

Research Article

Complex-Valued Flatness Grid-Feeding Controller for CSI-Based Grid-Tied Converters

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Abstract: This paper presents a non-linear controller for a grid-connected power electronics converter working in grid-following mode. The topology to be controlled consists of a current source inverter fed by a dc power source through an inductive DC-link, and connected to the grid via a capacitive-inductive output filter. This results in a system that allows converting the power from the input source into a controlled three-phase ac output current that is injected into the grid. To control this system, a complex-valued flatness-based strategy is designed. The performance of the proposed control system is illustrated through simulation results.

Keywords: grid-following inverter, current source inverter, complex-valued flatness controller, non-linear control

1. Introduction

When considering the design of the electrical grids currently used to generate, transmit, distribute, and consume electrical energy, a new paradigm has emerged in the 21st century that aims to improve environmental friendliness, process efficiency, and reliability in service delivery when compared to the standards imposed during much of the 20th century. This paradigm is named smart grid [1]. For this reason, design concepts and devices have emerged that are used daily in small and large electrical grids installed in the industrialized world. Among others, it is possible to mention smart meters, microgrids, flexible Alternating Current (AC) transmission systems, high voltage dc, and electric drives [2–5]. One device that is widely used, to the point that, for example, it has forced a reclassification of stability standards in electrical grids, is the Power Electronics Converter (PEC) [6].

The main characteristic of the PEC is that, unlike a classic transformer, it can be used to change the amplitude and frequency of the output voltage and current, with respect to those of the input. For this reason, it is used in the construction of equipment with one or more conversion stages, such as inverter-based resources, electric drives, and solid-state transformers, which can convert dc voltage or current signals into AC voltage or current signals, and vice versa.

Although the Voltage Source Inverter (VSI) (see, among others, [7] and several references therein) is the most commonly used topology, sometimes the Current Source Inverter (CSI) is preferred due to some of its advantages [8, 9]. This is why several researchers have proposed topologies including CSIs to be used in different applications. Among others, the use of the CSI operating in stand-alone mode and controlled using a PI passivity-based strategy was proposed in

[10]. The behavior of a three-phase sinusoidal voltage source consisting of a CSI with an output capacitor filter feeding a resistive load, controlled via a complex-valued sliding mode control, was studied in [11]. There, a sliding surface of relative degree 1 was proposed for controlling the conversion scheme. Due to this relative degree, a study is needed to ensure the stability of the zero-dynamics. An Inverter Based Resource (IBR), whose primary source is photovoltaic, using a CSI was presented in [12]. In this case, the proposed controller is linear. Another example is the one presented in [13], where the primary source is a small wind turbine. The CSI is also used to build drive trains for electric vehicles [14]. As with the VSI, the CSI has been used in back-to-back topologies [15–17].

As is the case for the VSI [18], there are topologies that use the CSI to build voltage and/or current sources that allow power to be injected into the grid [19]. From a control point of view, two types of algorithms are distinguished [20], which aim to set the behavior as a voltage source (grid-forming [21, 22]) or as a current source (grid-following [23, 24]). In turn, grid-following algorithms are usually subdivided into two classes. One subclass is the grid-feeding algorithm, where a current source is synchronized with the grid to inject power at the Point of Common Coupling (PCC). The other is grid-supporting algorithm, where the injected current is used to improve the grid power quality, for example, to build an active power filter [25, 26].

In order to generate a defined waveform at their output, in topologies that use the CSI, the switches are linked to the primary Power Source (PS) and to the grid through filters, composed of different combinations of inductors and/or capacitors [27, 28]. The topology resulting from this combination of switches and filters can be modeled by non-linear differential equations. For this reason, the use of non-linear control strategies is often the most suitable approach for controlling these systems when high performance and disturbance rejection capabilities are sought. These strategies are similar to those used in topologies containing a VSI [29, 30].

This paper proposes to design a non-linear grid-feeding control strategy based on flatness for a three-phase power electronic conversion system. From a DC PS and an inductive DC-link, this system obtains an AC current that is injected into the grid using a CSI-type switching stage and a capacitive-inductive (CL) output filter. As the three-phase system is modeled using complex variables, the selected flat output is a function of the complex energy, whose time derivative is the difference between the DC power of the input source and the complex power injected into the grid. The main contributions of the paper are twofold:

- It shows that the proposed conversion topology, consisting of a CSI, an inductive DC-link, and an output CL filter, controlled using a flatness-based nonlinear strategy, offers the advantage of ensuring stability and excellent performance when it is necessary to track rapidly varying references and reject abruptly varying disturbances, without restricting their magnitudes.
- The design considers the non-linear model of the system, and a non-linear controller is designed for the complete model. This strategy allows for excellent performance with a relatively smaller DC-link inductor than that required to achieve the same performance using classical control strategies, such as a cascade of decoupled control loops (one slow and one fast loop). This is illustrated through an example in the revised version of the manuscript.

In addition, the proposed strategy is implemented with a reduced number of sensors since two Reduced Order Observers (ROO) are designed for estimating the input power and the output current.

2. System's model

The schematic diagram of the system under study is shown in Figure 1a. The energy is supplied to the system by a voltage source which varies its value according to P_i/i_{dc} . This behavior models the series connection of the output converters of renewable PS, such as Photovoltaic (PV) panels, Wind Energy Conversion Systems (WECS), hydrogen fuel-cells, batteries, etc. These converters act as power sources, usually due to maximum power point tracking algorithms. The total input power P_i is then injected to the grid by the CSI in the form of current. This setup allows the injection of active power as it becomes available (when $P_i > 0$) and the injection of reactive power on demand.

