

Research Article

Comparative Evaluation of PI, PR, and LQR Controllers for Decentralized Primary Control of Islanded AC Microgrids

Md Masudur Rahman^{ID}, Nezar M. Alyazidi^{*ID}

Department of Control and Instrumentation Engineering, King Fahd University of Petroleum and Minerals (KFUPM), Dhahran 31261, Saudi Arabi

* Correspondence: nalyazidi@kfupm.edu.sa

Received: 21 March 2026; **Revised:** 24 May 2026; **Accepted:** 5 June 2026; **Published:** 29 June 2026

Abstract: With the increasing integration of power electronics and renewable energy sources in modern power systems, an efficient control system is critical to ensure stable operation in the islanded mode of a microgrid. This paper presents a comparative evaluation of various control strategies for decentralized primary control of an islanded Alternating Current (AC) microgrid. The study investigates Proportional-Integral (PI), Proportional-Resonant (PR), and Linear Quadratic Regulator (LQR) controllers implemented in single-loop and dual-loop voltage/current control schemes. Two microgrid configurations are considered: a single three-phase inverter and a parallel dual-inverter system, both with an inductor-capacitor (LC) output filter and both subjected to time-varying load disturbances. This paper analyzes 90 control configurations for each inverter case and determines the most effective controller design in terms of various performance criteria. For Case 1 (single-inverter), the best-ranked configuration employs a PI controller in the voltage loop with high gains and a PI controller in the current loop with low gains, achieving a voltage deviation Root-Mean-Square Error (RMSE) of 2.76 V, a frequency deviation RMSE of 2.70×10^{-4} Hz, and a settling time of 40 ms. For Case 2 (dual-inverter), the optimal configuration employs a PR controller in the voltage loop and a PI controller in the current loop with low gains, achieving a voltage deviation RMSE of 2.90 V, a frequency deviation RMSE of 2.50×10^{-4} Hz, and a settling time of 24.85 ms. Overall, these findings provide practical recommendations for selecting the appropriate control combination and its gain settings, and they can serve as a foundation for future supervisory control developments.

Keywords: islanded microgrid, primary control, Proportional-Integral (PI) controller, Proportional-Resonant (PR) controller, Linear Quadratic Regulator (LQR) controller, dual-loop control, voltage regulation, frequency regulation, total harmonic distortion

1. Introduction

Microgrids, which integrate advanced control systems, efficient power converters, and renewable energy sources, have become the primary component of the modern power system. They have become essential for fulfilling the exponentially increasing power demand, reducing global warming, and enhancing power quality, sustainability, reliability, environmental friendliness, and economy [1]. Figure 1 depicts a typical microgrid structure, including energy resources, storage units, loads, and utility-grid interaction.

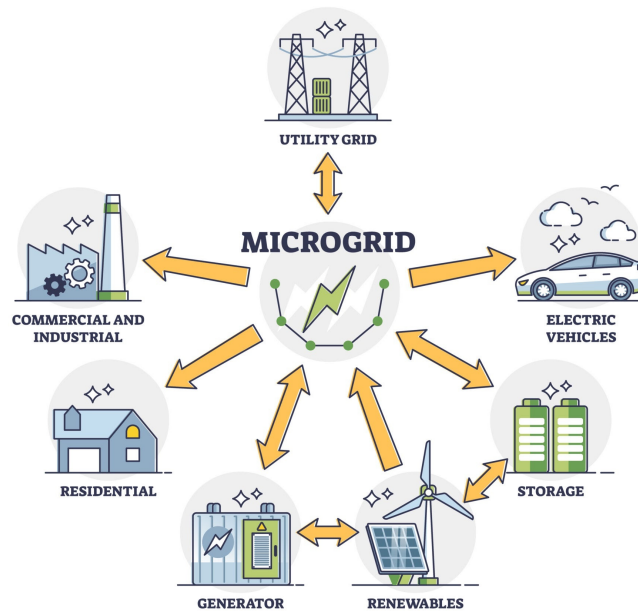


Figure 1. General architecture of a microgrid

Microgrids can be classified as Alternating Current (AC), Direct Current (DC), and hybrid AC/DC microgrids. AC microgrids are employed when the loads are primarily AC, and a grid connection is required. They need DC-DC and DC-AC converters [2]. When the loads are primarily DC, and connection to the main grid is not necessary, the DC microgrids are used [3]. Hybrid AC/DC microgrids are employed when the system must supply both AC and DC loads and is connected to the main grid [4].

Microgrids may operate in either islanded or grid-connected modes depending on the loads and the power generation situation [5]. In the grid-connected mode, the main grid controls the voltage and frequency, and the primary focus of control is the regulation of the grid current. In the islanded mode, there is no main grid. So, the primary control focus is the regulation of the system voltage and frequency [2].

Voltage control in islanded microgrids is commonly achieved by Voltage Source Inverters (VSIs) connected with inductor-capacitor (LC) or inductor-capacitor-inductor (LCL) filters. The control of these inverters has been extensively studied using various linear control techniques. Proportional Integral (PI) controller, Proportional-Resonant (PR) controller, and Linear Quadratic Regulator (LQR) are the most common ones. The performance measuring criteria can be reference tracking capability, Root-Mean-Square Errors (RMSE) in voltage and frequency deviation, Total Harmonic Distortion (THD), and transient responses (settling time, rise time, overshoot, and undershoot) against time-varying loads [5, 6].

Microgrid control architectures are generally classified as follows. In the centralized control, one controller is used to control all the converters or the Distributed Generation (DG) units. In the decentralized control, each converter or DG unit uses its own controller, and they take local load measurements, but there is no communication among them. In the distributed control, each converter or DG unit uses its own controller, and there is communication among them [2, 6].

Microgrid control is typically organized in a hierarchical structure, consisting of three levels as follows. (i) Primary control operates fast, and it is responsible for regulating voltage and frequency. (ii) Secondary control operates at moderate speed to restore frequency and voltage to their nominal values. (iii) Tertiary control manages economic dispatch and optimization over longer time period [2, 6].

The paper focuses on decentralized primary control of an islanded AC microgrid. Adaptive gain supervisory controllers offer improved performance over a wide range of operating conditions. However, a systematic understanding of how gain variations affect system behavior is a prerequisite for designing such supervisory controllers [7, 8]. Therefore, different

linear controller configurations are examined in this paper to identify the most suitable control scheme for an islanded AC microgrid under time-varying load conditions. Before the analysis, the related previous works are investigated in the following.

1.1 Literature review

The control of islanded AC microgrids has been studied extensively in recent years. Various linear controllers (PI, PR, LQR) and their improved designs were proposed to get the best results in different performance criteria. Some of the studies are listed as follows.

1.1.1 PI controller-based approaches

PI controllers are the most widely used controller in microgrid applications. A control scheme with PI controllers in the primary layer was established by [2] and [9] demonstrated the effectiveness of PI control strategies. An adaptive gain scheduling for PI controllers was examined in [10], showing improvements over fixed-gain. Moreover, it did not provide a comparison with PR and LQR controllers. However, they lack a comprehensive performance comparison for different controller combinations. Han et al. [11] compared PI with deadbeat and hysteresis controllers, but the study was limited to single-loop configuration analysis.

1.1.2 PR controller-based approaches

PR controllers are popular for islanded microgrid operation. A design guideline for resonant frequency selection and superior harmonic rejection was achieved in [12] and [13]. However, both focused on grid-connected operation rather than islanded microgrids. An adaptive PR controller with variable resonant frequency was proposed in [14] for the improved performance under frequency variations. However, it did not provide comparisons with PI or LQR controllers under islanded operation with time-varying loads.

1.1.3 LQR controller-based approaches

LQR control provides guaranteed stability margins. Yazdani and Iravani [15] developed LQR-based voltage control and [16] developed LQR-based load frequency control. A frequency-adaptive LQR-based current controller for a grid-connected inverter was proposed in [17], augmenting full-state feedback control with integral and resonant terms to compensate harmonic distortion under grid frequency variation. However, this work implements a single current-control loop rather than a cascaded voltage-current architecture, and together with [15] and [16], does not explore LQR combinations with other controllers in dual-loop architectures.

1.1.4 Hybrid and comparative studies

PI and PR dual-loop (voltage-current) architectures for a stand-alone three-phase inverter were compared in [18], with performance evaluated solely through total harmonic distortion under linear and nonlinear loads. A similar PI-versus-PR comparison for a grid-forming inverter in an unbalanced microgrid was presented in [19], evaluated under islanding, step-load, and fault disturbances. However, both studies implemented PI and PR as separate, internally consistent architectures rather than combining the two controller types within a single voltage-current loop pair, leaving such hybrid combinations, as well as a full investigation of all possible voltage-current loop combinations, unexplored. Table 1 provides a detailed comparison of the reviewed literature, highlighting the specific limitations of each work relative to the comprehensive approach adopted in this paper.

Table 1. Comparison of reviewed literature with proposed work

Reference	Controller Analysis				System		Performance Metrics
	Single			All Combin-ations	Dual-Loop Archi-tecture	Single & Dual Inverter	
	PI	PR	LQR				
[2]	✓	×	×	×	Partial	×	×
[9]	✓	×	×	×	✓	×	×
[11]	✓	×	×	×	×	×	Partial
[10]	✓	×	×	×	Partial	×	×
[13]	×	✓	×	×	✓	×	×
[12]	×	✓	×	×	✓	×	Partial
[14]	×	✓	×	×	✓	×	×
[20]	×	✓	×	×	✓	×	×
[17]	×	×	✓	×	✓	×	Partial
[16]	×	×	✓	×	×	×	×
[15]	×	×	✓	×	Partial	×	×
[21]	×	×	✓	×	✓	×	×
[18]	✓	✓	×	×	✓	×	Partial
[19]	✓	✓	×	×	×	×	Partial
[22]	×	✓	×	×	✓	×	Partial
[23]	✓	✓	×	×	✓	×	Partial
Proposed	✓	✓	✓	✓	✓	✓	✓

Note: ✓, ×, and “Partial” indicate the feature is addressed, not addressed, and incomplete or limited coverage, respectively

1.2 Research gap and contribution

To the best of our knowledge, no existing study simultaneously addresses all of the following research gaps:

1. Previously, some specific control configurations were studied, but no study investigated all 90 possible combinations of PI, PR, and LQR controllers across both the voltage and current loops with their associated gain conditions (Table 2). Moreover, comprehensive gain selection guidelines for each control scheme under time-varying loads are absent in the literature.

