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Research Article

Thermal Vibration of Thick FGM Circular Cylindrical Shells by Using TSDT

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Abstract: The generalized differential quadrature (GDQ) method is used to study the thermal vibration of thick functionally graded material (FGM) circular cylindrical shells. The nonlinear coefficient effect of third-order shear deformation theory (TSDT) model of displacements on the thermal equations of motion is considered. The parametric effects of FGM power law index and environment temperature on the time responses of stresses and displacements are considered. The linear varied effect of shear correction coefficients on the calculation of stiffness integration is also considered. The simply homogeneous equation is used to find the values of vibration frequency, also the dynamic GDQ discrete equation in matrix form are used to find the values of time response and transient response for the thick FGM circular cylindrical shells.

***Keywords*:** FGM; TSDT; thermal vibration; shells; nonlinear

1. Introduction

There are some studies of shear deformation effects in the functionally graded material (FGM) shells. In 2018, Cong et al. [1] used the Reddy's third-order shear deformation theory (TSDT) for the nonlinear displacements to study the time response of displacements of double curves shallow shells, the effecting numerical solutions for honeycomb materials in geometrical parameters, material properties and damping loads are presented. In 2017, Sobhaniaragh et al. [2] used the TSDT for the displacements to study the buckling loads of FGM Carbon Nano-Tube (CNT)-reinforced shells in the environment (room temperature 300 K) without thermal strains, parametric effects on material properties and critical buckling loads are presented by using the generalized differential quadrature (GDQ) method. In 2017, Dung and Vuong [3] used an analytical method with TSDT to study the buckling of FGM shells in elastic foundation under thermal environment and external pressure. In 2016, Dai et al. [4] presented a 2000–2015 reviewing focused on coupled mechanics, e.g., thermo-mechanical responses with the first-order shear deformation theory (FSDT) models, HSDT models in widely used TSDT to study the bending, buckling, free and forced vibrations of FGM cylindrical shells by using various theoretical, analytical and numerical methods. In 2016, Fantuzzi et al. [5] used the numerical GDQ methods to study the free vibration of FGM spherical and cylindrical shells, some frequency solutions in FGM exponent number and thickness ratio are included. There are some numerical studies in the thick shells. In 2016, Kar and Panda [6] used the code of finite element method (FEM) and the TSDT displacements to obtain the numerical static bending results of deflections and stresses for the heated FGM spherical shells under thermal load and thermal environment. In 2015, Kurtaran [7] used the

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methods of GDQ and FSDT to obtain the numerical transient results of moderately thick laminated composite spherical and cylindrical shells. In 2012, Viola et al. [8] presented static analyses of FGM cylindrical shells under mechanical loading by using the GDQ method and a 2D unconstrained third order shear deformation theory (UTSDT), the numerical solutions for stresses without thermal effect are obtained. In 2010, Sepiani et al. [9] used the FSDT formulation to obtain the numerical free vibration and buckling results for the FGM cylindrical shells without considering the thermal effect. The advantages and disadvantages of above studies are listed as follows. The advantages are the FGM shells well studied by considering the effects of many types of shear deformation theories in displacements. The disadvantages are the FGM shells does not well studied by considering the effects of varied shear correction coefficient in the shear stresses. The importance/impact of the FGM cylindrical shells in the present study are considering the effects of varied effects of shear correction coefficient and nonlinear terms of TSDT involved into the homogeneous equation. Usually, the engineering applications towards the thermal vibration of FGM shells are used in the fields of engine, propulsion and structures for the vehicle, plane and missile.

Some GDQ computational experiences are presented in the composited FGM shells and plates. In 2017, Hong [10] presented the numerical thermal vibration results of FGM thick plates by considering the FSDT model and the varied shear correction factor effects. In 2017, Hong [11] presented the numerical thermal vibration and flutter results of a supersonic air flowed over FGM thick circular cylindrical shells. In 2017, Hong [12] presented the numerical displacement and stresses results of FGM thin laminated magnetostrictive shells by considering with velocity feedback and suitable control gain values under thermal vibration. In 2016, Hong [13] presented the thermal vibration of Terfenol-D FGM circular cylindrical shells by considering the FSDT model and the constant modified shear correction factor effects. It is interesting to investigate the thermal stresses and center displacement of GDQ computation in this nonlinear TSDT vibration approach and the varied effects of shear correction coefficient of FGM circular cylindrical shells with four edges in simply supported boundary conditions. Parametric effects of environment temperature and FGM power law index on the thermal stress and center displacement of FGM circular cylindrical shells including the effect of varied shear correction coefficient are also investigated with the vibration frequency approach of simply homogeneous equation. The main contribution and novelty of paper is to provide and investigate the numerical solutions of thermal vibrations in thick FGM circular cylindrical shells by considering the linear varied values of shear correction coefficient and nonlinear terms of TSDT.

