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Research Article

The Elkonin-Davydov Curricular Approach: How Cognitive Development can be Driven by Cultural Tools

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Abstract: This conceptual paper examines the aspects of human development that are foundational to higher mental functions and how particular features of cognition essential for relational and other types of complex thought are activated and developmentally driven through experiential exposure, specifically with others, to intentionally designed cultural tools of teaching and learning. After taking a closer look at relational thinking, which is a foundation for more complex types of thought such as computational thinking, and a developmentally appropriate pedagogical method for fostering these aspects of cognitive development, we focus on the Elkonin-Davydov early "prenumerical" phase, presenting this curricular approach as a cultural tool which can foster cognitive development. Based on this, we further conclude that the (pedagogical) tools, to which children are exposed within an interactional pedagogy, can affect and shape children's cognitive development. Further, we perceive this work has shown promise as an instructional process for fostering abstract comprehension and relational reasoning in children, and thus, we recommend implementing the Elkonin-Davydov curricular approach in early childhood daily practice.

***Keywords*:** Elkonin-Davydov curricular approach, sociocultural theory, higher mental functions, cognitive development, relational thinking, early childhood

1. Introduction

Learning and development in young children are topics that have been widely studied (Konner, 2010; Piaget, 1962; Vygotsky, 1978; Vygotsky & Luria, 1993). While the influence of others and the environment are undeniable, learning and development are complex processes that are in motion from the very beginning of life. Embedded in social activity, children under five years old are growing and learning at a faster pace than at any other stage in their lives. Many assume that young children are not (yet) capable of abstract comprehension. However, in examining the research from multiple fields, including language learning, cognitive sciences, and developmental sciences, many examples contradict this universal assumption (Coles, 2021; Dougherty & Sloven, 2004; Venenciano et al., 2015; Wang & Black Delfin, 2021). Based on the literature, it is obvious that although a great amount of research has been investigating young children's capability on higher mental functions such as abstract thinking and comprehension, from various theoretical stands, there is less research highlighting the Elkonin-Davydov curricular approach from the analysis of Vygotsky's

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sociocultural perspective. Therefore, this conceptual paper presents information about a curricular approach based on the sociocultural theory that is counter to many in the Western world, but that shows promise in facilitating children's early development in areas of higher mental functions.

Vygotsky's (1978; 1991) examination of the processes that give rise to higher mental functions provides a framework for the explanation of young children's cognitive development. In a Vygotskian theoretical framework, the developmental lens is applied when looking at instructional processes that promote cognitive development and, particularly, relational thinking abilities in young children. While there are many ways to define and differentiate types of thinking, it seems that relational thinking, which is based on abstraction and symbolic representation, is foundational for more complex types of thought such as computational thinking.

From the subsequent work of Vygotsky's colleagues, D. B. Elkonin and V. V. Davydov, a developmentally appropriate pedagogical approach for fostering these aspects of cognitive development that serve as precursors for later computational thinking have been created and implemented in some parts of the world (Davydov, 1975; Sidneva, 2020; Schmittau & Morris, 2004). As this curricular approach emerged from the work of Vygotsky, there are many aspects and features of this pedagogy that comprise what is understood as developmental education (van Oers, 2012). Centered on interactive relationships and working through topics in a process orientation, this work has great potential as an instructional method for fostering abstract comprehension and relational reasoning in children whom many may see as too young to perceive or think in this way.

2. Literature review and theoretical framework

The Elkonin-Davydov curriculum is grounded in Vygotsky's developmental education framework and as such, focuses on cognitive development (Sidneva, 2020). What follows is a review of the literature with attention to the development of higher mental functions in human children, taking into account the Vygotskian sociocultural theoretical framework, as well as drawing from research on development coming from cognitive science. Both viewpoints support that environmental exposure can drive development, particularly exposure to salient, interactive cultural tools. Therefore, this review provides an analysis of the Elkonin-Davydov curricular approach from the perspective of sociocultural theory and cognitive science.

2.1 *Children's development and sociocultural theory*

For many years, development was seen as biologically pre-programmed and separate from culture or social context, occurring in prescripted stages in predictable sequences. For example, Piaget's (1962) views regarding cognitive development require that cognitive abilities are dependent on general development in areas of symbolic representation. Piaget sees play as an activity in which a child can engage after a certain amount of development has occurred that affords symbolic representation. In contrast, Vygotsky (1962) sees sociocultural interaction, often manifesting for children as symbolic play, as a driving force of development across multiple areas of cognition. In other words, in the Vygotskian theoretical frame, cognition is activated and emerges through sociocultural interaction. This includes the environment, language, culture, and people interacting in inseparable ways. With exposure to these things, development is initiated and propelled in a mutually-influencing reiterative process.