2. No study has evaluated these control configurations across both of the following two plant scenarios:

- Case 1 consists of a single three-phase inverter with a Pulse width Modulation PWM actuator and an LC output filter, supplied by a constant DC voltage source and subjected to time-varying loads.

- Case 2 consists of two identical inverter units connected in parallel at a Point of Common Coupling (PCC), where each unit comprises a constant DC power supply, a three-phase PWM inverter, and an LC output filter, with time-varying loads connected at the PCC.

3. Prior studies did not evaluate all critical performance metrics, including reference tracking capability, voltage and frequency deviation RMSE, THD, settling time, rise time, overshoot, and undershoot, across both plant cases and all controller combinations under time-varying loads.

To address the above gaps, this paper makes the following contributions:

1. All 90 possible combinations of PI, PR, and LQR controllers across both the voltage and current loops, together with their associated gain conditions, are systematically investigated and compared, as summarized in Table 2.
2. Comprehensive gain selection guidelines are established for each control scheme under time-varying loads.
3. A single three-phase inverter (Case 1) and a parallel dual-inverter system (Case 2) are investigated and compared across all controller combinations.

4. All critical performance metrics are jointly evaluated, including reference tracking capability, voltage and frequency deviation RMSE, THD, and transient response characteristics (settling time, rise time, overshoot, and undershoot), across both cases and all controller combinations under time-varying loads.

The rest of this paper is organized as follows: Section 2 presents the mathematical modeling of the single and dual-inverter systems and the controllers. Section 3 presents the simulation results and discussion. Finally, Section 4 concludes the paper with a future plan.

Table 2. Controller combinations for each case

Voltage Loop			Current Loop			
Controller	Gain combin ation for		Controller	Gain combination for		Total Combination
	Proportional/State Weight Part	Integral/Resonant/Control Weight Part		Proportional/State Weight Part	Integral/Resonant Control Weight Part	
PI	H/L	H/L				4
PI	H/L	H/L	PI	H/L	H/L	16
PI	H/L	H/L	PR	H/L	H/L	16
PR	H/L	H/L				4
PR	H/L	H/L	PI	H/L	H/L	16
PR	H/L	H/L	PR	H/L	H/L	16
LQR	H/L	L		H/L	H/L	2
LQR	H/L	L	PI	H/L	H/L	8
LQR	H/L	L	PR	H/L	H/L	8
(H = High gain, L = Low gain)			Grand Total:			90

2. Mathematical models of the systems and the controllers

Mathematical models of the systems and controllers that are required to perform simulations in MATLAB Simulink are derived in this section. Two distinct scenarios or cases of the plant are investigated in this paper, as stated in the previous section. The block diagram of the system with two cases is shown in Figure 2, where case 1 consists of a single three-phase inverter with an LC filter, whereas case 2 consists of two parallel inverter-LC units connected at the Point of Common Coupling (PCC) and supplying local loads.

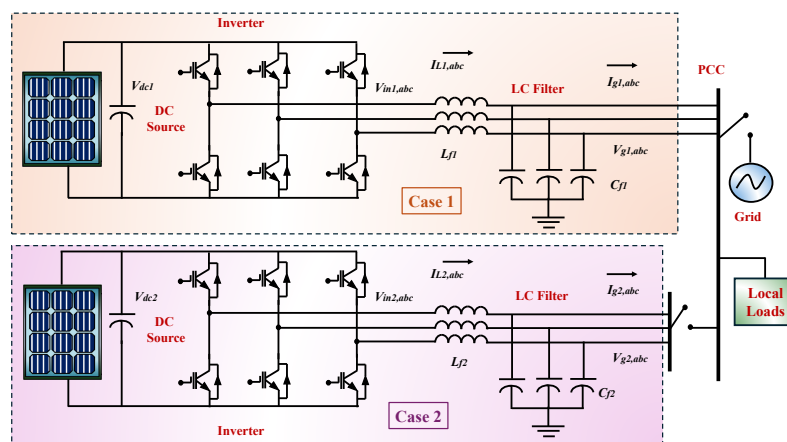


Figure 2. The block diagram of the system with two cases

2.1 Case 1: one three-phase inverter and one LC filter

The ordinary differential equations in the ABC reference frame of the system with one three-phase inverter and one LC filter can be expressed from Figure 2 as [23]:

$$L_f \frac{dI_{L,abc}}{dt} = V_{in,abc} - V_{g,abc} \quad (1)$$

$$C_f \frac{dV_{g,abc}}{dt} = I_{L,abc} - I_{g,abc}, \quad (2)$$

where L_f is the filter inductance, C_f is the filter capacitance, $V_{in,abc}$ are the inverter output voltages, $I_{L,abc}$ are the inductor currents, $V_{g,abc}$ are the grid voltages, and $I_{g,abc}$ are the grid currents. These three-phase ABC quantities are required to be transformed into the synchronous DQ frame to simplify the control design. Thus, equations (1) and (2) can be expressed as:

$$\frac{dI_{L,d}}{dt} = -\omega_g I_{L,q} - \frac{1}{L_f} V_{g,d} + \frac{1}{L_f} V_{in,d} \quad (3)$$

$$\frac{dI_{L,q}}{dt} = \omega_g I_{L,d} - \frac{1}{L_f} V_{g,q} + \frac{1}{L_f} V_{in,q} \quad (4)$$

$$\frac{dV_{g,d}}{dt} = \frac{1}{C_f} I_{L,d} - \omega_g V_{g,q} - \frac{1}{C_f} I_{g,d} \quad (5)$$

$$\frac{dV_{g,q}}{dt} = \frac{1}{C_f} I_{L,q} + \omega_g V_{g,d} - \frac{1}{C_f} I_{g,q}, \quad (6)$$

where $\omega_g = 2\pi f_g$ is the grid angular frequency. Table 3 lists all the parameter values of this three-phase inverter system. The state-space representation of the system can be expressed as follows [23]:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{B}_1 \mathbf{u} + \mathbf{B}_2 \mathbf{e} \quad (7)$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du}, \quad (8)$$

where $\mathbf{x}$ is the state vector, $\mathbf{y}$ is the output vector, $\mathbf{u}$ is the control input vector, $\mathbf{e}$ is the disturbance vector, $\mathbf{A}$ is the system matrix, $\mathbf{B}_1$ is the input matrix, $\mathbf{B}_2$ is the disturbance input matrix, $\mathbf{C}$ is the output matrix, and $\mathbf{D}$ is the direct feed-through matrix. From equations (3)–(8), the following matrices can be found as:

$$\mathbf{x} = \begin{bmatrix} I_{L,d} \\ I_{L,q} \\ V_{g,d} \\ V_{g,q} \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} V_{in,d} \\ V_{in,q} \end{bmatrix}, \quad \mathbf{e} = \begin{bmatrix} I_{g,d} \\ I_{g,q} \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} I_{L,d} \\ I_{L,q} \\ V_{g,d} \\ V_{g,q} \end{bmatrix},$$

$$\mathbf{A} = \begin{bmatrix} 0 & -\omega_g & -\frac{1}{L_f} & 0 \\ \omega_g & 0 & 0 & -\frac{1}{L_f} \\ \frac{1}{C_f} & 0 & 0 & -\omega_g \\ 0 & \frac{1}{C_f} & \omega_g & 0 \end{bmatrix}, \quad \mathbf{B}_1 = \begin{bmatrix} \frac{1}{L_f} & 0 \\ 0 & \frac{1}{L_f} \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\mathbf{B}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -\frac{1}{C_f} & 0 \\ 0 & -\frac{1}{C_f} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{D} = 0. \quad (9)$$

The matrix $\mathbf{A}$ gives four purely imaginary eigenvalues: $\lambda_{1,2} = (0.0000 \pm 6.0877i) \times 10^3$ and $\lambda_{3,4} = (0.0000 \pm 5.4593i) \times 10^3$. So, the system is marginally stable. Moreover, the system order is 4 and the controllability matrix is of rank 4. So, the system is controllable. Similarly, the observability matrix is of rank 4, so the system is also observable.

Table 3. Three-phase inverter system parameters

Parameter	Symbol	Value
DC-link voltage	V_{dc}	700 V
Filter 1 inductor	L_{1f}	2 mH
Filter 1 capacitor	C_{1f}	15 μ F
Filter 2 inductor	L_{2f}	1.8 mH
Filter 2 capacitor	C_{2f}	16 μ F
Grid frequency	f_g	50 Hz
Grid angular frequency	ω_0	314.16 rad/s
Resonant angular frequency	ω_c	5 rad/s
Active power	P_{rated}	10 kW

2.2 Case 2: two three-phase inverters and two LC filters

The ordinary differential equations in the ABC reference frame of the system with two three-phase inverters and two LC filters connected at the PCC can be expressed from Figure 2 as [23]:

$$L_{1f} \frac{dI_{1L,abc}}{dt} = V_{1in,abc} - V_{1g,abc} \quad (10)$$

$$L_{2f} \frac{dI_{2L,abc}}{dt} = V_{2in,abc} - V_{2g,abc} \quad (11)$$

$$C_{f,total} \frac{dV_{g,abc}}{dt} = I_{1L,abc} + I_{2L,abc} - I_{g,abc}. \quad (12)$$

where $C_{f,total} = C_{1f} + C_{2f}$, L_{1f} and L_{2f} are the filter inductance, C_{1f} and C_{2f} are the filter capacitance, $I_{1L,abc}$ are the first filter inductor currents, $I_{2L,abc}$ are the second filter inductor currents, $V_{1in,abc}$ are the first inverter output voltages, $V_{2in,abc}$ are the second inverter output voltages, $V_{g,abc}$ are the grid voltages, and $I_{g,abc}$ are the grid currents. For DQ frame, equations (10)–(12) can be expressed as:

$$\frac{dI_{L1,d}}{dt} = -\omega_g I_{L1,q} - \frac{1}{L_{1f}} V_{g,d} + \frac{1}{L_{1f}} V_{in1,d} \quad (13)$$

$$\frac{dI_{L1,q}}{dt} = \omega_g I_{L1,d} - \frac{1}{L_{1f}} V_{g,q} + \frac{1}{L_{1f}} V_{in1,q} \quad (14)$$

$$\frac{dI_{L2,d}}{dt} = -\omega_g I_{L2,q} - \frac{1}{L_{2f}} V_{g,d} + \frac{1}{L_{2f}} V_{in2,d} \quad (15)$$

$$\frac{dI_{L2,q}}{dt} = \omega_g I_{L2,d} - \frac{1}{L_{2f}} V_{g,q} + \frac{1}{L_{2f}} V_{in2,q} \quad (16)$$

$$\frac{dV_{g,d}}{dt} = \frac{I_{L1,d} + I_{L2,d}}{C_{f,total}} - \omega_g V_{g,q} - \frac{I_{g,d}}{C_{f,total}} \quad (17)$$

$$\frac{dV_{g,q}}{dt} = \frac{I_{L1,q} + I_{L2,q}}{C_{f,total}} + \omega_g V_{g,d} - \frac{I_{g,q}}{C_{f,total}}. \quad (18)$$