2. Procedure of Formulations

For a two-material thick FGM circular cylindrical shell is used with thickness h_1 of inner layer FGM material 1 and thickness h_2 of outer layer FGM material 2, L is the axial length of FGM shells. The material properties of power-law function of FGM shells are considered with Young's modulus E_{fgm} of FGM in standard variation form of power law index R_n, the others are assumed in the simple average form by Chi and Chung, in 2006 [14]. The properties P_i of individual constituent material of FGMs are functions of environment temperature T.

The time dependent of displacements u, v and w of thick FGM circular cylindrical shells are assumed in the TSDT equations including the nonlinear terms in z^3 with coefficient c_1 by Lee et al. in 2004 [15] as follows [Equations (1)–(3)]:

$$u = u_0(x,\theta,t) + z\Phi_x(x,\theta,t) - c_1 z^3 (\Phi_x + \frac{\partial w}{\partial x}) \tag{1}$$

$$v = v_0(x,\theta,t) + z\Phi_\theta(x,\theta,t) - c_1 z^3 (\Phi_\theta + \frac{\partial w}{R\partial x}) \tag{2}$$

$$w = w\left(x,\theta,t\right) \tag{3}$$

where u_0 and v_0 are tangential displacements in the in-surface coordinates x- and θ- axes direction, respectively, w is transverse displacement in the out of surface coordinates. z-axis direction of the middle-plane of shells, ϕ_x and ϕ_θ are the shear rotations, R is the middle-surface radius of FGM shells, t is time. The nonlinear coefficient for $c_1 = 4/[3(h^*)^2]$ is given as in TSDT approach, in which h^* is the total thickness of FGM shells.

For the normal stresses (σ_x and σ_θ) and the shear stresses ($\sigma_{x\theta}$, $\sigma_{\theta z}$ and σ_{xz}) in the thick FGM circular cylindrical shells under temperature difference ΔT can be presented relatively to the stiffness $\bar{Q}_{ij}$, in-plane strains ε_x, ε_θ and $\varepsilon_{x\theta}$, not negligible shear strains $\varepsilon_{\theta z}$ and ε_{xz}, coefficients of thermal expansion α_x and α_θ, coefficient of thermal shear $\alpha_{x\theta}$ by Lee and Reddy in 2005 [16]; by Whitney in 1987 [17]. The thermal load value of ΔT assumed in linear function of z between the thick FGM circular cylindrical shell and curing area, also satisfied the heat conduction equation in simple form for the thick FGM circular cylindrical shell by Hong in 2016 [13].

The dynamic equations of motion with TSDT for a thick FGM circular cylindrical shell can be assumed and given by Reddy in 2002 [18]. The Von Karman type of strain-displacement relations with $\frac{\partial v_0}{\partial z}=\frac{-v_0}{R}, \frac{\partial u_0}{\partial z}=\frac{-u_0}{R}$ and $\frac{\partial w}{\partial z}=\frac{\partial \phi_x}{\partial z}=\frac{\partial \phi_\theta}{\partial z}=0$ are assumed and used for the strains ε_x, ε_θ, ε_{zz}, $\varepsilon_{x\theta}$, $\varepsilon_{\theta z}$, and ε_{xz} as follows:

$$\varepsilon_x=\frac{\partial u}{\partial x}+\frac{1}{2}(\frac{\partial w}{\partial x})^2=\frac{\partial u_0}{\partial x}+z\frac{\partial \varphi_x}{\partial x}-c_1 z^3(\frac{\partial \varphi_x}{\partial x}+\frac{\partial^2 w}{\partial x^2})+\frac{1}{2}(\frac{\partial w}{\partial x})^2 \tag{4}$$