To better explain factors such as the environment, language, culture, and people's interactional impact on children's development, Table 1 is displayed below, which not only attempts to present the relationship among those components, but also provides an overall structure of the literature review and theoretical framework. However, it is acknowledged that the boundaries between each item are porous and that there is dovetailing and overlap between items. Further, a more accurate depiction may involve envisioning the chart lining the inside of a sphere to demonstrate the dynamic, circular and (re)iterative nature of the components of the framework.

Table 1. 2D graphic depiction of Vygotsky's sociocultural framework

The Environment	
Culture	
Self	Others
Intramental Knowledge	Intermental Knowledge
• Internalized integration of self	• External discursive information
Learning	
Active Knowledge	Understood Knowledge
Spontaneous Concept Acquisition	Scientific Concept Acquisition
• Individual learning that occurs in encounters with the material world	• Transmission of societal knowledge through direct instruction
Activity	
Individual	Societal
Tools	
Technical Tools	Psychological Tools
Cognition	
Concrete	Abstract

As can be seen, the commonality is that knowledge is gained alone and with others in a reproductive cycle. It would seem that learning occurs as continuous entangled relations between the self and the environment. Learning transpires between the self and others involving intermental knowledge (external discursive information) and intramental knowledge (internalized integration of self), taking the form of individual learning that occurs in encounters with the material world and the transmission of societal knowledge through direct instruction. In describing the processes of the Zone of Proximal Development (ZPD), Eun (2019) says, "Discursive practices, and cultural tools employed by people engaged in collaborative activities, will all be reflected in the individual mental processes. This is why even the innermost sphere of human consciousness retains a dialogical nature" (p. 22). Tools mediate learning in both avenues of self and others, and in turn, facilitate further tool generation (Vygotsky, 1978).

2.1.1 *Zone of proximal development impels learning potential*

Within the Vygotskian framework, several theoretical features address aspects of cognitive development that underpin relational thinking. Vygotsky's sociocultural theories primarily posit education (i.e. teaching and learning, cultural information and tools, social interaction) as a driver of development, specifically cognitive development of higher mental functions (1991). In devoting his attention to understanding the origins, differences, and potentialities of human higher mental functions, Vygotsky and his followers put much focus on development and learning and how these things cannot be separated from culturally discursive information that one produces, is exposed to, and immersed in through the course of being a human in society.

The culmination of Vygotskian theory is the idea that learning comes by way of others, whether through an instructional relationship with a more knowledgeable person or via a cultural tool or product, optimally within the ZPD

(Vygotsky, 1978). Three different, but related, concepts have come to describe ZPD, as shown in Table 2. The first is the distance between what a child can do alone and what a child can do with the assistance of a more knowledgeable other. The second is the distance between what a child can understand, or come to know about, on their own and what the child can learn from direct instruction from a more knowledgeable other. And the third is the difference between individual activity and collective, or societal, activity (Eun, 2019).

Table 2. Zone of proximal development

Zone of proximal development		
The distance between what a child can do alone and what a child can do with the assistance of a more knowledgeable other	The distance between what a child can understand, or come to know about, on their own and what the child can learn from direct instruction from a more knowledgeable other	The difference between individual activity and collective, or societal, activity

The second function of the ZPD is described by Eun (2019) as "the distance between understood and active knowledge" (p. 20). This definition nods to the Vygotskian principle of concept acquisition, which is often referred to as "spontaneous" and "scientific" concept acquisition. Eun (2019) equates understood knowledge with scientific concepts and formal direct instruction; active knowledge with spontaneous, "every day", informal interactions with the environment. Spontaneous conceptual understanding occurs as we interact within the day-to-day experiences of our lives. Spontaneous concepts originate in the sensory information of the material world; often referred to as "concrete" cognition. Vygotsky's concept acquisition model presents spontaneous concepts as emerging in the concrete and, with increased interaction with others in the environment, incorporating the abstract (Fleer & Raban, 2006). Conceptual understanding involves deeper comprehension, due to discernment of relations and relational significance between and including components. Vygotsky (1978) suggested that the two types of concept acquisition, spontaneous and scientific, mutually underpin comprehension of the concrete and the abstract, cleaving the two means of conceptual acquisition together and resulting in deeper understanding.

The last conceptual dimension of the ZPD, i.e. the difference between individual activity and collective, or societal, activity, has to do with the idea that a complex and adaptive system becomes more than the sum of its parts. In this case, a qualitative change occurs when individual human beings grow, learn and develop, and in the course of doing so create a culturally discursive system that is constantly transformed into greater levels of complexity and (re)iteratively further driving individual development. Humans engaged in joint sociocultural activities generate discourses that are constituted by human interaction, while at the same time discursive information is co-constitutive of humans, shaping perceptions and development. Eun (2019) states, "ZPD is a developmental mechanism... ZPD reflects development itself" (p. 20).