From equations (7), (8), and (13)–(18), the state-space representation of the system can be expressed as follows:

$$\mathbf{x} = \begin{bmatrix} I_{L1,d} \\ I_{L1,q} \\ I_{L2,d} \\ I_{L2,q} \\ V_{g,d} \\ V_{g,q} \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} V_{in1,d} \\ V_{in1,q} \\ V_{in2,d} \\ V_{in2,q} \end{bmatrix}, \quad \mathbf{e} = \begin{bmatrix} I_{g,d} \\ I_{g,q} \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} I_{L1,d} \\ I_{L1,q} \\ I_{L2,d} \\ I_{L2,q} \\ V_{g,d} \\ V_{g,q} \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 0 & -\omega_g & 0 & 0 & -\frac{1}{L_{1f}} & 0 \\ \omega_g & 0 & 0 & 0 & 0 & -\frac{1}{L_{1f}} \\ 0 & 0 & 0 & -\omega_g & -\frac{1}{L_{2f}} & 0 \\ 0 & 0 & \omega_g & 0 & 0 & -\frac{1}{L_{2f}} \\ \frac{1}{C_{f,total}} & 0 & \frac{1}{C_{f,total}} & 0 & 0 & -\omega_g \\ 0 & \frac{1}{C_{f,total}} & 0 & \frac{1}{C_{f,total}} & \omega_g & 0 \end{bmatrix}$$

$$\mathbf{B}_1 = \begin{bmatrix} \frac{1}{L_{1f}} & 0 & 0 & 0 \\ 0 & \frac{1}{L_{1f}} & 0 & 0 \\ 0 & 0 & \frac{1}{L_{2f}} & 0 \\ 0 & 0 & 0 & \frac{1}{L_{2f}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\frac{1}{C_{f,total}} & 0 \\ 0 & -\frac{1}{C_{f,total}} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{D} = 0. \quad (19)$$

The matrix $\mathbf{A}$ gives six purely imaginary eigenvalues: $\lambda_{1,2} = (-0.0000 \pm 6.1494i) \times 10^3$, $\lambda_{3,4} = (-0.0000 \pm 5.5211i) \times 10^3$, and $\lambda_{5,6} = (0.0000 \pm 0.3142i) \times 10^3$. It means the system is marginally stable. However, the system order is $n = 6$. The controllability matrix is of full rank 6, so the system is controllable. Similarly, the observability matrix is of full rank 6, so the system is also observable.

2.3 Time-varying load current calculation

The dq-axis load currents are calculated as [24]:

$$I_{g,d} = \sqrt{2}I_{rated} \cdot \text{pf} \quad (20)$$

$$I_{g,q} = \sqrt{2}I_{rated} \cdot \sin(\arccos(\text{pf})) \quad (21)$$

where pf is power factor, I_{rated} is the rated RMS current, and S_{rated} is the rated apparent power, given by:

$$I_{rated} = \frac{S_{rated}}{3 \cdot V_{g,rms}} = \frac{P_{rated}}{3 \cdot \text{pf} \cdot V_{g,rms}} \quad (22)$$

For time-varying load conditions, the load currents are scaled proportionally with $k = 0.25, 0.5, 1.0$ for 25%, 50%, 100% loads, respectively:

$$I_{g,d}^{(k\%)} = k \cdot I_{g,d}^{(100\%)}; \quad I_{g,q}^{(k\%)} = k \cdot I_{g,q}^{(100\%)} \quad (23)$$

2.4 Controllers

The cascaded control strategy for current and voltage loops is depicted in Figure 3. The PI controller can be expressed in the s-domain as follows:

$$C_{PI}(s) = K_{ip} + \frac{K_{ii}}{s}, \quad (24)$$

where K_{ip} and K_{ii} proportional and integral gains. The PR controller can be expressed in the s-domain as follows:

$$C_{PR}(s) = K_{rp} + K_{ri} \frac{2\omega_c s}{s^2 + 2\omega_c s + \omega_0^2} \quad (25)$$

where K_{rp} and K_{ri} proportional and resonant gains. The LQR controller can be expressed as follows:

$$J = \int_0^\infty (x^\top Q x + u^\top R u) dt \quad (26)$$

$$A^\top P + PA - PB_1 R^{-1} B_1^\top P + Q = 0 \quad (27)$$

$$u(t) = -K_q x(t) + K_{qr} r(t) \quad (28)$$

$$K_q = R^{-1} B_1^\top P, \quad (29)$$

where the equations (26), (27), and (28) are the cost function, the Riccati equation, and the control law, respectively. $Q = \text{diag}(q_1, q_2, \dots, q_n)$ is the state weighting matrix ($Q \geq 0$), $R = \text{diag}(r_1, r_2, \dots, r_m)$ is the control weighting matrix ($R > 0$), and K_q is the LQR feedback gain. Equation (30) can be derived from equation (7) as

$$\dot{x}(t) = (A - B_1 K_q) x(t) = A_{cl} x(t) \quad (30)$$

$$K_{qr} = (C_{track} (-A_{cl})^{-1} B_1)^{-1}, \quad (31)$$

where C_{track} presents the tracked states and K_q and K_{qr} denotes the LQR feedback and feedforward gains, respectively.

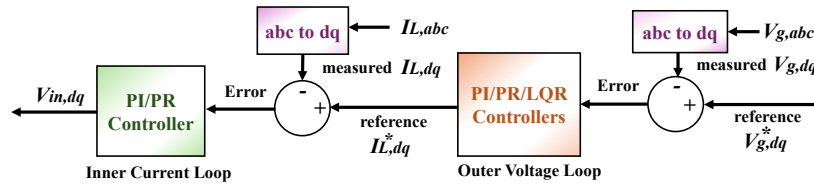


Figure 3. The block diagram of the control system

3. Results and discussion

3.1 MATLAB/Simulink implementation

The developed state-space models **A**, $[B_1, B_2]$, **C**, **D** from equations (9) and (19) are implemented in MATLAB/Simulink using the State-Space block, where all matrix values are defined in the MATLAB workspace script and loaded into the model. The PI and PR controllers are defined by equations (24) and (25), with their gains defined in the workspace script and implemented via Transfer Function blocks. For the LQR, the feedback gain matrix K_q and the feedforward gain K_{qr} are computed offline in the workspace using equations (29) and (31) (MATLAB command: `lqr(A,B1,Q,R)`), then applied in Simulink through a Gain block; the resulting control input **u** follows equation (28). For the PI and PR controllers, a

Saturation block with limits $\pm V_{dc}$ is applied before the control input $\mathbf{u}$ to prevent integrator windup. All gain values used in the simulations are listed in Tables 4 and 5. The time-varying load current $\mathbf{e}$ is generated using Repeating Sequence blocks with a step period of 5 s, following equation (23). The droop controller is implemented via a MATLAB Function block according to the algorithm described in the Appendix. A limitation of this setup is that it does not account for the nonlinear switching behavior of the IGBT devices in the three-phase inverter. To partially account for this limitation, uniformly distributed random noise in the range $[-0.1, 0.2]$ A is superimposed on the load current, as depicted in Figure 4.

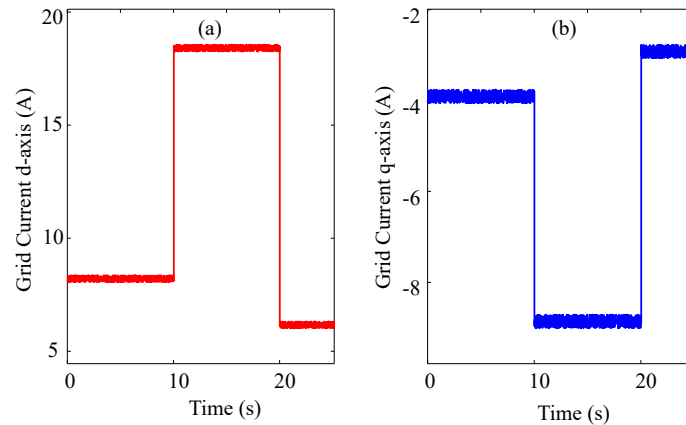


Figure 4. Applied load-current profile in the dq frame for Case 1: (a) d -axis current and (b) q -axis current

3.2 Performance evaluation metrics

The performance of each control configuration is evaluated for the following metrics:

- *Reference tracking*: the capability of the d -axis grid voltage to track its reference of 326.6 V;
- *Voltage deviation RMSE*: the root mean square error of the d -axis grid voltage deviation from its reference;
- *Frequency deviation RMSE*: the root mean square error of the grid frequency deviation, evaluated using a PLL block configured with a proportional gain of 150, an integral gain of 2500, a filter coefficient of 0.8, a maximum rate of change of frequency of 12 Hz, a filter cut-off frequency of 75 Hz, and a minimum frequency of 45 Hz;
- *Transient response*: settling time, rise time, overshoot, and undershoot, all evaluated with respect to the set point of 326.6 V; and
- *Total Harmonic Distortion (THD)*: computed over the full simulation duration of 50 s, corresponding to 2500 cycles of the 50 Hz signal.

The simulation runs for 50 s. The grid voltage $V_{g,d}$ and grid frequency f_g are logged to the MATLAB workspace via To Workspace blocks. The voltage and frequency deviations are computed by subtracting the respective reference values. The RMSE of voltage and frequency deviations and the transient metrics are extracted using a post-processing MATLAB script, and the THD is computed using the Powergui block.

3.3 Discussion

3.3.1 Performance analysis of single inverter configuration

The comparative analysis of 90 different controller configurations for case 1 using PI, PR, and LQR controllers is presented in Table 4. Weights ranging from 90 to 1 are assigned to each combination based on how well it performs in each performance metric. Next, 16, 16, 8, 4, 2, 2, and 1 are multiplied by the weights of the voltage deviation RMSE, frequency deviation RMSE, THD, rise time, settling time, undershoot, and overshoot, respectively, reflecting the relative importance of each metric in practical microgrid operation. The ranking schemes shown in Tables 4 and 5 are then determined by calculating the total amount of weight.