$$\varepsilon_\theta=\frac{1}{R}\frac{\partial v}{\partial \theta}+\frac{1}{2}(\frac{1}{R}\frac{\partial w}{\partial \theta})^2=\frac{1}{R}\frac{\partial v_0}{\partial \theta}+z\frac{1}{R}\frac{\partial \varphi_\theta}{\partial \theta}-c_1 z^3(\frac{1}{R}\frac{\partial \varphi_\theta}{\partial \theta}+\frac{1}{R^2}\frac{\partial^2 w}{\partial \theta^2})+\frac{1}{2}\frac{1}{R^2}(\frac{\partial w}{\partial \theta})^2 \tag{5}$$

$$\varepsilon_{zz}=0 \tag{6}$$

$$\varepsilon_{x\theta}=\frac{1}{R}\frac{\partial u}{\partial \theta}+\frac{\partial v}{\partial x}+(\frac{\partial w}{\partial x})(\frac{1}{R}\frac{\partial w}{\partial \theta})=\frac{1}{R}\left[\frac{\partial u_0}{\partial \theta}+z\frac{\partial \varphi_x}{\partial \theta}-c_1 z^3(\frac{\partial \varphi_x}{\partial \theta}+\frac{\partial^2 w}{\partial x \partial \theta})\right]+\frac{\partial v_0}{\partial x}+z\frac{\partial \varphi_\theta}{\partial x}-c_1 z^3(\frac{\partial \varphi_\theta}{\partial x}+\frac{1}{R}\frac{\partial^2 w}{\partial x \partial \theta})+(\frac{\partial w}{\partial x})(\frac{1}{R}\frac{\partial w}{\partial \theta}) \tag{7}$$

$$\varepsilon_{\theta z}=\frac{\partial v}{\partial z}+\frac{1}{R}\frac{\partial w}{\partial \theta}=\frac{-v_0}{R}+\varphi_\theta-3c_1 z^2(\varphi_\theta+\frac{1}{R}\frac{\partial w}{\partial \theta})+\frac{1}{R}\frac{\partial w}{\partial \theta} \tag{8}$$

$$\varepsilon_{xz}=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}=\frac{-u_0}{R}+\varphi_x-3c_1 z^2(\varphi_x+\frac{\partial w}{\partial x})+\frac{\partial w}{\partial x} \tag{9}$$

By substituting the stress equations and strain-displacement relations [Equations (4)–(9)] into the dynamic equations of motion, the five numbers of dynamic equilibrium differential equations in the cylindrical coordinates with TSDT of thick FGM circular cylindrical shells in terms of partial derivatives of five unknown displacements (u_0, v_0 and w) and shear rotations (ϕ_x and ϕ_θ) subjected to partial derivatives of given external loads ($f_1,\ldots,f_5$) containing thermal loads ($\bar{N},\tilde{\bar{M}},\bar{P}$), mechanical loads ($p_1,p_2,q$) and inertia terms can be derived and expressed in matrix forms including coefficient c_1 elements of stiffness integrals ($A_{i^s j^s}$, $B_{i^s j^s}$, $D_{i^s j^s}$, $E_{i^s j^s}$, $F_{i^s j^s}$, $H_{i^s j^s}$) and ($A_{i^* j^*}$, $B_{i^* j^*}$, $D_{i^* j^*}$, $E_{i^* j^*}$, $F_{i^* j^*}$, $H_{i^* j^*}$) by assuming that mid-plane strain terms $\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \left(\frac{\partial w}{\partial x}\right)\left(\frac{1}{R}\frac{\partial w}{\partial \theta}\right)$ and $\frac{1}{2}\left(\frac{\partial w}{\partial \theta}\right)^2$ are in constant values, in which

$$\left(A_{i^s j^s}, B_{i^s j^s}, D_{i^s j^s}, E_{i^s j^s}, F_{i^s j^s}, H_{i^s j^s}\right)=\int_{-\frac{h^*}{2}}^{\frac{h^*}{2}} \bar{Q}_{i^s j^s}(1,z,z^2,z^3,z^4,z^6)dz,(i^s, j^s=1,2,6) \tag{10}$$

$$\left(A_{i^* j^*}, B_{i^* j^*}, D_{i^* j^*}, E_{i^* j^*}, F_{i^* j^*}, H_{i^* j^*}\right)=\int_{-\frac{h^*}{2}}^{\frac{h^*}{2}} k_\partial \bar{Q}_{i^* j^*}(1,z,z^2,z^3,z^4,z^5)dz,(i^*, j^*=4,5) \tag{11}$$

where k_α is the shear correction coefficient. The computed and varied values of linear k_α are usually functions of total thickness of shells, FGM power law index and environment temperature presented by Hong in 2014 [19]. The stiffness

$\bar{Q}_{i^s j^s}$ and $\bar{Q}_{i^* j^*}$ for thick FGM circular cylindrical shells with *z/R* terms cannot be neglected are used in the simple forms by Sepiani et al. in 2010 [9] and by Hong in 2014 [19].