2.1.2 *Higher mental functions of learning on two planes*

In looking at the first two functions of the ZPD focusing on teaching and learning, it is important to understand an underlying principle within the Vygotskian theoretical framework that concerns the relationship that we have with ourselves and others. In this school of thought, it is understood that we acquire learning on two planes; intermental learning and intramental learning (Vygotsky & Luria, 1993). This process fosters the development of higher mental functions as an expansion in thought occurs between people engaged in the joint sociocultural activity, called intermental functioning and intramental functioning where the child internalizes the conceptual learning that has occurred in social relations (Vygotsky & Luria, 1993). In the course of internalizing the conceptual learning, the information is transformed and reorganized, as well as the child who has intramentally apprehended the concepts. Pakdaman-Savoji et al. (2019) state "reorganization implies qualitative changes in how learners perceive and operate in their world" (p. 3).

As babies, we first experience the environment as sensory input that originates external to us. In making meaning of that sensory input, we process the information of external origin intermentally. In the course of understanding the information, we internalize the information intramentally. Eun (2019) explains, “All individual mental functions originated as actual human relations and interactions between two or more people. Through assistance, collaboration and mediation, what was once carried out in the interpsychological plane becomes internalized and begins to perform intrapsychological functions” (Asmolov, 1998, p. 21).

In the course of internalization, intermental knowledge (i.e. sociocultural and interactional information) becomes intramental knowledge (i.e. integrated into the self). Following intramentalization, the self is expanded with the understanding of new facets of previous knowledge. As people interact, internalized knowledge becomes externalized (i.e. shared with others) and in doing so, becomes reorganized as something new. From this cycle of external to internal, back to external, human beings produce cultural content and products; tools. Vygotsky (1978) described two types of tools that emerge from social interactions; technical tools, such as books, objects, and digital devices, and psychological tools such as signs, symbols, and language. Tools serve as evidence of the third function of the ZPD, which is the difference between what can be produced by an individual and what can be produced by society.

2.1.3 *Cultural tool generation*

As such, technology is a cultural tool, as are mathematics, curricula, pedagogies and other methods devised by humans to increase the generalizability and transference of information. In human history, problem solving in naturalistic settings became limited by complexity, giving rise to abstract representations of culturally constructed means of mediating complexity (Schmittau, 2004). An example of this can be seen in the algorithm as a cultural tool. Algorithms are powerful models that were culturally constructed throughout the history of mathematics to offer a generalized conception of a complex problem. Schmittau (2004) explains, “The fact that the human mind in the course of history freed itself from methods workable in more restricted contexts but cumbersome in dealing with situations of greater problem complexity, by developing methods increasingly general in their application is important” (p. 23). Tools, whether technical or psychological, emerge out of societal interactions. In this way, they are built from accumulative discursive information and serve as an archive, of sorts, of human knowledge. Tools, then, empower learners to engage with complexity by enabling the “wisdom of previous generations'' (Pakdaman-Savoji et al., 2019, p. 2). The internalization of psychological tools is a transformational process that changes and expands cognitive development and abilities. An example of this can be seen in the work of Schwartz et al. (2005), which demonstrates that when children have opportunities to explore mathematics as a cultural tool, the development of physical understanding is propelled. Thus, the tools that we choose to use or, even, place in front of children for their use, become important in later development. The same can be said of the experiences that children have. This refers to what children are exposed to within their lived experience.

2.2 *Children’s development and cognitive science*

In the last decades of the twentieth century, cognitive science emerged from interdisciplinary research, including in the fields of psychology, computer science, and linguistics. This interdisciplinary study of cognition and the mind emphasized the examination of neural architecture afforded by modern technologies of brain imaging, as well as how the brain can process and represent information. Foundational to cognitive science is that “thinking can be best understood in terms of representational structures in the mind and computational procedures that operate on those structures” (Thagard, 2020, p. 2). This reference to mental representations acted upon by computational operations is central to the development of higher mental functions.