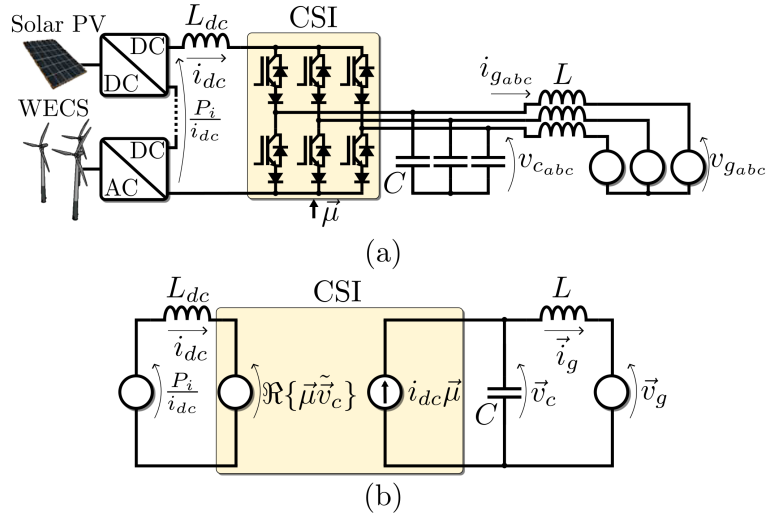


Figure 1. (a) Schematic diagram of the system; (b) Equivalent complex space-vector model

The system of Figure 1a can be modeled using complex space-vectors through the following set of differential equations

$$L_{dc}\dot{i}_{dc} = \frac{P_i}{i_{dc}} - \Re\{\vec{\mu}\vec{v}_c\}, \quad (1)$$

$$C\dot{\vec{v}}_c = i_{dc}\vec{\mu} - \vec{i}_g, \quad (2)$$

$$L\dot{\vec{i}}_g = \vec{v}_c - \vec{v}_g, \quad (3)$$

where $\Re\{\cdot\}$ is the real part of the variable, $\vec{\cdot}$ denotes a complex space-vector, and $\bar{\cdot}$ its complex conjugate. In this model $\vec{\mu}$ is a complex signal representing the modulation index. The block diagram representation of (1)–(3) is shown in Figure 1b. The complex space-vectors are obtained using the Clarke's power invariant transform: $\vec{x} = x_\alpha + jx_\beta = \sqrt{\frac{2}{3}}(x_a + x_b e^{j\frac{2\pi}{3}} + x_c e^{-j\frac{2\pi}{3}})$, where x_a , x_b and x_c are the phase components of variable “ x ” and $\vec{x}$ is the resulting complex space-vector. The original components can be recovered from the complex space-vector as follows: $x_a = \sqrt{\frac{2}{3}}\Re\{\vec{x}\}$, $x_b = \sqrt{\frac{2}{3}}\Re\{\vec{x}e^{-j\frac{2\pi}{3}}\}$ and $x_c = \sqrt{\frac{2}{3}}\Re\{\vec{x}e^{j\frac{2\pi}{3}}\}$. Additionally, if space vector modulation is used, then $|\vec{\mu}| \leq \sqrt{3}/2$ (see [31]). It is assumed that i_{dc} , $\vec{v}_c$ and $\vec{v}_g$ are measured, while P_i and $\vec{i}_g$ will be obtained using observers.

3. Full Feedback Linearization (FFL)

3.1 Selection of the flat output

It is proposed to linearize the system by means of the non-linear transformation known as Feedback Linearization [32, 33]. For a single input system such as the one under study ($\vec{\mu}$ is the only control input), the transformation requires defining a new variable $\vec{z}$, usually a non-linear function of the original states. This variable can be used, along with its time derivatives, to fully describe the trajectories of the input and the states of the system, and is known as a flat output. To be a flat output, the time derivatives of $\vec{z}$ up to the order of the original system must not be a function of the control input. Then, the set of time derivatives of $\vec{z}$ define a linear system that completely describes the dynamics of the original system,

with no hidden zero dynamics. This is known as Full Feedback Linearization (FFL). In this work it is proposed to use the complex energy of the system as the output, which, as will be shown below, is a flat output and results in the FFL of the system. Let $\vec{\xi}_1$ be the complex energy of the system

$$\vec{\xi}_1 = \frac{1}{2}(L_{dc}\dot{i}_{dc}^2 + C\vec{v}_c\dot{\vec{v}}_c + L\vec{i}_g\dot{\vec{i}}_g) + j\eta, \quad (4)$$

$$\dot{\eta} = \Im\{\vec{i}_g\dot{\vec{v}}_g\} = q, \quad (5)$$

where $\Im\{\cdot\}$ denotes imaginary part. Note that η may be divergent, for example, for constant q . This issue can be solved in the controller by computing $e_\eta = \eta - \eta^* = \int(q - q^*)dt$, where η^* and q^* are the references of η and q , respectively. If $q \rightarrow q^*$ then e_η is bounded. Differentiating (4) with respect to time three times and using (1)–(3), yields

$$\dot{\vec{\xi}}_1 = P_i - \vec{i}_g\dot{\vec{v}}_g = \vec{\xi}_2, \quad (6)$$

$$\dot{\vec{\xi}}_2 = \dot{P}_i - \left[\frac{\dot{\vec{v}}_g}{L}(\vec{v}_c - \vec{v}_g) + \vec{i}_g\ddot{\vec{v}}_g \right] = \vec{\xi}_3, \quad (7)$$