The differential quadrature (DQ) method is presented firstly by Bert et al. in 1989 [20]. The GDQ method is presented and improved firstly by Shu and Du in 1997 [21]. The GDQ method is also still well used and presented in the numerical investigation for some thermal vibration of thick FGM shells. The boundary conditions in dynamic GDQ discrete equations approach are to be considered for four sides simply supported, not symmetric, orthotropic of laminated thick FGM circular cylindrical shells. For a two-dimensional function f(x,θ) at coordinates of arbitrarily typical grid point (x_i, θ_j), $i = 1,2,\ldots, N$ and $j = 1,2,\ldots,M$, in which N is the number of the total discrete grid points used in the x direction. M is the number of the total discrete grid points used in the θ direction. The dynamic GDQ discrete equations can be written into the matrix form as follows:

$$[A]\{W^*\} = \{B\} \tag{12}$$

where $[A]$ is a dimension of N^{**} by N^{**} coefficient matrix ($N^{**} = (N-2) \times (M-2) \times 5$)) containing the stiffness integrals and weighting parameters $\left(A_{i,l}^{(m)}, B_{j,m}^{(m)}\right)$, in which $A_{i,l}^{(m)}$ and $B_{j,l}^{(m)}$ denote the weighting coefficients for the mth-order derivative of the function $f(x,\theta)$ with respect to the x and θ directions. $\{W^*\}$ is a N^{**}th-order unknown column vector and can be expressed as follows:

$$\begin{aligned}\{W^*\} = \{&U_{2,2}, U_{2,3}, \cdots, U_{2,M-1}, U_{3,2}, U_{3,3}, \cdots, U_{3,M-1} \cdots, U_{N-1,2}, U_{N-1,3}, \cdots, U_{N-1,M-1}, V_{2,2}, V_{2,3}, \cdots, V_{2,M-1},\\ &V_{3,2}, V_{3,3}, \cdots, V_{3,M-1}, \cdots, V_{N-1,2}, V_{N-1,3}, \cdots, V_{N-1,M-1}, W_{2,2}, W_{2,3}, \cdots, W_{2,M-1}, W_{3,2}, W_{3,3}, \cdots, W_{3,M-1} \cdots,\\ &W_{N-1,2}, W_{N-1,3}, \cdots, W_{N-1,M-1}, \Phi_{x2,2}, \Phi_{x2,3}, \cdots, \Phi_{x2,M-1}, \Phi_{x3,2}, \Phi_{x3,3}, \cdots, \Phi_{x3,M-1} \cdots, \Phi_{xN-1,2}, \Phi_{xN-1,3},\\ &\cdots, \Phi_{N-1,M-1}, \Phi_{\theta 2,2}, \Phi_{\theta 2,3}, \cdots, \Phi_{\theta 2,M-1}, \Phi_{\theta 3,2}, \Phi_{\theta 3,3}, \cdots, \Phi_{\theta 3,M-1} \cdots, \Phi_{\theta N-1,2}, \Phi_{\theta N-1,3}, \cdots, \Phi_{\theta N-1,M-1}\}^t\end{aligned} \tag{13}$$

and $U = u_0/L$, $V = v_0 / R$, $W = w / h^*$. {B} is a N^{**}th-order row external loads vector with each element can be written as follows, $\{B\}=\{F_1 \ldots F_1, F_2 \ldots F_2, F_3 \ldots F_3, F_4 \ldots F_4, F_5 \ldots F_5\}^t$. The columns $\{F\}$ can be represented in the discretized equation under thermal load ΔT, e.g., sinusoidal temperature with the applied heat flux frequency γ. The cosine expressions of coordinates x_i and θ_j for the grid points numbersNandMof thick FGM circular cylindrical shells are used to get the GDQ results.