2.2.1 *Exposure as a developmental driver*

One way to view Vygotsky’s conceptual framework of development, particularly the co-constitutive aspect of the sociocultural entanglement that serves as a developmental driver, can be seen in physical (cell) growth. In the cognitive sciences, the process is described as exposure, which is also understood as a driver of development. As sensory beings, our exposure to, and with, the environment, each other, and even ourselves in the environment, can be seen as a driver of development. Take only one of our sensory-receptor modalities, such as hearing, as an example. Hearing, or exposure to auditory sounds, causes a flow of information to be directed to the brain. As the brain cells “receive” this information, they begin to "process"; as in, seek to make meaning of the auditory sensory information. This entails cells growing and connecting together so that the sensory information can pass from neuron to neuron. The act of "receiving" auditory sensory information, whether a sound or a word, causes cells to develop and grow in such a way as to process that type of sensory information. This results in the development of neural "pathways" (metaphorically speaking) for processing (i.e. making meaning) of sensory information resulting from environmental exposure. Tierney and Nelson (2009) state, "Although the brain may come equipped with biases for certain perceptual information, such as for speech, language, or faces, it is the specific speech, language, and range of faces they are exposed to that drives subsequent development" (p. 13).

Exposure occurs as lived experience within the particular environment in which a being lives. For human beings, lived experience always already occurs within nature and culture with others who are also engaged in a lived experience within nature, culture, and society. As our brains and bodies process simultaneous sensory information within the sociocultural frame, cellular development occurs to make meaning of that exposure. A major part of a typical lived experience for a human child involves exposure to others. This inevitably introduces symbols and abstractions which activate cognitive development in the process of the brain seeking to make sense of the new stimuli. Based on a (re) iterative process of experiential exposure and "use" over time, meaning emerges from organized cell growth and connection (Cantor et al., 2019; Tierney & Nelson, 2009).

The notion of exposure as a developmental driver has been intensely studied, revealing numerous effects of experience on cellular structure and connection. Exposure to language has been studied as to its effect on children's language development, vocabulary (scope and size), and age/s at which development is driven by exposure is most likely. Romeo et al. (2018) find that "children's language exposure is related to activation specifically in left prefrontal cortical regions… the (parts of the) language network that are particularly sensitive to early linguistic input" (p. 7875). Likewise, the effects of exposure to adversity in the early years have been widely studied. Blair and Raver (2012) emphasize that "(early stress is a) primary canalizer or mechanism through which cognitive and social-emotional development in early childhood is shaped by experience" (p. 2). Further, they point out that, "stress physiology serves as a primary conduit or source of information in a developmental system" (p. 5).

Sheridan and McLaughlin (2014) highlight "mechanisms through which experiences (of threat and deprivation) affect neural structure and function in humans" (p. 580) in demonstrating the effect of adversity on cognitive structure and function. Research has also found that not only do sociocultural factors influence cognitive development, but exposure to nature and "green spaces" (Dadvand et al., 2015) and exposure to novelty (Tang et al., 2006) also have an influence on development. Konner (2010) summarizes, "genes guide neuronal movement and build connections, yet the final circuitry depends on transactions with the environment" (p. 626). Much like Vygotsky's insight into the role that the environment, particularly sociocultural interactions, plays in activating development, the idea of experiential exposure describes external and internal influences that likewise activate, and drive, development.

2.2.2 *Interactions with the human and the non-human*

Human children now have exposure to digital devices from early in life. In keeping with findings from the cognitive sciences, (experiential) exposure to electronic devices has an influence on children's development. This is especially significant if the child is operating or interacting with a device. The act of interacting with a digital medium drives the developing structure of the brain's architecture. As an interaction, either receptive or expressive, will cause the brain to grow in such a way as to make meaning of the interaction, one could say that contemporary children's neural organization is shaped by interactions with humans and non-humans (Galvan, 2010).

As humans interact more and more with and through non-human, digital interfaces, such as computational devices, the abilities necessary for creating and communicating with these technical/psychological tools become highly valued and important for children to acquire. Computational thinking has been taken up as the term that describes a particular type of thought process in human beings. In Vygotskian terms, computational thinking constitutes one of the most complex higher mental functions. This thought process is specific to human beings in that it refers to cognitive qualities and skills that arise (are generated/constituted/accessed/utilized) in relation to technology, particularly computational devices. Wing (2011) provides this definition, "Computational thinking is the thought processes [*sic*] involved in formulating problems and their solutions so that the solutions are represented in a form that can be effectively carried out by an information-processing agent" (p. 20). This specifically describes human thought processes that are internalized and can also manifest in a relational interaction between a human and a non-human. However, human beings are not born with a fully developed and refined ability to think computationally, or even relationally. Computational thinking is a product of cognitive development over time. Further, as this paper asserts, it is dependent on exposure and can be fostered through interaction with cultural tools designed to facilitate abstract understanding and symbolic representation. Wing (2011) notes that "The most important and high-level process in computational thinking is the abstraction process" (p. 20). Abstraction can be used to capture relations or properties of things, where one can represent many, giving us the use of scale. Abstraction allows us to navigate complexity (Wing, 2011).