$$\dot{\vec{\xi}}_3 = a\ddot{\vec{\mu}} + b = \vec{r}, \quad (8)$$

where

$$a = -\frac{\dot{\vec{v}}_g i_{dc}}{LC}, \quad (9)$$

$$b = \dot{P}_i - 2\frac{\dot{\vec{v}}_g}{L}(\vec{v}_c - \vec{v}_g) + \frac{\dot{\vec{v}}_g\vec{i}_g}{LC} + \frac{\dot{\vec{v}}_g\dot{\vec{v}}_g}{L} - \vec{i}_g\ddot{\vec{v}}_g, \quad (10)$$

and $\vec{r}$ is the control action of the system described in the new variables $\vec{\xi}_1$ – $\vec{\xi}_3$. Since the control action $\ddot{\vec{\mu}}$ of the system in the original variables is present after differentiating the output $\vec{\xi}_1$ three times with respect to time, and the original system is of third order, it follows that $\vec{\xi}_1$ is a flat output, and the proposed non-linear transformation is a FFL of the system. It is now possible to implement a linear controller for the system (6)–(8) and obtain the auxiliary control action $\vec{r}$.

Once $\vec{r}$ has been computed, the control action on the original variables can be obtained from (8) by

$$\ddot{\vec{\mu}} = (\vec{r} - b)/a. \quad (11)$$

Notice that (10) requires the time derivatives of $\vec{v}_g$. Assuming $\vec{v}_g$ is a positive sequence signal with angular frequency ω , these derivatives can be approximated by

$$\dot{\vec{v}}_g = j\omega\vec{v}_g, \quad (12)$$

$$\ddot{\vec{v}}_g = j\omega \dot{\vec{v}}_g. \quad (13)$$

These approximations assume that $|\vec{v}_g|$ is constant. In the event of a magnitude variation, the resulting error will be compensated by the controller without significant deviations, as will be shown in the simulations results.

3.2 Feedback law in transformed coordinates

A linear Full State Feedback (FSF) law is used for controlling the new output defined in the transformed coordinates. This is implemented by

$$\vec{r} = -\vec{k}_1 \vec{e}_{\xi_1} - \vec{k}_2 \vec{e}_{\xi_2} - \vec{k}_3 \vec{e}_{\xi_3}, \quad (14)$$

where $\vec{k}_1, \vec{k}_2$ and $\vec{k}_3 \in \mathbb{C}$ are constants that define the dynamics of the error, $\vec{e}_{\xi_1} = \vec{\xi}_1 - \vec{\xi}_1^*$, $\vec{e}_{\xi_2} = \vec{\xi}_2 - \vec{\xi}_2^*$, $\vec{e}_{\xi_3} = \vec{\xi}_3 - \vec{\xi}_3^*$, with $\vec{\xi}_1^*, \vec{\xi}_2^*$ and $\vec{\xi}_3^*$ the references of $\vec{\xi}_1, \vec{\xi}_2$ and $\vec{\xi}_3$, respectively. These references are obtained in the next section. Combining (14) with (6)–(8), the error dynamics result

$$\begin{bmatrix} \dot{\vec{e}}_{\xi_1} \\ \dot{\vec{e}}_{\xi_2} \\ \dot{\vec{e}}_{\xi_3} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\vec{k}_1 & -\vec{k}_2 & -\vec{k}_3 \end{bmatrix}}_A \begin{bmatrix} \vec{e}_{\xi_1} \\ \vec{e}_{\xi_2} \\ \vec{e}_{\xi_3} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \dot{\vec{\xi}}_3^* \quad (15)$$

where $\dot{\vec{\xi}}_3^*$ is a disturbance. The gains can be chosen to position the eigenvalues of A to achieve the desired dynamics using any linear control placement technique. For instance, $K = \text{place}(A1, B1, \text{pole_locations})$ in MATLAB will yield the vector of gains $K = [\vec{k}_1 \ \vec{k}_2 \ \vec{k}_3]$, where $A1 = [0 \ 1 \ 0; 0 \ 0 \ 1; 0 \ 0 \ 0]$, $B1 = [0; 0; 1]$ and pole_locations is a vector of the desired poles locations.

3.3 Reference signals

Let q^* be an arbitrary reference of q [defined in (5)]. If it is considered that in steady state the active power balance must be zero, and the reactive power must converge to q^* , from (6), it follows

$$\vec{i}_g^* = \frac{P_i - j q^*}{\vec{v}_g}. \quad (16)$$

Applying the same reasoning to (7), but in this case equating the imaginary part to q^* yields

$$\vec{v}_c^* = L \frac{\dot{P}_i - \vec{i}_g^* \dot{\vec{v}}_g - j q^*}{\vec{v}_g} + \vec{v}_g. \quad (17)$$

Then, the references $\vec{\xi}_1^* - \vec{\xi}_3^*$ are obtained by replacing the variables in (4)–(7) by their reference values:

$$\vec{\xi}_1^* = \frac{1}{2} (L_{dc} \vec{i}_{dc}^{*2} + C \vec{v}_c^* \vec{v}_c^* + L \vec{i}_g^* \vec{i}_g^*) + j \eta^*, \quad (18)$$

$$\dot{\eta}^* = q^*, \quad (19)$$

$$\vec{\xi}_2^* = P_i - \vec{i}_g^* \vec{v}_g, \quad (20)$$

$$\vec{\xi}_3^* = \dot{P}_i - \left[\frac{\vec{v}_g^*}{L} (\vec{v}_c^* - \vec{v}_g) + \vec{i}_g^* \dot{\vec{v}}_g \right], \quad (21)$$

where i_{dc}^* is an arbitrary reference to i_{dc} .