3. Some Numerical Results and Discussions

To study the GDQ displacement results of shells layers in the stacking sequence (0° / 0°) with two constituent FGM material 1 and FGM material 2 under thermal loads as shown in Figure 1. The typical solution under four sides simply supported boundary condition is presented with no in-plane distributed forces ($p_1 = p_1 = 0$) and no external pressure load ($q = 0$). By using the simplified sinusoidal temperature $\Delta T = \frac{z}{h^*}\overline{T_1}\sin(\pi x / L)\sin(\pi\theta)\sin(\gamma t)$ for the thermal loads, in which γ is the frequency of applied heat flux, $\overline{T_1}$ is the amplitude of temperature, the value of γ can be calculated from the simplified heat conduction equation by Hong in 2016 [13]. Before the process of thermal vibrations of thick FGM circular cylindrical shells, it is needed to obtain the calculation values of vibration frequency ω_{mn} in two directional vibrations with mode shape subscript numbers m and n.

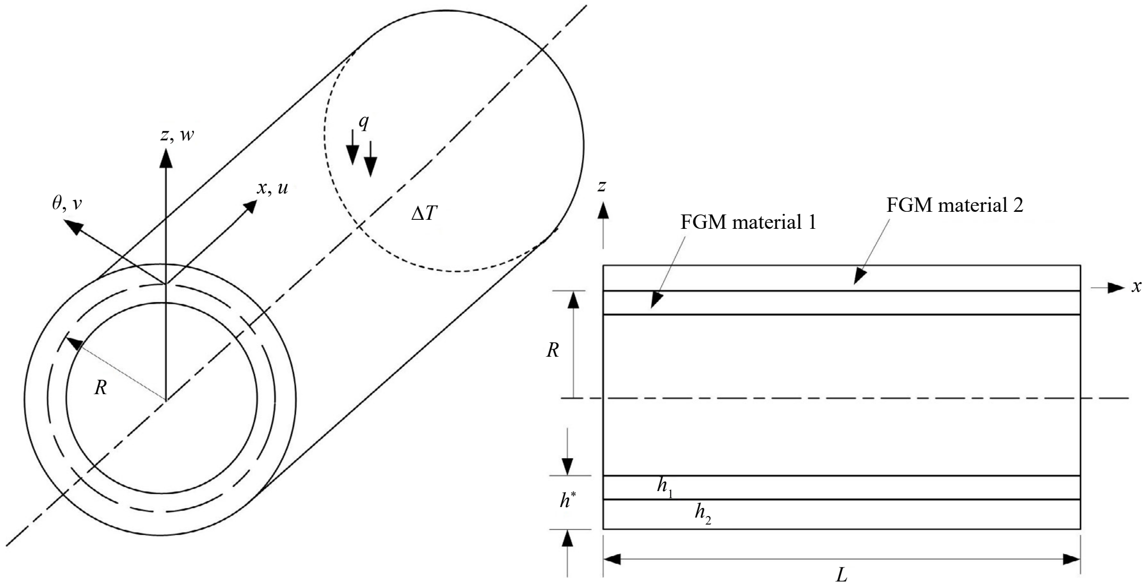

Figure 1. A thick FGM circular cylindrical shell under thermal loads.

It is usual and available by using the computer compiler software of Lahey-Fujitsu FORTRAN 7.8 Microsoft Visual Studio 2015 to implement the nonlinear TSDT program for thick TSDT FGM circular cylindrical shells. The FGM material 1 at inner position is SUS304, the FGM material 2 at outer position is Si3N4 used for the numerical GDQ computations. For the preliminary study of FGM cylindrical shells, it did not consider the effect of c_1 term on the calculation of varied k_α. Thus, the varied values of linear k_α are usually functions of h^*, R_n and T. Firstly, the dynamic convergence of center displacements amplitude w(L/2,2π/2) (unit: mm) in the thermal vibration of TSDT (with c_1= 0.925925/mm2) and linear (with c_1= 0) are studied. For thick FGM circular cylindrical shells L/h^* = 10 with applied heat flux γ = 0.2618004/s and L/h^* = 5 with γ = 0.2618019/s, are used respectively. The dynamic convergence $w(L/2,2\pi/2)$ results at t = 6s, L/R=1, h^*= 1.2 mm, $h_1 = h_2$ = 0.6 mm, T = 100 K, $\overline{T}_1$ = 100 K are presented in Table 1. It is considering the varied effects of linear k_α and ω_{11} for three values of R_n with the vibration frequency approach of simply homogeneous equation. The error accuracy is 8.8×10^{-7} for the nonlinear center displacements amplitude of R_n= 0.5 and L/h^*= 10 cases. The N×M = 13 × 13 grid points can be treated in the very good convergence result and used further in the following GDQ computations of time responses for displacements and stresses of the thermal vibration of nonlinear TSDT thick FGM circular cylindrical shells.