2.2.3 *The mutual ascent of the abstract and the concrete*

In looking to identify developmentally appropriate methods to promote relational thinking in early childhood, it is crucial to examine what aspects of development are fundamental to higher-order thinking and how particular features of cognition essential for higher mental functions are activated through experiential exposure. Cognitive development generally refers to processes involving thought, perception, or the mind. As an area of study, the focus is on young children since much cognitive development occurs early in life. During the early years, a development that enables making meaning of abstraction, pattern recognition, symbolic representation, and language learning is ongoing. In looking at these aspects of cognitive development, there appear to be multiple, synchronous influences that affect the timing and qualities of cognition.

Because development begins at conception and unfolds over time, it is commonly characterized as sequences of developmental activity that occur in order and do not precede each other or even co-occur. In this way, development is often misunderstood as following a direct line of cause-and-effect from the concrete to the abstract. Although some aspects of this may be accurate, this developmental model is too simplistic to capture the multiple and simultaneous influences that make up human growth and learning. Beginning with the conception, through gestation and birth, development centers, at first, on biological growth. As the zygote becomes a fetus, receptive interaction with the environment begins as the baby's sensory organs become more fully developed (Pujol et al., 1991).

Perhaps because of the biological origins of development, newborns, infants, and young children are considered to be in a "concrete" phase of cognition. Very young children come to know the world through their movements and early physical interactions with the environment. With little language or sense of time, concrete cognition consists of sensory perception of the material. The concrete is detailed, precise, sensuous, granular, situated, and actual. It is easy to see how very young children are grounded in the concrete or material world. However, it is difficult to point to a time when they are not always already interacting with an environment that is external to themselves. In this way, development seems to be influenced, and even driven by, interactions with the concrete and the abstract, with the internal and the external (Wellsby & Pexman, 2014). The environment itself may be material, but the interaction with it is relational, and this gives rise to abstraction and the possibility of symbolic representation. The relationality of lived experience in a material body and environment forms a multi-directional avenue for concrete and symbolic representations to emerge concurrently and to inform the understanding of each (other) (p. 7). Humans have a marked ability to recognize relations between things. In doing so, abstractions emerge, and in the understanding of them, symbolic meaning is produced. As a child grows and experiences more of the world, interaction with the environment produces more relations (i.e. abstractions), from which the child makes sense. The mental act of discerning meaning from relations ('this is like that', 'this comes from that' or 'this can represent that') is the beginning of abstract cognition. Unlike the concrete, which is actual, the abstract is virtual, relational, and usually a symbolically depicted/enacted generalization. Abstraction is a mental perception of the discursive (Hayes & Kraemer, 2017).

Ilyenkov's (1982) often cited quote regarding "the ascent of the abstract to the concrete" is usually interpreted to indicate that general knowledge occurs first in child development, with details (the concrete) filling in as a child has more experiences. However, this sequence of development is facile, as Radford (2021b) states, "knowledge appears first in a general or abstract manner. As our journey unfolds, knowledge acquires specific or concrete content" (p. 1). This would seem to contradict the arguments of the preceding paragraphs, but only if one were to look at it as a direct line of cause-and-effect. When we see the concrete and the abstract as simultaneously emergent and informing of each other, then the metaphor of "ascent" may be less applicable than one that describes the reiterative and recursive nature of development. Ilyenkov (1982) goes on to state, "one can say that the ascent from the concrete to the abstract and the ascent from the abstract to the concrete, are two mutually assuming forms of theoretical assimilation of the world, of abstract thinking. Each of them is realised only through its opposite and in unity with it" (p. 137).

As such, if the development of higher mental functions requires both mutually assuming forms of theoretical assimilation, i.e. concrete cognition and abstract thought, then it makes sense to foster children's comprehension of the material world at the same time that abstract ideas are introduced. In Vygotskian terms, the mutual instantiation of both types of understanding would necessarily be present in any cultural tool that drives development. In young children, the acquisition of concrete and abstract comprehension is essential for further developing other higher mental functions, such as relational and computational thinking.

Grover and Pea (2013) assert that abstraction is the "keystone" of computational thinking (p. 39). Further, included in the key elements comprising a curricular approach to promoting computational thinking, Grover and Pea (2013) specify that "abstractions and pattern generalizations, (including models and simulations)...symbol systems and representations" are essential to computational thinking (p. 39). Because of this, when seeking a curricular approach that fosters computational thinking, it is very important for both the concrete and the abstract to be supported and offered for consideration and learning because it is through both avenues of understanding that meaning can ultimately be made from either one.