Table 4. Performance comparison of different controller configurations for case 1

No	Controller Config.						Performance Metrics						
	Volt. Loop			Curr. Loop			Volt. Dev. RMSE[V]	Freq. Dev. RMSE [$\times 10^{-4}$ Hz]	Settling Time [ms]	Rise Time [μ s]	Over- shoot [%]	Under- shoot [%]	THD [%]
	Ctrl	K_{ip} K_{rp} Q_{V_g}	K_{ii} K_{rr} $R_{V_{in}}$	Ctrl	K_{ip} K_{rp} Q_{V_g}	K_{ii} K_{rr} $R_{V_{in}}$							
1	PI	H	H	PI	L	L	2.76	2.70	40.00	110.1	48.66	0.76	0.73
2	PI	H	H	PR	L	L	3.01	2.60	40.00	110.1	48.80	0.76	0.77
3	PI	H	L	PI	L	L	2.76	2.70	40.00	110.1	48.66	0.76	0.73
4	PI	H	L	PR	L	L	3.86	2.60	45.00	110.1	48.80	0.76	0.76
5	PR	H	L	PR	H	L	5.21	0.90	40.00	110.2	49.06	0.76	1.34
6	PR	H	L	PR	H	H	5.01	0.90	40.00	110.2	49.07	0.76	1.35
7	PI	H	H	PI	L	H	3.23	7.70	40.00	110.1	48.84	0.76	0.86
8	PR	H	H	PR	H	L	5.28	0.90	40.00	110.2	49.08	0.76	1.35
9	PR	H	H	PR	H	H	5.08	0.90	40.00	110.2	49.09	0.76	1.37
10	PI	H	L	PI	H	L	10.07	0.90	40.00	110.2	49.06	0.76	0.18
11	PI	H	L	PI	L	H	3.20	7.00	45.00	110.1	48.84	0.76	1.14
12	PR	H	H	PI	H	L	8.30	0.90	40.00	110.2	48.98	0.76	0.81
13	PR	H	L	PI	H	L	8.31	0.90	40.00	110.2	48.97	0.76	0.81
14	PI	H	H	PI	H	L	10.07	0.90	40.00	110.2	49.06	0.76	0.81
15	PI	H	H	—	—	—	4.24	10.70	40.00	110.1	48.85	0.76	1.26
16	PR	H	L	PI	H	H	8.20	0.90	40.00	110.2	48.97	0.76	0.82
17	LQR	H	L	PI	L	H	6.31	39.60	40.00	110.2	38.76	0.75	0.71
18	PR	H	H	PI	H	H	8.21	0.90	40.00	110.2	48.98	0.76	0.82
19	PI	H	L	—	—	—	4.81	9.70	40.00	110.1	48.85	0.76	1.22
20	PR	H	L	PI	L	L	4.36	2.90	40.00	110.4	49.05	0.72	1.42
21	LQR	H	L	PR	L	H	6.10	39.60	40.00	110.3	40.31	0.75	0.71
22	LQR	H	L	—	—	—	6.79	39.60	40.00	110.2	38.69	0.76	0.71
23	PR	H	H	PI	L	L	4.46	2.90	40.00	110.4	49.07	0.72	1.45
24	LQR	H	L	PR	L	L	6.80	39.70	40.00	110.2	38.82	0.76	0.71
25	PI	H	L	PI	H	H	25.11	0.90	40.00	110.2	48.98	0.76	0.18
26	PI	L	H	PI	H	H	5.59	60.70	45.00	110.5	29.64	0.76	0.57
27	PR	H	L	PR	L	L	6.63	3.20	45.00	110.4	49.19	0.72	1.72
28	PR	H	L	PI	L	H	7.31	3.20	40.00	110.4	49.17	0.72	1.75
29	PR	H	H	PR	L	L	6.82	3.10	45.00	110.4	49.21	0.72	1.75
30	PI	L	H	PR	H	H	7.35	60.70	45.00	110.5	29.68	0.76	0.18
31	PI	H	L	PR	H	L	26.12	0.90	40.00	110.2	49.06	0.76	0.18
32	LQR	H	L	PI	L	L	6.56	39.60	40.00	110.2	38.68	0.76	2.30
33	PI	H	H	PI	H	H	25.12	0.90	40.00	110.2	48.98	0.76	1.10
34	PR	H	H	PI	L	H	7.52	3.80	45.00	110.4	49.19	0.72	1.80
35	PI	L	H	PI	H	L	7.97	60.70	45.00	110.5	29.67	0.76	0.58
36	PI	L	L	PI	H	H	6.10	64.30	45.00	110.5	29.50	0.76	1.10
37	PI	H	H	PR	H	L	26.04	0.90	40.00	110.2	49.06	0.76	1.12

Table 4. (cont.)

No	Controller Config.						Performance Metrics						
	Volt. Loop			Curr. Loop			Volt. Dev. RMSE[V]	Freq. Dev. RMSE [$\times 10^{-4}$ Hz]	Settling Time [ms]	Rise Time [μ s]	Over- shoot [%]	Under- shoot [%]	THD [%]
	Ctrl	K_{ip} K_{rp} Q_{V_g}	K_{ii} K_{rr} $R_{V_{in}}$	Ctrl	K_{ip} K_{rp} Q_{V_g}	K_{ii} K_{rr} $R_{V_{in}}$							
38	PI	L	H	PR	H	L	7.88	60.70	45.00	110.5	29.67	0.76	1.12
39	PI	H	H	PR	H	H	26.09	0.90	40.00	110.2	49.07	0.76	1.13
40	PI	L	L	PR	H	H	7.75	64.30	45.00	110.5	29.55	0.76	0.58
41	PI	H	L	PR	H	H	26.18	0.90	40.00	110.2	49.07	0.76	1.13
42	PI	L	L	PR	H	L	8.26	64.30	45.00	110.5	29.54	0.76	0.58
43	PI	L	H	PI	L	H	6.08	59.50	45.00	238.7	26.09	1.61	1.59
44	PI	L	L	PI	H	L	8.29	64.30	45.00	110.5	29.54	0.76	0.81
45	PI	L	L	PI	L	H	8.07	61.90	45.00	239.3	26.07	1.61	0.88
46	PI	L	H	PI	L	L	8.09	59.50	45.00	239.3	26.08	1.61	1.52
47	PI	L	H	PR	L	H	8.36	57.60	45.00	233.2	25.91	1.61	1.59
48	PI	L	H	PR	L	L	8.36	59.20	45.00	238.7	26.08	1.61	1.59
49	PR	L	H	PI	H	H	13.32	73.30	45.00	110.4	30.95	0.76	1.35
50	PR	L	L	PI	H	H	13.32	69.60	45.00	110.5	29.60	0.76	1.35
51	PR	L	H	PI	H	L	14.86	73.60	*	110.4	30.94	0.76	1.37
52	PR	L	H	PR	H	L	14.52	73.80	*	110.4	31.03	0.76	1.39
53	PR	L	H	PR	H	H	14.46	73.80	*	110.4	31.05	0.76	1.39
54	PR	L	L	PR	H	H	14.46	70.00	*	110.5	29.69	0.76	1.39
55	PR	L	L	PI	H	L	14.86	69.90	*	110.5	29.60	0.76	1.37
56	PR	L	L	PR	H	L	14.52	70.00	*	110.5	29.68	0.76	1.39
57	LQR	L	L	PI	L	L	16.76	94.10	45.00	112.3	18.45	0.76	0.71
58	PR	L	H	PI	L	H	15.39	64.60	45.17	232.9	20.06	1.54	2.05
59	PR	L	L	PI	L	H	15.40	64.70	45.17	239.0	19.84	1.54	2.05
60	PI	L	L	—	—	—	26.31	16.60	46.08	217.1	28.69	1.61	7.45
61	PI	L	L	PI	L	L	16.73	62.50	45.00	239.9	26.08	1.61	3.57
62	LQR	L	L	PI	H	L	15.97	93.70	45.00	112.4	20.13	0.76	2.32
63	PI	L	L	PR	L	H	24.13	60.70	45.01	233.7	25.80	1.61	4.89
64	LQR	L	L	PI	L	H	16.67	93.80	45.00	112.4	18.36	0.75	2.32
65	LQR	L	L	PR	H	H	16.59	93.90	45.00	112.3	20.14	0.76	2.32
66	LQR	L	L	PR	H	L	16.55	93.90	45.00	112.3	20.13	0.76	2.32
67	LQR	L	L	PI	H	H	15.79	93.90	45.00	112.4	20.13	0.76	2.32
68	PI	L	L	PR	L	L	23.37	62.50	45.00	239.3	25.97	1.61	4.89
69	LQR	L	L	PR	L	H	19.10	94.00	*	112.2	19.73	0.75	2.32
70	LQR	L	L	—	—	—	19.18	93.90	*	112.2	18.60	0.76	2.30
71	PR	L	H	PI	L	L	30.03	66.00	46.98	233.7	20.10	1.54	4.75
72	LQR	L	L	PR	L	L	19.29	94.00	*	112.3	18.54	0.76	2.32
73	PR	L	L	PI	L	L	30.03	65.50	46.98	239.5	20.01	1.54	4.75
74	PR	L	H	PR	L	H	166.6	83.10	*	228.4	0.00	1.54	1.80

Table 4. (cont.)