3.4 Output current observer

The measurements of $\vec{v}_c$ and $\vec{v}_g$ can be used to estimate the output current. Since these measurements have good Signal to Noise Ratio (SNR), since their magnitudes vary relatively little around their nominal values, a ROO is proposed to estimate $\vec{i}_g$. From (2) the following dynamics for the estimate of $\vec{v}_c$ is proposed

$$C\dot{\vec{v}}_c = i_{dc}\vec{\mu} - \hat{\vec{i}}_g, \quad (22)$$

where $\hat{\vec{i}}_g$ is an estimate of $\vec{i}_g$. Then, from (3) the dynamics of $\hat{\vec{i}}_g$ are computed as follows

$$L\dot{\hat{\vec{i}}}_g = \vec{v}_c - \vec{v}_g - gLC(\dot{\vec{v}}_c - \dot{\hat{\vec{v}}}_c) = \vec{v}_c - \vec{v}_g + gL(\vec{i}_g - \hat{\vec{i}}_g), \quad (23)$$

where (2) and (22) were used to obtain the last equality of (23), and $g \in \mathbb{R}$ is a constant that defines the dynamics of the observer. From (3) and (23) the error dynamics result

$$\dot{\vec{e}}_{i_g} = -g\vec{e}_{i_g}, \quad (24)$$

from which $g = 4.6/\tau$, with τ the desired settling time.

To avoid computing the time derivative of $\vec{v}_c$, the following auxiliary variable is defined by adding $gLC\dot{\vec{v}}_c$ to the left and right sides of the first equality of (23), and replacing $C\dot{\vec{v}}_c$ with (22):

$$\dot{\vec{e}} = L\dot{\hat{\vec{i}}}_g + gLC\dot{\vec{v}}_c = \vec{v}_c - \vec{v}_g + gL(i_{dc}\vec{\mu} - \hat{\vec{i}}_g). \quad (25)$$

The estimate $\hat{\vec{i}}_g$ can be obtained integrating both sides of the first equality of this equation. The ROO is summarized below

$$\hat{\vec{i}}_g = (\vec{e} - gLC\vec{v}_c)/L, \quad (26)$$

$$\dot{\vec{e}} = \vec{v}_c - \vec{v}_g + gL(i_{dc}\vec{\mu} - \hat{\vec{i}}_g). \quad (27)$$

3.5 Input power observer

The input power P_i can be estimated from the measurements of i_{dc} and $\vec{v}_c$. As stated before, $\vec{v}_c$ has a good SNR. Additionally, in a CSI application, i_{dc} is kept around a fixed reference value, and therefore it can be assumed it also has a good SNR. Therefore, a ROO can be used here as well. Assuming that the input power can vary between constant values with constant slope, the signal model is given by

$$\dot{P}_i = m, \quad (28)$$

$$\dot{m} = 0. \quad (29)$$

Defining the energy in L_{dc} as

$$E_{dc} = \frac{1}{2} L_{dc} i_{dc}^2, \quad (30)$$

computing its time derivative, and using (1) yields

$$\dot{E}_{dc} = P_i - i_{dc} \Re\{\vec{\mu} \tilde{\vec{v}}_c\}. \quad (31)$$

Then, using this equation, the following dynamics is proposed to estimate this energy

$$\dot{\hat{E}}_{dc} = \hat{P}_i - i_{dc} \Re\{\vec{\mu} \tilde{\vec{v}}_c\}, \quad (32)$$

where $\hat{P}_i$ is an estimate of P_i . Now, using (28)–(29) the dynamics of $\hat{P}_i$ and $\hat{m}$ are computed as follows

$$\dot{\hat{P}}_i = \hat{m} + \lambda_1 (\dot{E}_{dc} - \dot{\hat{E}}_{dc}) = \hat{m} - \lambda_1 (P_i - \hat{P}_i), \quad (33)$$

$$\dot{\hat{m}} = \lambda_2 (\dot{E}_{dc} - \dot{\hat{E}}_{dc}) = \lambda_2 (P_i - \hat{P}_i), \quad (34)$$

where (31)–(32) were used in the last equality of (33)–(34), and λ_1 and $\lambda_2 \in \mathbb{R}$ are constants that define the dynamics of the observer.

Subtracting (33)–(34) from (28)–(29), the error dynamics result

$$\underbrace{\begin{bmatrix} \dot{e}_{P_i} \\ \dot{e}_m \end{bmatrix}}_{A_o} = \underbrace{\begin{bmatrix} -\lambda_1 & 1 \\ -\lambda_2 & 0 \end{bmatrix}}_{A_o} \begin{bmatrix} e_{P_i} \\ e_m \end{bmatrix}, \quad (35)$$

where $e_{P_i} = P_i - \hat{P}_i$, $e_m = m - \hat{m}$ and λ_1 and λ_2 can be selected equating the characteristic equation of A_o ($s^2 + \lambda_1 s + \lambda_2$) to one with the desired observer dynamics.