Table 1. Convergence of FGM circular cylindrical shells with TSDT and linear k_α.

c_1(1/mm^2)	L/h^*	GDQ method	$w(L/2,2\pi/2)$ (unit: mm) at t =6 s		
		$N \times M$	R_n= 0.5	R_n= 1	R_n= 2
0.925925	10	7 × 7	1.130075	1.114773	1.144947
		9 × 9	1.130954	1.130920	1.131019
		11 × 11	1.130950	1.130927	1.131012
		13 × 13	1.130949	1.130915	1.131017
	5	7 × 7	0.086110	0.085986	0.085963
		9 × 9	0.085948	0.085880	0.085908
		11 × 11	0.085949	0.085882	0.085909
		13 × 13	0.085947	0.085882	0.085908
0	10	7 × 7	15.752050	15.084518	17.771356
		9 × 9	2.119939	2.332193	2.577847
		11 × 11	2.122492	2.342385	2.566322
		13 × 13	2.122820	2.335284	2.583867
	5	7 × 7	0.174378	0.197195	0.233112
		9 × 9	0.168509	0.190151	0.222200
		11 × 11	0.168513	0.190158	0.222217
		13 × 13	0.168511	0.190156	0.222212

Secondly, the amplitude of center displacement $w(L/2,2\pi/2)$ (unit mm) for the thermal vibration of nonlinear TSDT and linear of FGM thick circular cylindrical shells are calculated with the varied γ of applied heat flux and the vibration frequency approach of simply homogeneous equation, $m = n =1$, $q = 0$. The γ values are decreasing from γ =15.707960/s at t = 0.1 s to γ =0.523601/s at t = 3.0 s used for L/h^* = 5, from γ = 15.707963/s at t = 0.1s to γ =0.523599/s at t =3.0s used for L/h^*= 10. Figure 2 shows the response values of center displacement amplitude $w(L/2,2\pi/2)$ (unit mm) versus time t of nonlinear TSDT and linear under the values of c_1 = 0.925925/mm2 and c_1= 0 in FGM circular cylindrical shells for thick L/h^*= 5 and 10, respectively, L/R=1, h^* = 1.2 mm, $h_1 = h_2$ = 0.6 mm, R_n=1, k_α = 0.120708, T = 600 K, $\overline{T}_1$= 100 K and t = 0.1–3.0 s with time step is 0.1s. The maximum value of center displacements amplitude is 12.746453 mm occurs at t = 0.1 s for thick L/h^* = 5 with c_1 = 0.925925/mm2 and γ = 15.707963/s. The maximum value of center displacement amplitude is 160.021988 mm occurs at t = 0.1s for thick L/h^* = 10 with c_1 = 0/mm2 and γ = 15.707963/s. The values of center displacement amplitudes are all decreasing with time in the both cases c_1 = 0.925925/mm2 and c_1 = 0/mm2 for thick L/h^*= 5 and 10, respectively. The values of center displacement amplitudes in linear case of c_1 = 0/mm2 are greater than that nonlinear case of c_1 = 0.925925/mm2 at the corresponding time for thick L/h^*= 5 and 10, respectively.