No	Controller Config.						Performance Metrics						
	Volt. Loop			Curr. Loop			Volt. Dev. RMSE[V]	Freq. Dev. RMSE [$\times 10^{-4}$ Hz]	Settling Time [ms]	Rise Time [μ s]	Over- shoot [%]	Under- shoot [%]	THD [%]
	Ctrl	K_{ip} K_{rp} Q_{V_g}	K_{ii} K_{rr} $R_{V_{in}}$	Ctrl	K_{ip} K_{rp} Q_{V_g}	K_{ii} K_{rr} $R_{V_{in}}$							
75	PR	L	L	PR	L	H	166.6	83.80	*	233.2	0.00	1.54	1.77
76	PR	L	H	PR	L	L	166.6	84.20	*	233.2	0.00	1.54	1.77
77	PR	L	L	PR	L	L	166.6	84.80	*	238.9	0.00	1.54	1.75
78	PI	H	H	PR	L	H	19569	2.32e5	*	110.1	1.44e4	1.45e4	1.2e5
79	PI	H	L	PR	L	H	20506	4.62e5	*	110.1	1.45e4	1.46e4	2.42e5
80	PI	L	H	—	—	—	1923	2.54e4	*	216.7	1.76e3	1.70e3	746.38
81	PR	H	L	—	—	—	2116.9	2.36e6	*	110.4	1.53e3	1.61e3	4.55e3
82	LQR	H	L	PR	H	H	3.9e11	1.77e5	*	110.4	3.4e12	3.4e12	7.55e4
83	PR	H	H	—	—	—	7958.3	2.9e6	*	110.4	6.82e3	6.92e3	3.16e4
84	LQR	H	L	PR	H	L	4.2e11	2.13e5	*	110.4	4.5e12	4.5e12	9.49e4
85	PR	H	L	PR	L	H	1.46e6	3.16e5	*	110.4	1.39e6	1.39e6	7.31e6
86	PR	H	H	PR	L	H	1.47e6	2.96e5	*	110.4	1.39e6	1.39e6	1.75e7
87	LQR	H	L	PI	H	L	1e12	1.27e5	*	110.4	1.5e13	1.6e13	1.70e5
88	LQR	H	L	PI	H	H	8.2e11	3.85e5	*	110.4	7.4e12	7.8e12	1.14e5
89	PR	L	L	—	—	—	75992	3.23e6	*	217.1	8.24e4	8.25e4	8.39e5
90	PR	L	H	—	—	—	8.51e5	3.39e6	*	213.2	7.26e5	7.26e5	1.92e7

Note: Ctrl = Controller; Gain settings: H = High ($K_{ip} = 100, K_{rp} = 100, K_{ii} = 10, K_{rr} = 10, Q_{V_g} = 1, Q_{V_g} = 10000, R_{V_{in}} = 0.01$), L = Low ($K_{ip} = 1, K_{rp} = 1, K_{ii} = 1, K_{rr} = 1, Q_{V_g} = 100, R_{V_{in}} = 0.01$); * = did not settle within the desired time-frame

Table 5. Performance comparison of different controller configurations for case 2

No	Controller Config.						Performance Metrics						
	Volt. Loop			Curr. Loop			Volt. Dev. RMSE[V]	Freq. Dev. RMSE [$\times 10^{-4}$ Hz]	Settling Time [ms]	Rise Time [μ s]	Over- shoot [%]	Under- shoot [%]	THD [%]
	Ctrl	K_{ip} K_{rp} Q_{V_g}	K_{ii} K_{rr} $R_{V_{in}}$	Ctrl	K_{ip} K_{rp} Q_{V_g}	K_{ii} K_{rr} $R_{V_{in}}$							
1	PR	H	L	PI	L	L	2.90	2.50	24.85	110.2	48.29	0.17	0.75
2	PR	H	H	PI	L	L	2.93	3.50	35.00	110.2	48.30	0.17	0.77
3	PI	H	H	—	—	—	3.61	10.20	20.00	110.1	48.48	0.17	1.20
4	LQR	H	L	—	—	—	4.10	27.06	40.00	121.3	12.94	0.17	0.47
5	PI	L	H	PR	H	H	2.09	37.96	40.00	110.6	28.96	0.17	0.12
6	PI	H	L	PI	L	L	4.06	36.13	24.51	110.3	47.53	0.16	0.77
7	LQR	H	L	PI	H	L	5.12	27.04	40.00	121.3	11.66	0.17	0.47
8	PI	H	L	—	—	—	4.31	9.73	45.00	110.1	48.48	0.17	1.20
9	PI	L	H	PR	H	L	2.10	37.96	40.00	110.6	28.94	0.17	0.12

Table 5. (cont.)

No	Controller Config.						Performance Metrics						
	Volt. Loop			Curr. Loop			Volt. Dev. RMSE[V]	Freq. Dev. RMSE [$\times 10^{-4}$ Hz]	Settling Time [ms]	Rise Time [μ s]	Over- shoot [%]	Under- shoot [%]	THD [%]
	Ctrl	K_{ip} K_{rp} Q_{V_g}	K_{ii} K_{rr} $R_{V_{in}}$	Ctrl	K_{ip} K_{rp} Q_{V_g}	K_{ii} K_{rr} $R_{V_{in}}$							
10	PI	L	H	PI	H	L	2.10	37.96	40.00	110.6	28.94	0.17	0.12
11	LQR	H	L	PI	L	H	4.61	27.10	40.00	121.3	12.93	0.17	0.47
12	PI	L	H	PI	H	H	2.10	37.96	40.00	110.6	28.94	0.17	0.12
13	PI	L	L	PI	H	H	2.43	37.93	40.00	110.7	28.82	0.17	0.30
14	PI	H	L	PI	L	H	4.14	35.84	110.52	110.3	47.62	0.16	0.81
15	LQR	H	L	PI	L	L	4.61	27.09	40.00	121.3	12.85	0.17	0.47
16	LQR	H	L	PR	H	L	5.12	27.05	40.00	121.3	12.85	0.17	0.47
17	PI	H	H	PI	L	L	4.37	36.09	24.40	110.3	47.53	0.16	0.79
18	PI	L	L	PR	H	H	2.44	37.94	40.00	110.7	28.84	0.17	0.30
19	PR	H	L	PI	H	H	9.97	0.41	24.92	110.2	48.59	0.17	0.82
20	PI	L	L	PR	H	L	2.44	37.94	40.00	110.7	28.82	0.17	0.30
21	PI	L	L	PI	H	L	2.44	37.94	40.00	110.7	28.82	0.17	0.30
22	PR	H	H	PI	H	H	9.96	0.41	25.22	110.2	48.61	0.17	0.83
23	LQR	H	L	PR	L	H	4.89	27.14	40.00	121.0	13.82	0.17	0.47
24	LQR	H	L	PR	L	L	4.89	27.14	40.00	121.3	12.90	0.17	0.47
25	PI	H	H	PI	L	H	4.49	35.85	170.11	110.3	47.62	0.16	0.82
26	PR	H	H	PR	H	L	9.37	0.41	26.52	110.2	48.61	0.17	0.83
27	PR	H	L	PR	H	L	9.40	0.41	25.91	110.2	48.59	0.17	0.83
28	LQR	H	L	PI	H	H	5.12	27.04	40.00	121.3	11.68	0.17	0.47
29	LQR	H	L	PR	H	H	5.13	27.05	40.00	121.3	12.86	0.17	0.47
30	PR	H	H	PI	H	L	10.68	0.41	25.67	110.2	48.61	0.17	0.83
31	PR	H	L	PI	H	L	10.72	0.41	25.38	110.2	48.59	0.17	0.83
32	PI	H	H	PR	L	L	4.54	36.15	10.00	110.3	47.62	0.16	0.86
33	PR	H	L	PR	H	H	10.32	0.42	26.58	110.2	48.60	0.17	0.85
34	PR	H	H	PR	H	H	10.30	0.42	27.09	110.2	48.62	0.17	0.86
35	PI	H	L	PR	L	L	4.61	36.43	40.00	110.3	47.62	0.16	0.82
36	PR	H	L	PI	L	H	8.52	2.04	289.46	110.6	48.77	0.14	1.69
37	PR	H	H	PI	L	H	8.79	3.01	295.28	110.6	48.78	0.14	1.72
38	PR	H	L	PR	L	L	9.89	2.13	40.00	110.6	48.77	0.14	1.53
39	PR	H	H	PR	L	L	10.14	2.50	40.00	110.6	48.78	0.14	1.56
40	PI	H	H	PR	H	L	7.83	38.16	43.94	110.3	48.50	0.17	1.05
41	PI	H	H	PI	H	L	7.82	38.16	43.85	110.3	48.50	0.17	1.05
42	PI	H	H	PR	H	H	7.80	38.18	44.02	110.3	48.50	0.17	1.05
43	PI	H	L	PI	H	L	7.83	38.17	43.85	110.3	48.49	0.17	1.05
44	PI	H	L	PR	H	H	7.80	38.19	44.02	110.3	48.50	0.17	1.05

Table 5. (cont.)

No	Controller Config.						Performance Metrics						
	Volt. Loop			Curr. Loop			Volt. Dev. RMSE[V]	Freq. Dev. RMSE [$\times 10^{-4}$ Hz]	Settling Time [ms]	Rise Time [μ s]	Over- shoot [%]	Under- shoot [%]	THD [%]
	Ctrl	K_{ip} K_{rp} Q_{V_g}	K_{ii} K_{rr} $R_{V_{in}}$	Ctrl	K_{ip} K_{rp} Q_{V_g}	K_{ii} K_{rr} $R_{V_{in}}$							
45	PI	H	L	PR	H	L	7.84	38.17	43.94	110.3	48.50	0.17	1.05
46	PI	H	H	PI	H	H	7.90	38.17	43.39	110.3	48.50	0.17	1.05
47	PI	H	L	PI	H	H	7.91	38.17	43.31	110.3	48.50	0.17	1.05
48	PI	L	H	PR	L	H	7.37	34.58	45.00	245.9	12.48	0.34	1.53
49	PR	L	L	PI	H	H	10.33	39.99	40.00	110.5	29.14	0.17	0.66
50	PR	L	H	PI	H	H	10.33	40.71	40.00	110.4	30.38	0.17	0.66
51	PR	L	L	PR	H	L	11.27	40.28	45.00	110.5	29.12	0.17	0.68
52	PR	L	L	PI	H	L	11.70	40.20	45.00	110.5	29.13	0.17	0.67
53	PI	L	H	PI	L	H	13.77	34.52	45.00	244.6	12.48	0.37	1.10
54	PR	L	H	PR	H	L	11.28	41.03	45.00	110.4	30.36	0.17	0.68
55	PR	L	H	PI	H	L	11.70	40.95	45.00	110.4	30.37	0.17	0.67
56	PR	L	L	PR	H	H	12.02	40.28	45.00	110.5	29.14	0.17	0.68
57	LQR	L	L	PI	L	H	8.71	68.10	40.00	116.6	9.34	0.17	1.40
58	LQR	L	L	PI	L	L	8.72	68.13	40.00	116.7	9.30	0.17	1.39
59	PR	L	H	PR	H	H	12.02	41.03	45.00	110.4	30.38	0.17	0.68
60	PI	L	L	—	—	—	25.39	18.49	45.94	222.8	13.79	0.37	7.03
61	LQR	L	L	PI	H	L	9.02	68.26	40.00	116.5	9.87	0.17	1.41
62	PI	L	L	PI	L	H	19.24	34.58	45.00	245.3	12.47	0.37	1.49
63	LQR	L	L	PR	H	L	9.02	68.29	40.00	116.5	9.87	0.17	1.41
64	PI	L	H	PI	L	L	19.82	34.93	45.00	245.4	12.47	0.37	1.47
65	PI	L	H	PR	L	L	17.61	34.97	45.00	244.6	12.48	0.37	1.54
66	PR	L	L	PI	L	H	15.12	35.39	45.00	244.4	9.37	0.34	1.67
67	LQR	L	L	PI	H	H	9.00	68.33	40.00	116.5	9.87	0.17	1.41
68	LQR	L	L	PR	H	H	9.03	68.29	40.00	116.5	9.87	0.17	1.41
69	PR	L	H	PI	L	H	15.10	36.18	45.01	238.9	9.41	0.34	1.67
70	PI	L	L	PR	L	H	23.11	34.69	45.00	246.4	12.43	0.34	4.87
71	PI	L	L	PI	L	L	30.52	34.96	45.00	246.1	12.48	0.37	3.52
72	PR	L	L	PI	L	L	24.42	35.89	45.01	244.6	9.45	0.37	4.76
73	PI	L	L	PR	L	L	28.47	35.09	45.00	245.3	12.43	0.37	4.87
74	PR	L	H	PI	L	L	24.41	36.73	45.01	238.9	9.44	0.37	4.76
75	LQR	L	L	—	—	—	11.75	68.36	45.00	116.4	9.83	0.17	1.39
76	LQR	L	L	PR	L	H	12.13	68.72	45.00	116.2	10.24	0.17	1.41
77	LQR	L	L	PR	L	L	12.13	68.73	45.00	116.6	9.39	0.17	1.41
78	PR	L	H	PR	L	H	1.65e2	63.23	*	234.4	0.00	0.34	0.99
79	PR	L	H	PR	L	L	1.65e2	63.20	*	238.9	0.00	0.34	0.95
80	PR	L	L	PR	L	H	1.65e2	61.52	*	239.1	0.00	0.34	0.95