To avoid computing the time derivative of E_{dc} , the following auxiliary variables are defined by subtracting $\lambda_1 E_{dc}$ from both sides of the first equality in (33), subtracting $\lambda_2 \dot{E}_{dc}$ from both sides of the first equality in (34), and replacing $\dot{E}_{dc}$ with (32):

$$\psi_1 = \dot{\hat{P}}_i - \lambda_1 \dot{E}_{dc} = \dot{m} - \lambda_1 (\dot{\hat{P}}_i - i_{dc} \Re\{\bar{\mu} \tilde{v}_c\}) \quad (36)$$

$$\psi_2 = \dot{\hat{m}} - \lambda_2 \dot{E}_{dc} = -\lambda_2 (\dot{\hat{P}}_i - i_{dc} \Re\{\bar{\mu} \tilde{v}_c\}). \quad (37)$$

The estimates of P_i and m can be obtained integrating both sides of the first equalities in (36) and (37), respectively. The ROO is summarized below

$$\hat{P}_i = \psi_1 + \lambda_1 E_{dc}, \quad (38)$$

$$\hat{m} = \psi_2 + \lambda_2 E_{dc}, \quad (39)$$

$$\dot{\psi}_1 = \dot{\hat{m}} - \lambda_1 (\dot{\hat{P}}_i - i_{dc} \Re\{\bar{\mu} \tilde{v}_c\}) \quad (40)$$

$$\dot{\psi}_2 = -\lambda_2 (\dot{\hat{P}}_i - i_{dc} \Re\{\bar{\mu} \tilde{v}_c\}). \quad (41)$$

4. Double loop control

It is well known that linear double-loop cascade control is most often used (see, among others [34–37], and several references in those papers) to control these types of systems and their dual (capacitive DC-link plus VSI). In such cases, the design considers the dynamics of the inner and outer loops to be decoupled. The inner loop is made fast, and its reference is set by the outer loop, which must be slow to satisfy the decoupling condition. By slowing the outer loop, which in this case controls i_{dc} , it is necessary to use an inductor L_{dc} of a much larger value than that required if the proposed non-linear strategy, which considers the entire system dynamics, is used. To compare the performance of the proposed control against a classical control, this section briefly introduces a double-loop control strategy. This strategy consists of a fast control loop for $\vec{i}_g$ and a slow control loop for i_{dc} . The slow control loop will generate the reference for the fast control loop.

The following fast control loop for $\vec{i}_g$ is proposed:

$$\bar{\mu} = (-\bar{\gamma}_1 \vec{v}_c - \bar{\gamma}_2 \vec{i}_g - \bar{\gamma}_3 \vec{x}_r) / i_{dc}, \quad (42)$$

$$\dot{\vec{x}}_r = j\omega \vec{x}_r + \vec{i}_g - \vec{i}_g^*, \quad (43)$$

where $\bar{\gamma}_1$, $\bar{\gamma}_2$, and $\bar{\gamma}_3$ are complex gains, and $\vec{x}_r$ is a complex resonant state that allows tracking the reference $\vec{i}_g^*$ and rejecting the disturbance $\vec{v}_g$. The gains can be computed from the dynamic equation of the open loop system (2), (3) and (43):

$$\begin{bmatrix} \dot{\vec{v}}_c \\ \dot{\vec{i}}_g \\ \dot{\vec{x}}_r \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -1/C & 0 \\ 1/L & 0 & 0 \\ 0 & 1 & j\omega \end{bmatrix}}_{A_2} \begin{bmatrix} \vec{v}_c \\ \vec{i}_g \\ \vec{x}_r \end{bmatrix} + \underbrace{\begin{bmatrix} 1/C \\ 0 \\ 0 \end{bmatrix}}_{B_2} \vec{u}, \quad (44)$$

where $\vec{u} = i_{dc}\vec{\mu}$ is considered as the control input. Gains can be computed using, $\text{gamma} = \text{place}(A_2, B_2, \text{pole_locations})$ in MATLAB, where $\text{gamma} = [\vec{\gamma}_1 \ \vec{\gamma}_2 \ \vec{\gamma}_3]$, and pole_locations is a vector of the desired poles locations.

The following slow control loop is proposed for i_{dc} :

$$v_{CSI} = \frac{P_i}{i_{dc}} + \delta_1 i_{dc} + \delta_2 x_{dc}, \quad (45)$$

$$\dot{x}_{dc} = i_{dc} - i_{dc}^*, \quad (46)$$

where δ_1 and δ_2 are real gains, x_{dc} is an integral state, added to track i_{dc}^* in steady-state, and v_{CSI} is the control action of the controller, which represents the voltage that the CSI must impose at its dc side. In (1), this voltage would be $v_{CSI} = \Re\{\vec{\mu}\vec{v}_c\}$. However, since the slow control loop cannot be used to directly compute $\vec{\mu}$, it is related to the reference current for the fast control loop through power balance:

$$\vec{i}_g^* = (v_{CSI}i_{dc} - jq^*) \frac{\vec{v}_g}{|\vec{v}_g|^2}, \quad (47)$$

that is, the power drained by this voltage must be the active power injected to the grid. The gains for the slow loop can be computed from the dynamic equations of the open loop system (1) and (46):

$$\begin{bmatrix} \dot{i}_{dc} \\ \dot{x}_{dc} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}}_{A_3} \begin{bmatrix} i_{dc} \\ x_{dc} \end{bmatrix} + \underbrace{\begin{bmatrix} 1/L_{dc} \\ 0 \end{bmatrix}}_{B_3} v, \quad (48)$$

where $v = P_i/i_{dc} - v_{CSI}$ is considered as the control input. Gains can be computed using, $\text{delta} = \text{place}(A_3, B_3, \text{pole_locations})$ in MATLAB, where $\text{delta} = [\delta_1 \ \delta_2]$, and pole_locations is a vector of the desired poles locations. Note that the poles of the slow loop must be between 5 and 10 times slower than the poles of the fast control loop.