3. Analysis of Elkonin-Davydov curricular approach

The Elkonin-Davydov math curriculum and pedagogy were developed in the USSR following World War II by Vasily Davydov and E. B. Elkonin. This was the first widespread attempt to teach math to children using a curriculum that centered on real numbers through measurement and relations, versus most math curricula that focus on whole numbers and counting. Davydov and Elkonin based their curricular process on Vygotsky's ideas concerning developmental education, whereby Vygotsky asserts that cognitive development occurs when a learner encounters a problem for which the learner's previous methods of solution do not apply or work in the current situation. Based on this, Davydov and Elkonin designed a curricular approach that "consists of a series of carefully sequenced problems that require progressively more powerful insights and methods for their solution" (Devlin, 2009, p. 5). In this section, we not only focus on comprehending the Elkonin-Davydov curricular approach from the sociocultural perspective, but also explain how each key component of the Elkonin-Davydov 'prenumerical' phase is integrated with the primary principles of sociocultural theory.

3.1 *Engagement with 'topics' drives children's cognitive development*

Due to the institutional structures in place at the time this curriculum was developed, the original program was ordered by semesters with the first two semesters focused on what Davydov (1975) called " 'prenumerical' period of mathematics instruction". During this phase, children are introduced to numerous objects and begin to study the properties of the objects, such as color, shape, and size, and then quantities such as length, volume, area, and mass (Sidneva, 2020). While children are urged to explore the objects, they are presented, not as toys or manipulatives, but as symbols that embody physical properties and representations of mathematical concepts. Donovan and Alibabi (2021) find that when presented and used in this way, children construe the objects as tools and instead of playing with the objects, they can create various ways to measure and relate them. Davydov (1975) explains, "The unity (of the manipulative and the logical approach; concrete and abstract) may include being introduced to the operation of measuring" (p. 115). The operation of measuring establishes criteria for comparison. Terms such as "quantity" are shown to be related to and given a specific interpretation of "equal to", "more than", and "less than". By establishing criteria of comparison, the set is transformed into a quantity. Davydov (1975) notes, "This introductory section could consist of an introduction to physical objects and criteria for comparing them, with quantity being distinguished as a subject for mathematical consideration" (p. 133).

The early activities of the "prenumerical" section, according to Davydov (1975), should include an "introduction for methods of comparison, symbolic means for designating the results, and methods of analyzing the general properties of quantities related to actions the child can perform in order to learn the material" (p. 133). As the children in the "prenumerical" phase of the curriculum are often pre-literate, it is of interest that they are expected to begin to document the observed relationships via symbols. Devlin (2009) explains that once children begin to perceive the size and whole/

part relationships they are guided to express this in writing, “documenting these relationships begins with variables, not necessarily numbers (amounts)”. Documentation of variable relationships is the introduction of representing through abstraction. This promotes “sophisticated reasoning about part/whole relationships” (p. 7).

In Topic I (the term and structure that Davydov employed when providing an explanation of the curricular process), the “prenumerical” phase introduces physical objects to the children, allowing them to discover the properties through concrete, tactile exploration (please see Appendix A for an outline of the topical structure of this curricular approach). This deliberate inclusion of physical objects for exploration is a way of activating prior knowledge of the physical world and connecting these early understandings to introductions of abstractions. By encouraging children to manipulate the objects of study at the same time that they begin to explore the relational properties of the objects, children begin to form a bi-directional understanding (concrete/abstract, abstract/concrete) of reality. Then the children begin to use specified symbols to designate the properties and relations (Topic II). The specified symbols are called “the schematic” (representation) and include “=”, “<”, “>” and variables to designate the objects being compared (Davydov, 1975, p. 135). With the introduction of the schematic as the way to symbolically represent the problem, scaffolding begins. It is important to note that symbolic representation of the material objects and their relations is introduced at the beginning of the curriculum. The abstract is introduced with the concrete in what Radford (2021a) calls “a sensuous developmental process” (p. 329). In this way, “(children) begin to conceive of the situation with objects as a concrete illustration of an abstract idea, as opposed to viewing the relationship only as a property of the specific objects themselves” (Schmittau & Morris, 2004, p. 63). Introducing the concrete with the abstract fosters connection to prior learning and builds a conceptual system that unites concrete and abstract cognition. This serves to “connect actions with objects to symbolic representations of those actions and the quantities of the objects (schematic) from the beginning” (Schmittau, 2004, p. 21).

Working through the activities in the ‘prenumerical’ portion of the curriculum, children engage with a small number of peers and at least one teacher. The physical interaction with the material objects is initially exploratory. However, with each successive activity, the teacher increases guidance and scaffolding, often in the form of prompting a child to explain an observation or thought process regarding the material objects being considered. The combination of physical manipulation of objects, verbal articulation of observations and the symbolic representation of the relations between objects all serve to introduce concrete information in conjunction with the abstract relations.