Table 5. (cont.)

No	Controller Config.						Performance Metrics						
	Volt. Loop			Curr. Loop			Volt. Dev. RMSE[V]	Freq. Dev. RMSE [$\times 10^{-4}$ Hz]	Settling Time [ms]	Rise Time [μ s]	Over- shoot [%]	Under- shoot [%]	THD [%]
	Ctrl	K_{ip} K_{rp} Q_{V_g}	K_{ii} K_{rr} $R_{V_{in}}$	Ctrl	K_{ip} K_{rp} Q_{V_g}	K_{ii} K_{rr} $R_{V_{in}}$							
81	PR	L	L	PR	L	L	1.65e2	62.39	*	244.4	0.00	0.34	0.92
82	PI	H	H	PR	L	H	3.23e2	341.3	*	113.6	161.93	124.05	138.21
83	PI	H	L	PR	L	H	3.40e2	546.1	*	113.6	146.49	134.96	239.92
84	PR	H	L	—	—	—	4.34e3	2.44e6	*	110.6	3.86e3	3.95e3	1.15e4
85	PR	H	H	—	—	—	1.48e4	2.80e6	*	110.6	1.50e4	1.51e4	6.86e4
86	PR	H	L	PR	L	H	1.63e6	4.51e5	*	110.6	1.51e6	1.51e6	5.18e6
87	PI	L	H	—	—	—	1.95e3	1.93e4	*	222.4	1.52e3	1.41e3	757.09
88	PR	H	H	PR	L	H	1.64e6	7.18e5	*	110.6	1.52e6	1.51e6	5.39e6
89	PR	L	L	—	—	—	1.24e5	2.92e6	*	222.7	1.24e5	1.24e5	1.78e6
90	PR	L	H	—	—	—	1.58e6	3.40e6	*	219.3	1.50e6	1.50e6	6.24e6

Note: Ctrl = Controller; Gain settings: H = High ($K_{ip} = 100, K_{rp} = 100, K_{ii} = 10, K_{rr} = 10, Q_{V_g} = 1, Q_{V_g} = 10000, R_{V_{in}} = 0.01$), L = Low ($K_{ip} = 1, K_{rp} = 1, K_{ii} = 1, K_{rr} = 1, Q_{V_g} = 1, Q_{V_g} = 100, R_{V_{in}} = 0.01$); * = did not settle within the desired time-frame

In terms of the lowest RMSE computed for the deviation in the d-axis grid voltage, configurations 1, 3, 2, 11, 7, 4, and 15 are the top seven performers. They have a PI controller with a high proportional gain in the voltage control loop and a PI/PR controller with a low proportional gain in the current control loop. In terms of the lowest RMSE computed for the deviation in the grid frequency, configurations 5, 14, 10, 6, 8, 13, 12, and 9 are the best eight. They have mostly a PR controller (and PI controller in two combinations) with a high proportional gain in the voltage control loop and a PI/PR controller with a high proportional gain in the current control loop.

Configurations 10, 25, 31, 30, 26, 35, 42, and 40, which include a PI/PR controller with a high proportional gain in the current control loop and a PI controller with a high/low proportional gain in the voltage control loop, achieve the best THD performance. Combinations 22, 17, 21, 24, and 57 indicate that LQR in the voltage loop, with a high state-weighting parameter Q_{V_g} in the LQR cost function, performs well. The PI controller in the voltage control loop with a large proportional gain often performs the best in terms of rise time. Combinations 7, 11, 2, 4, 1, 3, 19, and 15 represent this trend. Note that configurations 78 and 79, despite having low rise times, are unstable and are excluded from this assessment.

In terms of settling time, configurations 78–90 yield the worst results, with missing settling times indicating instability. Combinations 74–77 have a PR controller in both the voltage and current loops with low proportional gains for the least amount of overshoot. Combinations 34, 29, 28, 27, 23, and 20 feature a high proportional gain PR controller in the voltage loop for the lowest undershoot.

The best overall performance is demonstrated by combinations 1, 2, 3, and 4, all of which employ a PI controller with a high proportional gain in the voltage control loop and a PI/PR controller with a low proportional gain in the current control loop. The reference tracking capability of the three best-performing PI, PR, and LQR controller configurations for the single inverter under time-varying loads is depicted in Figure 5.

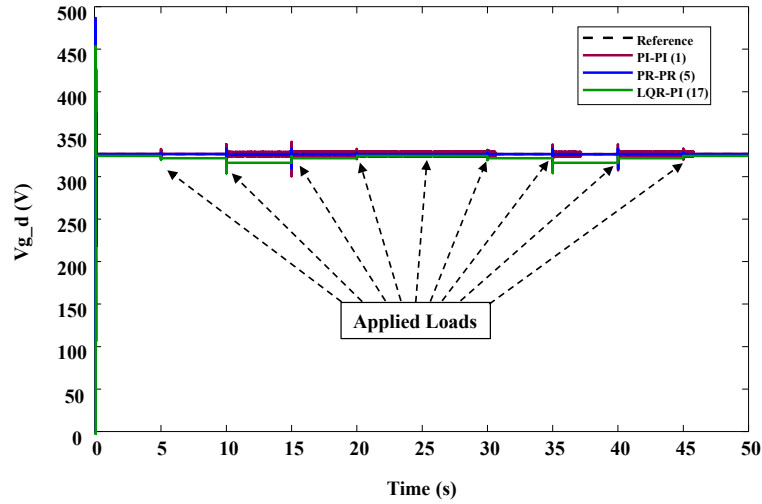


Figure 5. The output voltage tracking capability of the three selected configurations for case 1 under time-varying loads

3.3.2 Performance analysis of dual inverter configuration

The comparative analysis of 90 different controller configurations for case 2 using PI, PR, and LQR controllers is presented in Table 5. In terms of the lowest RMSE computed for the deviation in the d-axis grid voltage, configurations 5, 10, 9, 12, 13, 18, 21, and 20 are the top eight performers. They have a PI controller with a low proportional gain in the voltage control loop and a PI/PR controller with a large proportional gain in the current control loop. In terms of the lowest RMSE computed for the deviation in the grid frequency, configurations 19, 22, 31, 30, 27, 26, 33, and 34 are the best eight. They have a PR controller with a high proportional gain in the voltage control loop and a PI/PR controller with a high proportional gain in the current control loop.

Configurations 12, 10, 9, 5, 13, 21, 20, and 18, which include a PI/PR controller with a high proportional gain in the current control loop and a PI controller with a low proportional gain in the voltage control loop, achieve the best THD performance. Combinations 23, 4, 11, 15, 24, 16, 7, 29, and 28 indicate that LQR in the voltage loop with a high state-weighting parameter Q_{V_g} in the LQR cost function performs well. The PR controller in the voltage control loop with a large proportional gain often performs the best in terms of rise time. Nevertheless, the single-loop PI controller with a large proportional gain is one of the top two performers. Combinations 3, 8, 30, 31, 34, 33, 22, 19, 26, 27, 2, and 1 represent this trend.

In terms of settling time, configurations 78–90 yield the worst results, with missing settling times indicating instability. Combinations 78, 79, 80, and 81 have a PR controller in both the voltage and current loops with low proportional gains for the least amount of overshoot. Combinations 37, 39, 36, and 38 feature a high proportional gain PR controller in the voltage loop and a low proportional gain PI/PR controller in the current loop for the lowest undershoot.

The best overall performance is demonstrated by combinations 1 and 2, all of which employ a PR controller with a high proportional gain in the voltage control loop and a PI controller with a low proportional gain in the current control loop. The reference output voltage tracking capability of the three best-performing PI, PR, and LQR controller configurations for the dual-inverter case under time-varying load disturbances is shown in Figure 6.

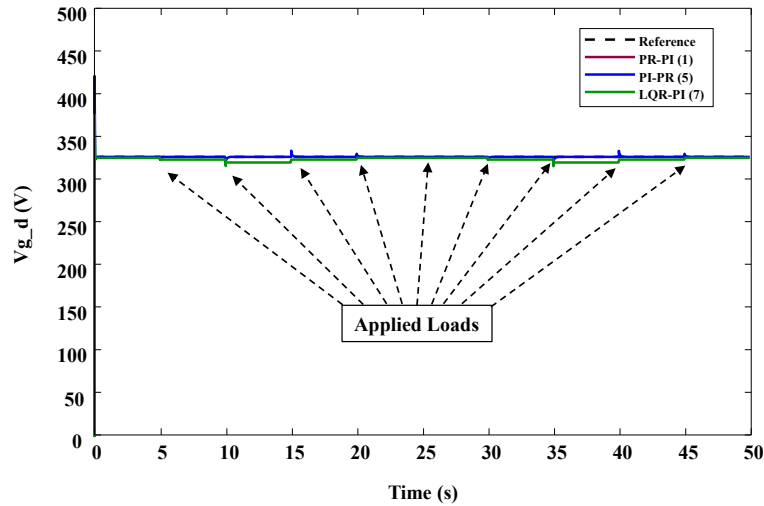


Figure 6. The output voltage tracking capability of the three selected configurations for case 2 under time-varying loads

3.3.3 Gain selection guidelines and significance of the results

The following guidelines are derived from the comprehensive analysis of all 90 configurations across both cases. The significance of these results for the design of islanded AC microgrids is also discussed as follows.