5. Simulation results

Simulation results are presented below. The system parameters are as follows: Rated power 2333 kVA, rated voltage 110 Vrms (rated $|\vec{v}_g| = 191$ V), rated current 7.07 Arms (rated $|\vec{i}_g| = 12.2$ A), rated angular frequency $2\pi 50$ rad/s, $L_{dc} = 10$ mH, $C = 22 \mu\text{F}$, $L = 2$ mH. The controller gains are calculated to obtain three real poles with settling times of 2, 4 and 6 ms, respectively, resulting in $\vec{k}_1 = 2028 \times 10^6$, $\vec{k}_2 = 5290 \times 10^3$ and $\vec{k}_3 = 4216$. The reference i_{dc}^* is set to 12 A. Additionally, the grid current observer is given a settling time of 2 ms, resulting in $g = 2300$, and the input power observer is given a settling time of 1 ms with optimal damping 0.707, resulting in $\lambda_1 = 9200$ and $\lambda_2 = 42.33 \times 10^6$.

The simulation starts with $P_i = 0$, $\vec{i}_g = 0$ and $\vec{v}_c = \vec{v}_g$, with $\vec{v}_g$ a positive sequence sinusoidal signal and nominal voltage and frequency. At $t = 20$ ms P_i is increased to $2333/\sqrt{2}$ W. As shown in Figure 2a, the power injected to $\vec{v}_g$ (shown in Figure 3), $p = \Re\{\vec{v}_g \vec{i}_g^*\}$, is practically equal to P_i , and the current $\vec{i}_g$ (Figure 4a) increases to $12.2/\sqrt{2}$ A. During the P_i transient, Figure 2b shows that i_{dc} deviates slightly from its reference value. This is due to the approximation made when obtaining the references. However, the deviation is of low magnitude and is quickly corrected with the dynamics established for the control.

Then, at $t = 80$ ms, q is increased to $2333/\sqrt{2}$ Var, bringing the system to nominal apparent power, as shown by the current $\vec{i}_g$ in Figure 4a, which reaches its nominal value. The reactive is then brought back to zero at $t = 140$ ms. As can be seen in Figure 2b, the transients in i_{dc} are of similar magnitude in both cases.

Afterwards, between $t = 200$ ms and $t = 260$ ms the magnitude of $\vec{v}_g$ is reduced to 70% of its nominal value, as shown in Figure 3a. As can be seen in Figure 2b and Figure 4a, this results in transients in i_{dc} and $\vec{i}_g$ similar to those observed in the previous events.

Additionally, Figure 3b and Figure 4b show $\vec{v}_c$ and $\vec{\mu}$. From these figures it is seen that throughout the simulation $\vec{v}_c$ is virtually equal to $\vec{v}_g$, while $|\vec{\mu}| < \sqrt{3}/2$, showing that the system does not saturate despite the fast dynamic response. The results shown validate the performance of the proposed control system, which is able to quickly follow large excursions of the reference q^* , as well as reject large excursions of the disturbances (P_i and $\vec{v}_g$ in this case).

Figure 5 shows the performance of the proposed grid current observer. As can be seen in Figure 5a–b, the actual and estimated currents are indistinguishable from one another, which is further verified by the error signals in Figure 5c–d. Note that the error is zero because the simulated system perfectly matches the model used in the observer development, where no assumptions were made regarding the dynamics of the involved variables, which are all measured. The performance of the input power observer is shown in Figure 6. The observer performs very well, with only a small error during the input power transient. Note that unlike the current observer, the power observer does exhibit error during transients. This is due to the dynamic model chosen to describe the input power, defined by (28) and (29). These results validate the performance of the proposed observers.

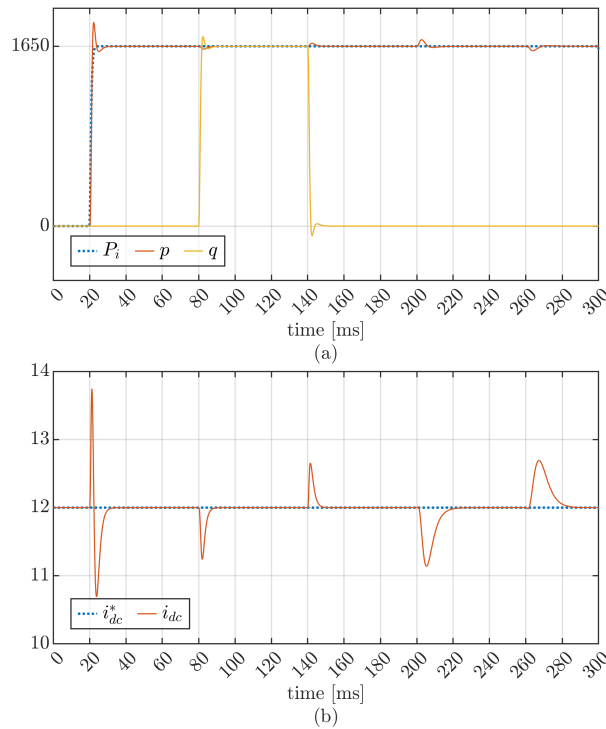


Figure 2. Simulation results (a) P_i [W], p [W] and q [Var]; (b) i_{dc} , i_{dc}^* [A]

