ for $i \in \{1,2,3,4,5\}$.

The results in Fig 3 confirm part of our proposition that the trajectories of the closed-loop system are bounded. We can clearly see that the distance between vehicles is close to $p^* = 5$, and that the convergence errors δv_i and the estimation errors $\bar{v}_{i-1}$ are bounded around the origin.

In Fig 4, we observe that the Global *Practical asymptotic string stability*, given in Definition 2 is satisfied. In fact, the more λ_1 is important, the less the convergence error towards the desired inter-vehicular distance $\delta p_{i,i-1}$ for all $i \in \{1,2,3,4,5\}$.

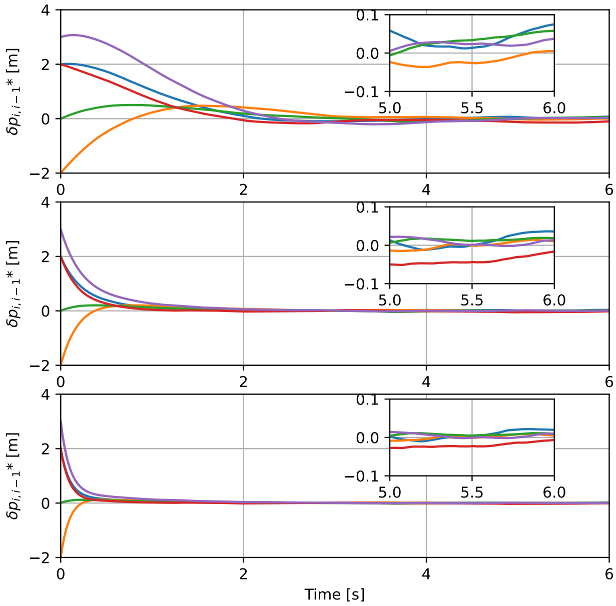

Figure 4. Trajectories of $\delta p_{i,i-1}$ for $\lambda_1 = 1$ (upper plot), $\lambda_1 = 5$ (middle plot) and $\lambda_1 = 10$ (bottom plot) for $i \in \{1,2,3,4,5\}$.

6. Conclusion

In this paper, we have introduced a novel approach to ensuring *Practical asymptotic string stability* in vehicular platoons. By designing a control law based on Backstepping and implementing a preceding vehicle velocity observer, we have demonstrated that the distance between neighboring vehicles can be stabilized to a desired value. Our control strategy effectively addresses the challenges posed by unknown leader velocities and external disturbances, ensuring that the platoon remains stable and safe under various conditions.

Through rigorous analysis, we have shown that the proposed control system is globally bounded and achieves Practical asymptotic string stability. The fixed-gain linear observer provides accurate estimations of the preceding vehicle's velocity, enhancing the robustness of the control design. This work contributes to the advancement of autonomous vehicle platooning by offering a reliable and efficient method for maintaining platoon stability.

Future research could explore the integration of more complex models and additional real-world constraints to further improve the performance and applicability of the proposed control system.

Conflicts of Interest

The authors declare no conflicts of interest.

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