Case 1 (Single Inverter):

- *PI voltage loop:* A high proportional gain ($K_{ip} = 100$) is essential regardless of the integral gain level. The current-loop gain is the most critical parameter: low gains ($K_{ip} = 1$, $K_{ii} = 1$) must be used, as excessively high current-loop bandwidth introduces undesirable cross-coupling between the inner and outer loops in the DQ frame, increasing the average voltage deviation RMSE from 3.14 V to 21.85 V. A PI current-loop controller minimizes voltage RMSE (2.76 V); a PR current-loop controller further reduces frequency deviation RMSE to 2.60×10^{-4} Hz.

- *PR voltage loop:* A high proportional gain ($K_{rp} = 100$) in the voltage loop is required. Unlike the PI case, high current-loop proportional gains are needed to exploit the PR controller's theoretically infinite gain at the resonant frequency ω_0 , which eliminates steady-state error for the 50 Hz sinusoidal reference and achieves frequency deviation RMSE as low as 9.00×10^{-5} Hz. A PR current-loop controller achieves superior frequency performance; a PI current-loop controller offers slightly better THD.

- *LQR voltage loop:* A high state-weighting parameter ($Q_{V_g} = 10000$) is critical—increasing Q_{V_g} from 100 to 10000 reduces the average voltage deviation RMSE from 16.39 V to 6.51 V at the cost of slightly increased overshoot. The control-weighting parameter should be kept low ($R_{V_{in}} = 0.01$). Low current-loop gains are preferred.

Case 2 (Dual Inverter):

- *PI voltage loop:* A low proportional gain ($K_{ip} = 1$) is required—opposite to Case 1. High current-loop gains (K_{ip} or $K_{rp} = 100$) are needed, which is again the reverse of Case 1. This combination achieves voltage deviation RMSE as low as 2.09 V and THD as low as 0.12%, though frequency regulation remains poor ($\text{RMSE} \approx 3.80 \times 10^{-3}$ Hz).

- *PR voltage loop:* A high proportional gain ($K_{rp} = 100$) is required. Low current-loop gains ($K_{ip} = 1$) with a PI current-loop controller yield the best overall performance—settling time of 24.85 ms and frequency deviation RMSE of 2.50×10^{-4} Hz (configuration 1). This is the recommended configuration for Case 2, as the PR controller's frequency-selective action handles the additional inter-inverter dynamics of the sixth-order plant more effectively than the PI controller's wideband integral action.

- *LQR voltage loop:* High $Q_{V_g} = 10000$ and low $R_{V_{in}} = 0.01$ are required. The current-loop gain has minimal influence. This combination yields a consistent voltage deviation RMSE of 4.10–5.13 V and THD of 0.47%, making LQR the most predictable but not the highest-performing choice for Case 2.

4. Conclusion

Stable voltage and frequency regulation in islanded AC microgrids depends critically on the selection of the primary controller and its gain conditions, yet no prior study had systematically evaluated all possible combinations of PI, PR, and LQR controllers across both voltage and current loops for multiple inverter topologies. This paper addressed this by comparing 90 PI, PR, and LQR controller configurations across all voltage–current loop combinations under high and low gain conditions for a single three-phase inverter (Case 1) and a parallel dual-inverter system (Case 2), evaluated against voltage and frequency deviation RMSE, THD, and full transient characteristics under time-varying load conditions.

For Case 1, a PI voltage-loop controller with high proportional gain and a PI current-loop controller with low gains achieved the best performance: voltage deviation RMSE of 2.76 V, frequency deviation RMSE of 2.70×10^{-4} Hz, and a settling time of 40 ms. For Case 2, a PR voltage-loop controller with high proportional gain and a PI current-loop controller with low gains gave the best results: voltage deviation RMSE of 2.90 V, frequency deviation RMSE of 2.50×10^{-4} Hz, and a settling time of 24.85 ms. A key finding is that the optimal current-loop gain direction reverses between the two cases, and the best voltage-loop controller shifts from PI to PR due to the increased plant order (4th- to 6th-order) and inter-inverter coupling in Case 2, confirming that control designs must not be transferred between topologies without re-tuning.

These results confirm that no single controller dominates all performance metrics, and provide topology-specific gain selection guidelines that directly support the design and commissioning of islanded AC microgrids. A droop controller is also implemented and demonstrated in the Appendix, serving as a foundation for future supervisory control developments. Future work will consider experimental validation on a laboratory-scale islanded AC microgrid platform to verify the simulation results under real hardware and switching constraints.

Conflicts of interest

The authors declare no conflicts of interest.

Appendix

The droop controller regulates the voltage and frequency references in the islanded mode by ensuring $P_{\text{generated}} = P_{\text{consumed}}$. The instantaneous active and reactive powers are computed in the synchronous dq reference frame as [15]:

$$P_{\text{meas}} = 1.5 \times (V_{gd}I_{gd} + V_{gq}I_{gq}) \quad (\text{A1})$$

$$Q_{\text{meas}} = 1.5 \times (V_{gq}I_{gd} - V_{gd}I_{gq}). \quad (\text{A2})$$

The inverter apparent power constraint $S \leq S_{\text{max}}$ is enforced by computing [15]:

$$S_{\text{demand}} = \sqrt{P_{\text{meas}}^2 + Q_{\text{meas}}^2}. \quad (\text{A3})$$

If $S_{\text{demand}} > S_{\text{max}}$, both active and reactive powers are scaled proportionally:

$$P_{\text{out}} = P_{\text{meas}} \cdot \frac{S_{\text{max}}}{S_{\text{demand}}}, \quad Q_{\text{out}} = Q_{\text{meas}} \cdot \frac{S_{\text{max}}}{S_{\text{demand}}}. \quad (\text{A4})$$

Otherwise, $P_{\text{out}} = P_{\text{meas}}$ and $Q_{\text{out}} = Q_{\text{meas}}$. The frequency droop reference is calculated as [25]:

$$\omega_{\text{ref}} = \omega_0 - m_P(P_{\text{out}} - P_{\text{set}}) \quad (\text{A5})$$

$$m_P = \frac{2\pi\Delta f_{\text{max}}}{P_{\text{rated}}}, \quad (\text{A6})$$

where ω_0 is the nominal angular frequency and P_{set} is the active power setpoint. The reference frequency is saturated within grid limits: $\omega_{\text{ref}} = \max(\omega_{\text{min}}, \min(\omega_{\text{ref}}, \omega_{\text{max}}))$. The voltage droop reference is calculated as [25]:

$$V_{\text{ref,desired}} = V_0 - n_Q(Q_{\text{out}} - Q_{\text{set}}) \quad (\text{A7})$$

$$n_Q = \frac{\Delta V_{\text{max}}}{Q_{\text{base}}}, \quad (\text{A8})$$

where V_0 is the nominal voltage amplitude and Q_{set} is the reactive power setpoint. The achievable AC voltage is limited by:

$$V_{\text{ac,max}} = \frac{m_{a,\text{max}} \cdot V_{\text{dc}}}{\sqrt{2}}. \quad (\text{A9})$$

The final voltage reference is $V_{\text{ref}} = \min(V_{\text{ref,desired}}, V_{\text{ac,max}})$, subsequently saturated within grid voltage limits: $V_{\text{ref}} = \max(V_{\text{min}}, \min(V_{\text{ref}}, V_{\text{max}}))$. The droop controller parameters are summarized in Table A1. The results after applying droop control with a PI voltage-loop controller under a noisy time-varying load are shown in Figures A1–A3.

Table A1. Droop controller parameters

Parameter	Value	Description
S_{max}	11 kVA	Maximum apparent power
$m_{a,\text{max}}$	1.0	Maximum modulation index
V_0	311 V	Nominal AC voltage amplitude
V_{min}	$0.85 \cdot V_0$ V	Minimum voltage limit
V_{max}	$1.10 \cdot V_0$ V	Maximum voltage limit
f_{min}	49.5 Hz	Minimum frequency limit
f_{max}	50.5 Hz	Maximum frequency limit
Δf_{max}	0.1 Hz	Maximum frequency deviation
m_P	6.28×10^{-5} rad/s/W	Frequency droop coefficient
Q_{base}	3000 VAR	Reactive power base
ΔV_{max}	3.11 V	Maximum voltage deviation
n_Q	1.04×10^{-3} V/VAR	Voltage droop coefficient
P_{set}	0 W	Active power setpoint
Q_{set}	0 VAR	Reactive power setpoint

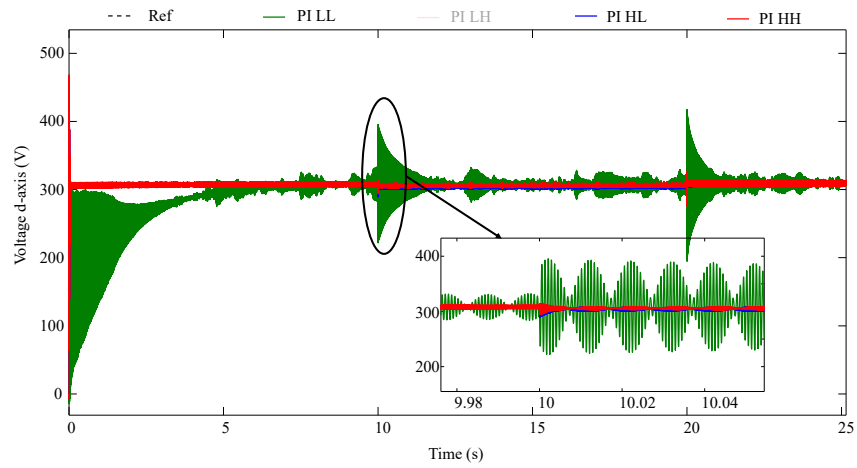


Figure A1. The output voltage tracking capability of the d-axis grid voltage for the single inverter case with time-varying load