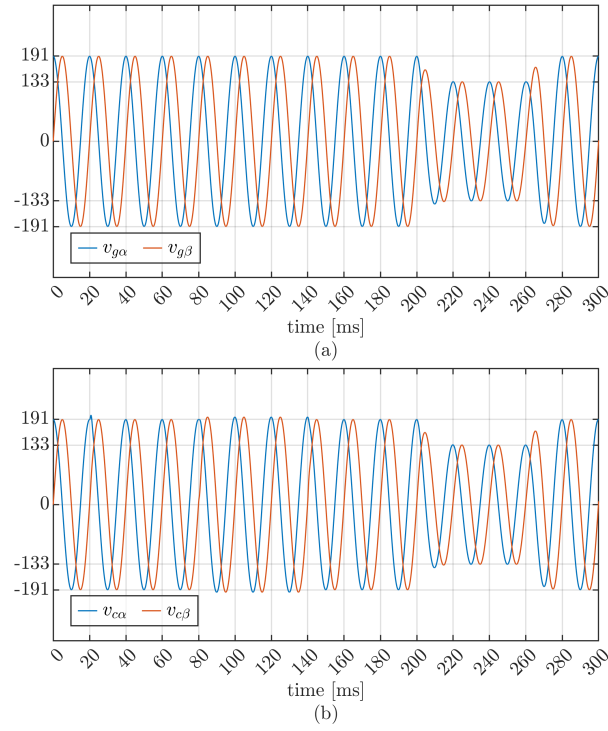


Figure 3. Simulation results (a) $\vec{v}_g = v_{g\alpha} + jv_{g\beta}$ [V]; (b) $\vec{v}_c = v_{c\alpha} + jv_{c\beta}$ [V]

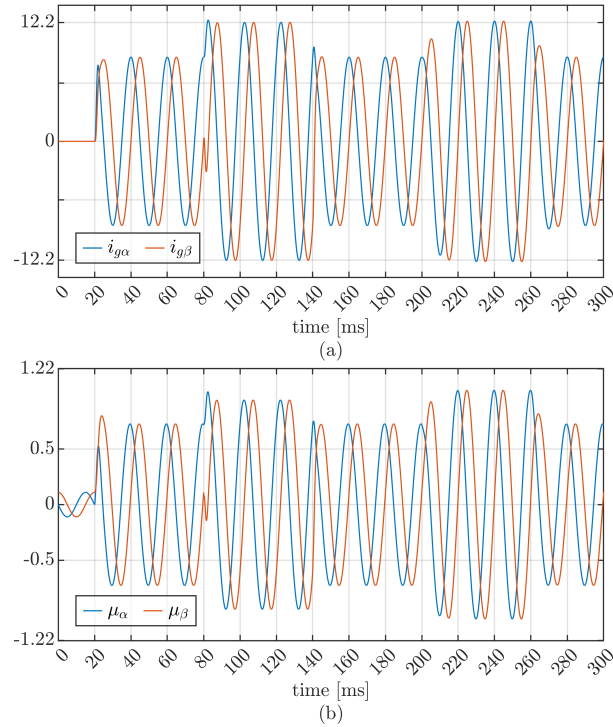


Figure 4. Simulation results (a) $\vec{i}_g = i_{g\alpha} + ji_{g\beta}$ [A]; (b) $\vec{\mu} = \mu_\alpha + j\mu_\beta$

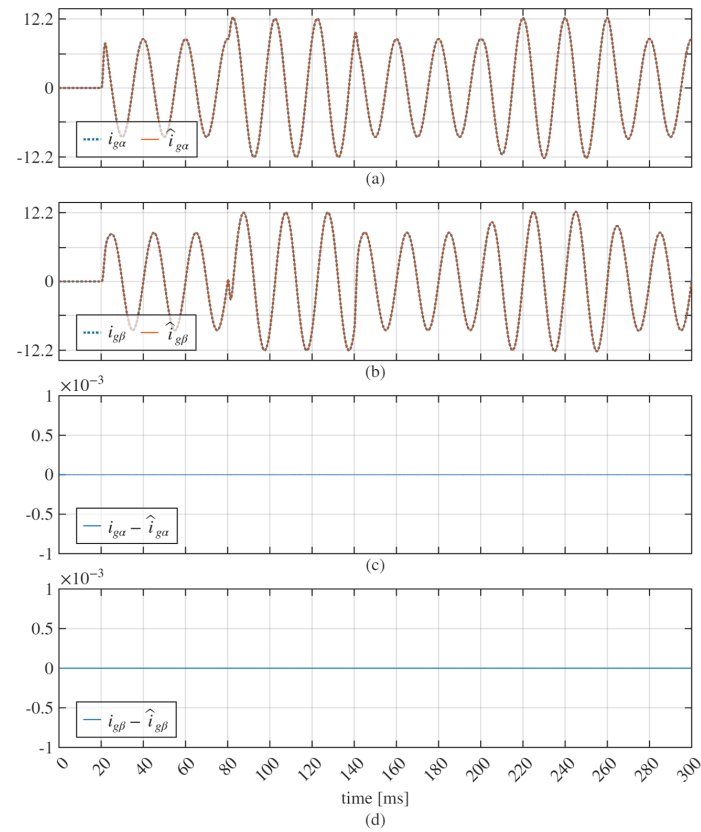


Figure 5. Simulation results (a) $i_{g\alpha}$ and $\hat{i}_{g\alpha}$ [A]; (b) $i_{g\beta}$ and $\hat{i}_{g\beta}$ [A]; (c) Error signal $i_{g\alpha} - \hat{i}_{g\alpha}$ [A]; (d) Error signal $i_{g\beta} - \hat{i}_{g\beta}$ [A]