As children advance through the “Topics”, the comparisons and actions become more complex. Bridging from actions with objects to the expression of the action with symbols, the “representational schematic of actions” takes on more complex representations, enabling children to distinguish actions on quantities from actions on numbers” (Schmittau, 2004, p. 27). The schematic offers an abstract path for carrying out mathematical operations and analysis of the relationships. A schematic is a tool that reveals the theoretical structure of mathematical problems regardless of differences in surface features due to generalization. The ability to generalize is to understand the relations. Generalization is the ability to abstract (Schmittau, 2004).

It is in Topic IV when the child is first introduced to the concept of number “as the expression of a relationship between the whole of some object and a part of it” (Davydov, 1975, p. 138). From the inception, the child is made aware that measuring, and even counting, may result in a ‘remainder’. Thus, this is also the beginning of the introduction of fractions.

Topics V and VI introduce operations of addition and subtraction, but this is not represented as “+” or “-”. Here the basic properties of quantity (equality/inequality, more than, less than) and the operation of addition (communicative and associative) are transferred to whole numbers. Children are asked to solve the problem in the easiest way. An equation may be presented as an addition problem, such as 6 + x = 11, but the schematic would present it as

```
  11
  ^
 / \
6   x
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When an equation is missing a first or second term, the term “^” is used in the schematic, highlighting the part/whole relationship within the equation. Reveals that the “easiest way” to solve the (addition) problem is to use subtraction, or remove a part from the whole to see what remains. Thus, the Elkonin-Davydov curricular approach introduces algebra, along with arithmetic, from the beginning of instruction (Schmittau, 2004, p. 28).

3.2 *Curriculum and pedagogy as cultural tools facilitate children's cognitive development*

Curriculum and pedagogy are examples of cultural tools created by individuals/society consisting of ideas and processes but are structured as tools for generalizing a knowledge set and/or computational processes. As such, the design of the curricular and pedagogical approach chosen as the educational tool to which children are exposed is fundamental in its effect on their development.

The Elkonin-Davydov curricular approach to teaching math employs a process orientation to solving problems through a simultaneous introduction to object properties and their relations in space and time. The relational aspect of this curricular approach means that an introduction to math does not begin with counting or discrete numbers of objects in a collection. The focus is on relationships between objects, which gives rise to abstractions and ways to express abstract relations. This begins with measurement, and this requires a different number system than the natural (or whole) numbers of counting. The measurement relies on real numbers needed to assess the continuous properties of objects and things. The emphasis on measurement and relations means that math concepts are introduced as abstractions even to the youngest learners (Devlin, 2009).

This curricular approach to teaching math was created as an "attempt to apply Vygotsky's main idea about the interconnections between learning and development to education" (Sidneva, 2020, p. 2). The Elkonin-Davydov curricular approach is considered "Developmental Education (DE)" with a focus on cognitive development in areas of abstract thinking and application of abstraction, as opposed to "Traditional Education (TE)" which is based on "empirical concepts". The curricular approach's intentional introduction of relational concepts, (such as part/whole and difference), to young children, gives rise to an early understanding of abstraction and symbolic representation, versus a focus on counting discrete units.

Another Vygotskian influence can be seen in the "prenumerical" portion of the Elkonin-Davydov curricular approach where manipulation of concrete objects alongside manipulation of abstract ideas produces a mix of everyday or spontaneous exposure to the properties of the objects at the same time that direct instruction of the schematic and other abstract representations occurs. Both introduction to the material objects and direct instruction on the relational properties and how to represent them occur through guided interaction with a more knowledgeable other, i.e. a teacher, in a small group setting with further interaction among peers. As with Vygotskian theories of spontaneous and scientific concept acquisition, the Elkonin-Davydov curriculum "fosters conditional learning that encompasses both naturalistic and algorithmic solution methods" (Schmittau, 2004, p. 23). However, it should be noted that following the "prenumerical" phase, the instructional method shifts away from the specific and concrete to a scientific-concepts method that encourages generalization (i.e. direct interactional instruction of abstract "rules" or operations). In designing the curriculum so, it was believed that mathematics using a general to specific, scientific approach would lead to better mathematical understanding in the long run than a predominantly spontaneous approach with material objects (Devlin, 2009).

3.3 *Concretion, relation, and abstraction foster children's cognitive development*