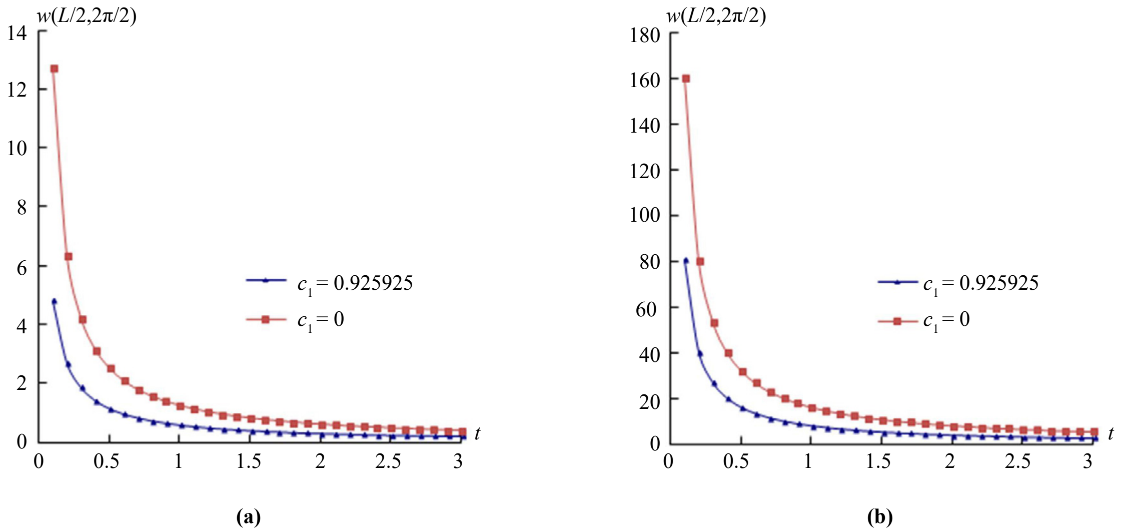

Figure 2. $w(L/2,2\pi/2)$ versus t for L/h^*= 5 (a) and 10 (b).

Figure 3 show the time responses of the dominated stresses σ_x (unit GPa) at center position of inner surface $z = -0.5h^*$ as the analyses of displacements case in Figure 2 for R_n = 1, thick L/h^* =5 and 10 with c1 = 0.925925/mm2, respectively. The maximum value of stresses σ_x is 1.7445×10^{-3} GPa occurs at t = 0.1 s in the periods t = 0.1–3 s for thick L/h^* = 10. The values of dominated stresses σ_x are all decreasing with time in the case c_1 = 0.925925/mm2 for thick L/h^* = 5 and 10, respectively.

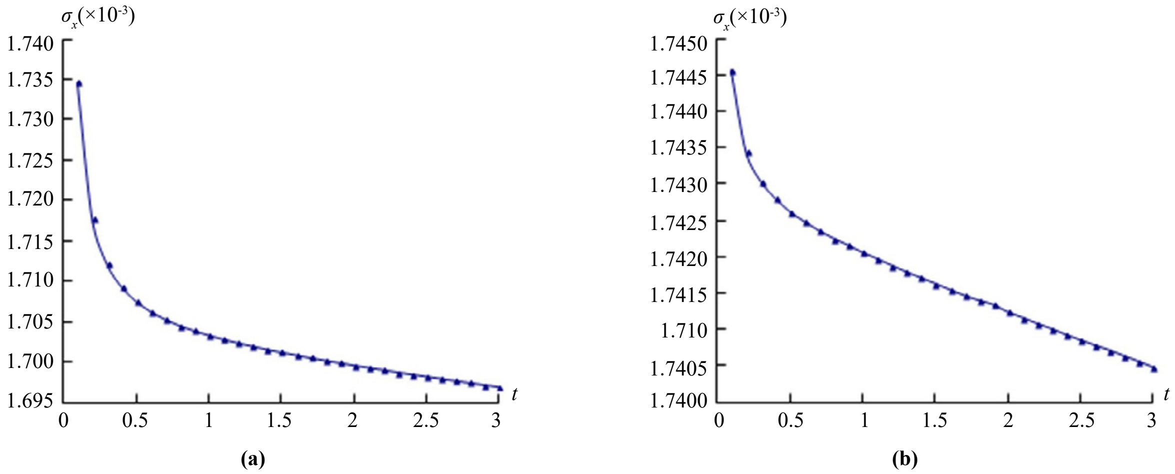

Figure 3. σ_x versus t for L/h^*= 5 (**a**) and 10 (**b**).

Figure 4 shows the response values of center displacements amplitude w(L/2,2π/2) (unit: mm) versus T (100 K, 600 K and 1000 K) with R_n (from 0.1 to 10) of nonlinear TSDT under the values of c_1 = 0.925925/mm2 in FGM circular cylindrical shells for thick L/h^* = 5 with the vibration frequency approach of simply homogeneous equation in L/R=1, h^* = 1.2 mm, $h_1 = h_2$ = 0.6 mm, $\overline{T}_1$=100 K, γ values of applied heat flux, calculated and varied values of k_α at t = 0.1 s. The maximum value of $w(L/2,2\pi/2)$ is 9.066230mm occurs at T = 1000 K for R_n= 0.1. The center displacements amplitude values are all increasing versus T from T = 100 K to T = 1000 K, for all value of R_n, the amplitude $w(L/2,2\pi/2)$ of the L/h^* = 5 cannot withstand for higher temperature (T=1000K) of environment