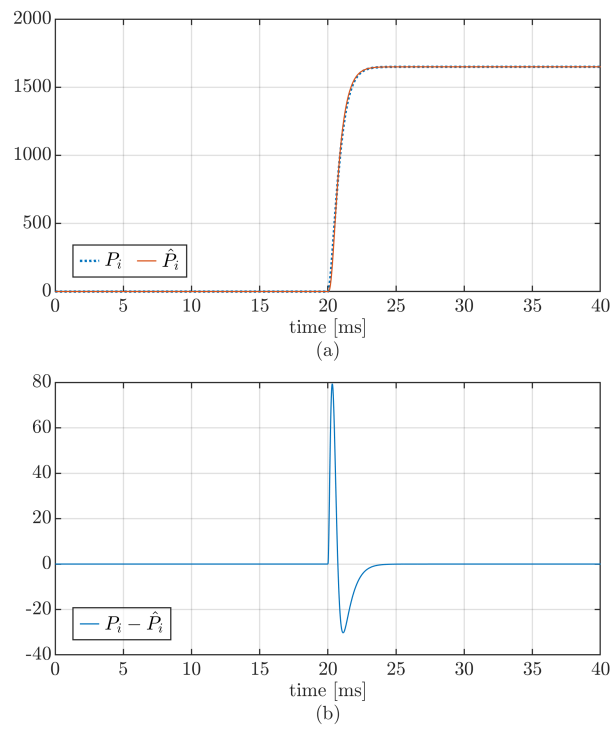


Figure 6. Simulation results (a) P_i and $\hat{P}_i$ [W]; (b) Error signal $P_i - \hat{P}_i$ [W]

To compare the proposed non-linear control strategy, the dual-loop control described by (42), (43), and (45)–(47) is simulated. In this case, no observers are used, measuring all signals instead. The system's parameters are those described at the beginning of this section. However, given the slowness of the control loop of i_{dc} , it was necessary to increase L_{dc} to 50 mH (5 times). The fast-loop poles were chosen to be the same as those of the proposed control, with settling times of 2, 4, and 6 ms, resulting in $\bar{\gamma}_1 = 0.101 + j0.00694$, $\bar{\gamma}_2 = -0.718 + j0.0638$, and $\bar{\gamma}_3 = 99.4 + j88.4$. For the slow loop, a pair of complex conjugate poles 5 times slower was chosen, with a settling time of 30 ms and damping of 0.7, resulting in $\delta_1 = 15.33$ and $\delta_2 = 2399$. The dual-loop control was simulated under the same conditions as the proposed control, and the results are shown in Figure 7. As can be seen, the performance of the fast loop is similar to that of the proposed control, achieving rapid injection of the reference active and reactive power. However, given the slowness required in the slow loop to decouple the dynamics, and despite having considerably increased L_{dc} , the variations in i_{dc} shown in Figure 7b are of greater amplitude than in the proposed control, shown in Figure 2b. These results demonstrate the significant improvement of the non-linear proposal, which allows for a significant reduction in the size of the DC link inductor.

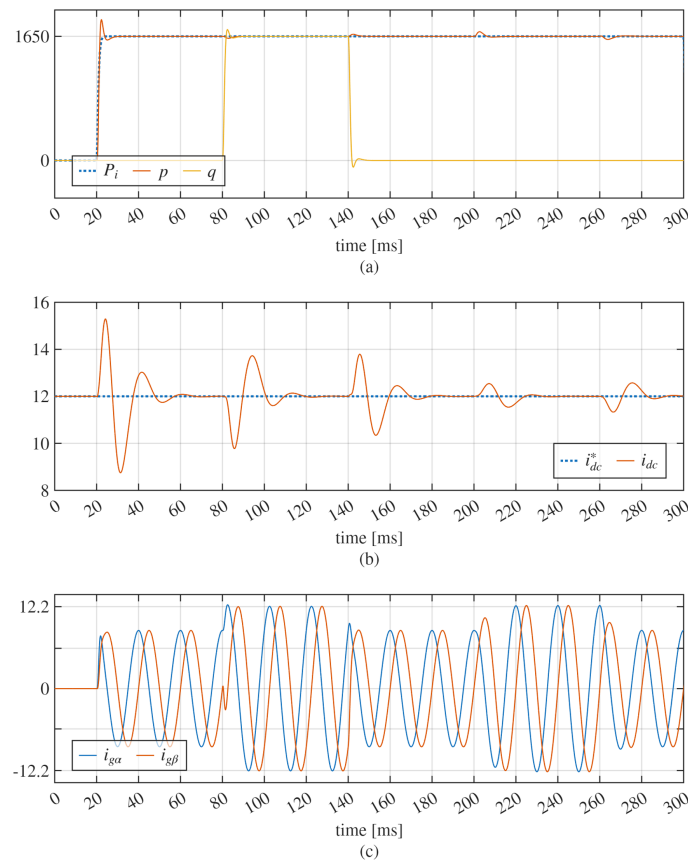


Figure 7. Simulation of double loop control (a) P_i [W], p [W] and q [VAr]; (b) i_{dc} , i_{dc}^* [A]; (c) $\vec{i}_g = i_{ga} + ji_{gb}$ [A]

6. Conclusions

In this paper, a complex-valued flatness-based control strategy was presented for a grid-feeding power injection system. The topology studied is made up of a primary power source, a CSI and a capacitive-inductive output filter. Because the system is modeled using complex-value states, the flat output selected to obtain the full input-output feedback linearization turns out to be the complex instantaneous energy. Then, the complex instantaneous power (active plus reactive) injected into the grid is controlled by designing a linear full-state feedback law at the auxiliary input. In addition, two reduced order

observers are proposed to estimate the input power and output current, reducing the total number of required sensors. The proposed non-linear controller is designed taking the non-linear nature of the system into consideration. This approach offers the advantage of ensuring stability and excellent performance when it is necessary to track rapidly varying references and reject abruptly varying disturbances, without restricting their magnitudes. In addition, the proposed strategy allows for excellent performance with a relatively small DC-link inductor when compared to the one required to achieve the same performance using classical control strategies, such as a cascade of decoupled control loops (one slow and one fast loop), as was illustrated through simulations. The behavior of the proposed control system is tested through simulation. The results show the good performance of the proposed grid power injection control scheme.

Conflicts of interest

The authors declare no conflicts of interest.

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