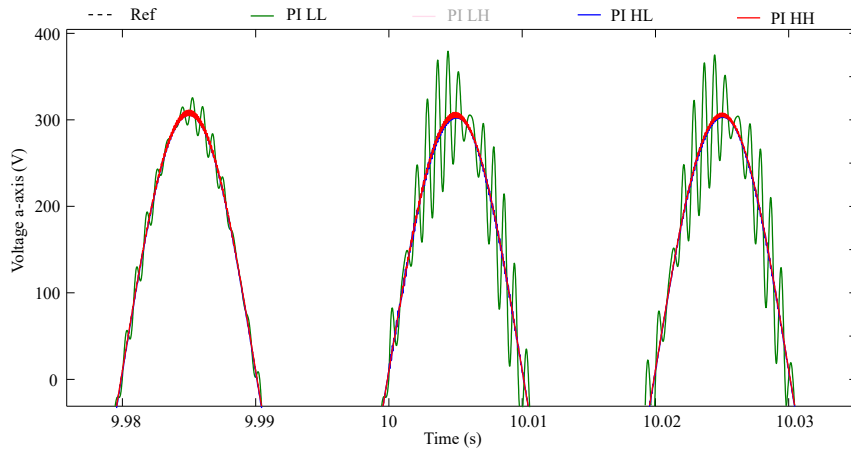


Figure A2. The reference tracking capability of the a-axis grid voltage for case 1

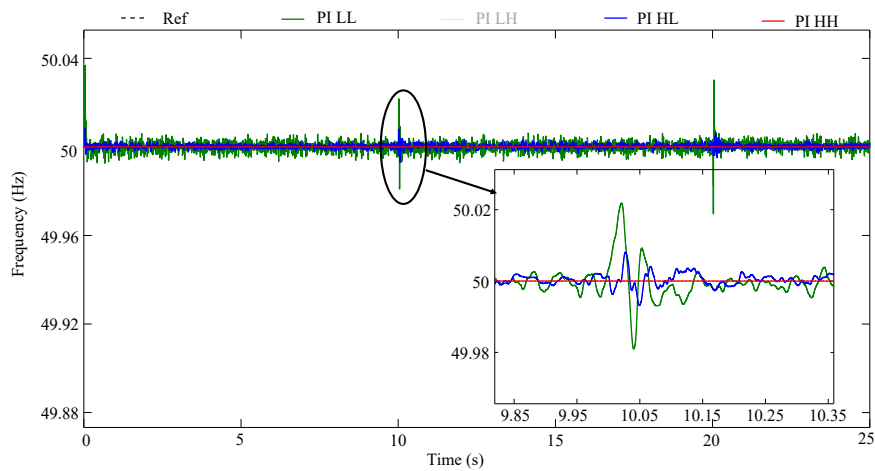


Figure A3. Grid-frequency response for case 1 under droop-based control and time-varying load disturbance

References

- [1] R. H. Lasseter, "Microgrids," In Proc. 2002 IEEE Power Engineering Society Winter Meeting, New York, NY, USA, Jan. 27-31, 2002, pp. 305-308. <https://doi.org/10.1109/PESW.2002.985003>.
- [2] J. M. Guerrero, J. C. Vasquez, J. Matas, L. G. de Vicuña, and M. Castilla, "Hierarchical control of droop-controlled AC and DC microgrids—a general approach toward standardization," *IEEE Transactions on Industrial Electronics*, vol. 58, no. 1, pp. 158-172, 2011. <https://doi.org/10.1109/TIE.2010.2066534>.
- [3] T. Dragičević, X. Lu, J. C. Vasquez, and J. M. Guerrero, "DC microgrids—part ii: A review of power architectures, applications, and standardization issues," *IEEE Transactions on Power Electronics*, vol. 31, no. 5, pp. 3528–3549, 2016. <https://doi.org/10.1109/TPEL.2015.2464277>.
- [4] E. Unamuno and J. A. Barrena, "Hybrid AC/DC microgrids—part i: Review and classification of topologies," *Renewable and Sustainable Energy Reviews*, vol. 52, pp. 1251-1259, 2015. <https://doi.org/10.1016/j.rser.2015.07.194>.
- [5] N. Hatziaargyriou, Ed., *Microgrids: Architectures and Control*. Hoboken, NJ: Wiley-IEEE Press, 2014.
- [6] A. Bidram and A. Davoudi, "Hierarchical structure of microgrid control systems," *IEEE Trans. Smart Grid*, vol. 3, no. 4, pp. 1963-1976, 2012. <https://doi.org/10.1109/TSG.2012.2197425>.
- [7] F. M. Shakeel and O. P. Malik, "On-line self-tuning adaptive control of an inverter in a grid-tied micro-grid," *Electric Power Systems Research*, vol. 178, p. 106045, 2020. <https://doi.org/10.1016/j.epsr.2019.106045>.
- [8] X.-y. Jiang, C. He, and K. Jermstiparsert, "Online optimal stationary reference frame controller for inverter interfaced distributed generation in a microgrid system," *Energy Reports*, vol. 6, pp. 134-145, 2020. <https://doi.org/10.1016/j.egy.2019.12.016>.
- [9] J. Rocabert, A. Luna, F. Blaabjerg, and P. Rodríguez, "Control of power converters in AC microgrids," *IEEE Transactions on Power Electronics*, vol. 27, no. 11, pp. 4734-4749, 2012. <https://doi.org/10.1109/TPEL.2012.2199334>.
- [10] H. Bevrani and T. Hiyama, *Intelligent automatic generation control*. Boca Raton, FL: CRC Press, 2012.
- [11] Y. Han, M. Luo, X. Zhao, J. M. Guerrero, and L. Xu, "Comparative performance evaluation of orthogonal-signal-generators-based single-phase PLL algorithms—a survey," *IEEE Transactions on Power Electronics*, vol. 31, no. 5, pp. 3932-3944, 2016. <https://doi.org/10.1109/TPEL.2015.2466631>.
- [12] D. N. Zmood and D. G. Holmes, "Stationary frame current regulation of PWM inverters with zero steady-state error," *IEEE Transactions on Power Electronics*, vol. 18, no. 3, pp. 814-822, 2003. <https://doi.org/10.1109/TPEL.2003.810852>.
- [13] R. Teodorescu, F. Blaabjerg, M. Liserre, and P. C. Loh, "Proportional-resonant controllers and filters for grid-connected voltage-source converters," *IEE Proceedings-Electric Power Applications*, vol. 153, no. 5, pp. 750-762, 2006. <https://doi.org/10.1049/ip-epa:20060008>.
- [14] J. M. Espi, J. Castello, R. García-Gil, G. Garcera, and E. Figueres, "An adaptive robust predictive current control for three-phase grid-connected inverters," *IEEE Transactions on Industrial Electronics*, vol. 58, no. 8, pp. 3537-3546, 2011. <https://doi.org/10.1109/TIE.2010.2089945>.
- [15] A. Yazdani and R. Iravani, *Voltage-sourced converters in power systems: modeling, control, and applications*. Hoboken, NJ: John Wiley & Sons, 2010. <https://doi.org/10.1002/9780470551578>.
- [16] H. Bevrani, F. Habibi, P. Babahajyani, M. Watanabe, and Y. Mitani, "Intelligent frequency control in an AC microgrid: Online PSO-based fuzzy tuning approach," *IEEE Transactions on Smart Grid*, vol. 3, no. 4, pp. 1935-1944, 2012. <https://doi.org/10.1109/TSG.2012.2196806>.
- [17] R. Bimarta, T. V. Tran, and K.-H. Kim, "Frequency-adaptive current controller design based on lqr state feedback control for a grid-connected inverter under distorted grid," *Energies*, vol. 11, no. 10, p. 2674, 2018. <https://doi.org/10.3390/en1102674>.
- [18] A. Cherifi, A. Hadjkaddour, A. Chouder, A. Aillane, A. Kessal, and K. Louassaa, "Control of a voltage source inverter in a microgrid architecture using pi and pr controllers," In Proc. 2022 19th International Multi-Conference on Systems, Signals & Devices (SSD), Sétif, Algeria, May 6-10, 2022, pp. 1471-1477. <https://doi.org/10.1109/SSD54932.2022.9955891>.
- [19] P. H. Gadde and S. Brahma, "Comparison of pr and pi controllers for inverter control in an unbalanced microgrid," In Proc. 2020 52nd North American Power Symposium (NAPS), Tempe, AZ, USA, Apr. 11-13, 2021, pp. 1-6. <https://doi.org/10.1109/NAPS50074.2021.9449699>.
- [20] A. G. Yepes, F. D. Freijedo, J. Doval-Gandoy, O. Lopez, J. Malvar, and P. Fernandez-Comesana, "Effects of discretization methods on the performance of resonant controllers," *IEEE Transactions on Power Electronics*, vol. 25, no. 7, pp. 1692-1712, 2010. <https://doi.org/10.1109/TPEL.2010.2041256>.

- [21] Y. A.-R. I. Mohamed and E. F. El-Saadany, "Adaptive decentralized droop controller to preserve power sharing stability of paralleled inverters in distributed generation microgrids," *IEEE Transactions on Power Electronics*, vol. 23, no. 6, pp. 2806-2816, 2008. <https://doi.org/10.1109/TPEL.2008.2005100>.
- [22] M. M. Rahman, S. P. Biswas, M. A. Rahman, M. R. Islam, and K. M. Muttaqi, "A second order high performance resonant controller for a three-phase islanded microgrid," In Proc. 2020 IEEE Industry Applications Society Annual Meeting, Detroit, MI, USA, Oct. 10-16, 2020, pp. 1-6. <https://doi.org/10.1109/IAS44978.2020.9334748>.
- [23] M. M. Rahman, S. P. Biswas, M. R. Islam, K. M. Muttaqi, and S. Mueen, "Design of a high performance resonant controller for improved stability and robustness of islanded three-phase microgrids," *IEEE Access*, vol. 10, pp. 119206-119220, 2022. <https://doi.org/10.1109/ACCESS.2022.3220621>.
- [24] F. Blaabjerg, R. Teodorescu, M. Liserre, and A. V. Timbus, "Overview of control and grid synchronization for distributed power generation systems," *IEEE Transactions on Industrial Electronics*, vol. 53, no. 5, pp. 1398-1409, 2006. <https://doi.org/10.1109/TIE.2006.881997>.
- [25] J. C. Vasquez, J. M. Guerrero, A. Luna, P. Rodriguez, and R. Teodorescu, "Adaptive droop control applied to voltage-source inverters operating in grid-connected and islanded modes," *IEEE Transactions on Industrial Electronics*, vol. 56, no. 10, pp. 4088-4096, 2009. <https://doi.org/10.1109/TIE.2009.2027921>.