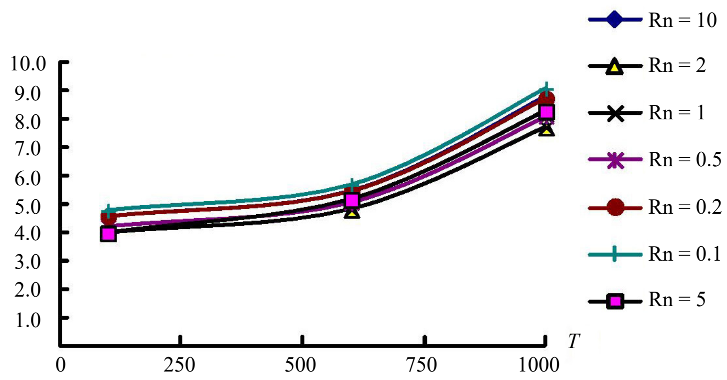

Figure 4. $w(L/2,2\pi/2)$ versus T for $L/h^*= 5$ with Rn from 0.1 to 10.

Usually, the higher frequency of applied heat flux gets higher amplitude of the transient responses. Finally, the transient responses of $w(L/2,2\pi/2)$ (unit mm) are presented in Figure 5 with c_1 = 0.925925/mm2, R_n = 1.0, T = 600 K, $\bar{T}_1$ = 100 K and k_α = 0.120708. The transient response values in the short period t = 0.002–0.05 s are calculated with lower frequencies γ = 785.3982/s, ω_{11} = 0.010717/s and γ = 0.523601/s, ω_{11} = 0.001947/s, respectively. The sinusoidal $w(L/2,2\pi/2)$ amplitude at t = 0.05s is nearly around 9.011686mm under γ = 785.3982/s and is greater than that 0.187629mm under smaller γ = 0.523601/s. The transient response values are all converging with time.

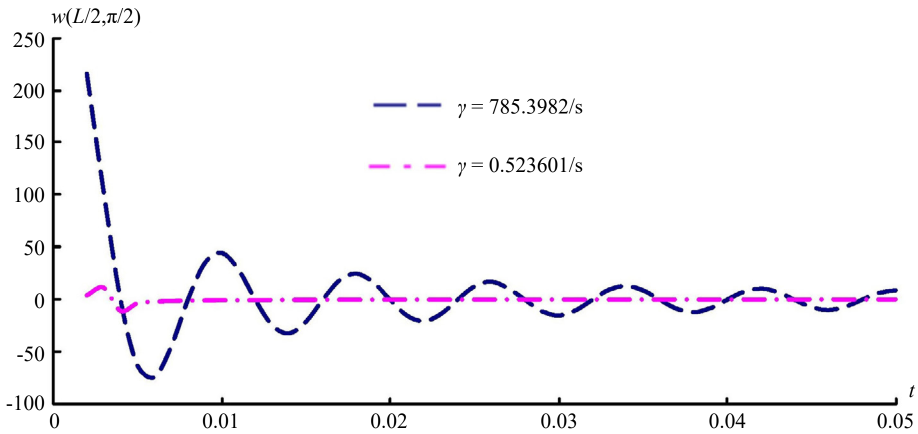

Figure 5. Transient responses of $w(L/2,2\pi/2)$(unit mm) for $L/h^*= 5$.

4. Conclusions

The GDQ solutions are calculated and investigated for the displacements and stresses in the thermal vibration of thick FGM circular cylindrical shells with the vibration frequency approach of simply homogeneous equation by considering the linear varied effects of shear correction coefficient and coefficient c_1 term of TSDT. The values of center displacements amplitudes in linear case of c_1= 0/mm2 are greater than that nonlinear case of c_1= 0.925925/mm2 at the corresponding time for thick L/h^*= 5 and 10, respectively. Thus, the values of center displacements can be modified into the more accuracy data by using the suitable c_1 terms. The values of dominated stresses σ_x are all decreasing with time in the case c_1= 0.925925/mm2 for thick L/h^*= 5 and 10, respectively. The amplitude $w(L/2,2\pi/2)$ of the L/h^*=5 cannot withstand for higher temperature (T = 1000 K) of environment. The higher frequency of applied heat flux gets more higher amplitude of displacements.

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Conflict of interest

There is no conflict of interest for this study.

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