This pedagogy challenges the one-way avenue of conceptual acquisition, from concrete to abstract, through the intentional introduction of concrete properties along with symbolic representations and abstract concepts. A big difference between this math curriculum and most of the math curricula offered in the West is the introduction of abstraction from the start of the program. Many instructional programs infuse math manipulatives throughout the first three years of instruction. In the Elkonin-Davydov curriculum, young children beginning the program have access to material learning objects while at the same time that abstraction and relations are introduced. Radford (2021a) explains, "The curriculum starts with a focus on a quantitative comparison of quantities (lengths, width, volume, and area), first on instructional materials, then on representations" (p. 329). The inclusion of abstract relationships from the onset is an intentional design component of this curricular approach that strays from the conventional concrete-to-abstract progression found in many pedagogies, such as advocated in the learning sciences (Sawyer, 2006).

Davydov's (1975) goal was the development of theoretical thinking in young children and he believed that prolonging concrete empirical learning would not drive the development of abstract abilities that afford theoretical thinking, and later, relational thinking. Introducing abstraction early on is thought to serve the purpose of activating and driving cognitive development. Additionally, asking children to not only recognize symbols, but also use them concurrently to represent actions and relations, helps children to not fixate on the material object but to see the abstract relations surrounding and encompassing the material object. Again, “theoretical thinking involves both the apprehension of a concept at its most general and abstract level and the application of generalized understandings to particular cases” (Schmittau & Morris 2004, p. 6). As Devlin (2009) notes, introducing abstraction to young children is an example of Vygotsky’s developmental education where “teaching and learning (of abstractions) drives the development of the abilities to comprehend, use and produce abstractions” (p. 6).

The influence of Vygotsky in the design of the Elkonin-Davydov curriculum can also be seen in the consideration of conditional influences in the approach to problem solving. This consideration of conditions present in the inception of the problem and recursively adjusted over time as the problem is computed, is intentionally built in as a part of the problem. In other words, children are asked to consider the conditions inherent to the problem when selecting and adjusting, the computational approach toward a solution. Schmittau (2004) suggests that conditional learning and reflective evaluation of the learning process (verbalized by the learner in articulation with another) assist learners with the construction of deep conceptual structures of computational processes and algorithms. Comprehension of the deeper conceptual structures with various conditions is more adaptable to situations (transference and generalization) than learning that occurs in “absolutist” ways (Schmittau, 2004, p. 23). Because abilities to comprehend and produce abstractions are foundational to computational thinking, it is in generalization that the concrete becomes abstract and teaching and learning can begin to yield computational thought processes. Radford (2021a) states, “generalization allows one to recognize the inner structure of scientific concepts and the system of their constitutive connections” (p. 329).

4. Implementations of the Elkonin-Davydov curriculum

The first implementation in the United States of the Elkonin-Davydov early grades mathematics program was overseen by Jean Schmittau, who also rendered the first English translation of the curriculum from Russian. She implemented the program at a private school in New York, named The Susquehanna School, after having spent several years in Russia observing the curriculum implemented there. Schmittau (2004) summarized after observing this program in implementation for many years that American children found the program to be difficult. But once the initial challenges were past, young children “consistently resolved computational errors conceptually and finally demonstrated the ability to solve high school level mathematics problems. The curriculum appeared to foster the development of theoretical thinking, an explicit goal of the program” (p. 19).

Following Schmittau’s implementation of the curriculum, the Curriculum, Research, and Development Group (CRDG) at the University of Hawai’i at Manoa began full implementation of the Elkonin-Davydov curricular and pedagogical approach to teaching mathematics in 2001 called the *Measure Up* project. Two Hawai’ian schools were the primary research sites for cohorts of learners beginning in the “prenumerical” period through five years of instruction in the curricular approach. This research project was initially a joint venture with the *Institute of Developmental Psychology and Pedagogy* in Krasnoyarsk, Russia, and *Best Practices in Education* in New York. Further collaborative efforts on this project were provided by the University of Mississippi. The 2005-06 academic year was a milestone as the first cohort completed grade five in the *Measure Up* program. Several articles have emerged from the *Measure Up* research project and other sites of implementation of the Elkonin-Davydov methods reporting a wide range of findings and implications for practices.

Venenciano et al. (2021) studied children’s early exposure to the Elkonin-Davydov method as part of the *Measure Up* project and found that students who were exposed to symbols early on (“=”, “<”, “>”) have a better understanding of relations and relational symbols. Further, they found that relational thinking was promoted through the learning activities in the Measure Up project, as children’s developing mental representations of actions began to be “mediated through representing relationships with physical materials, diagrams and symbols” (p. 416). They offered numerous examples of children’s actions and behaviors when engaged in the activities of the Elkonin-Davydov curricular approach that exhibit relational thinking, symbolizing and interpreting as:

- Symbols are offered to stand for actions or distinctions.
- Symbol use is governed by mathematical rules or constraints embedded in the structuring environment.
- Operations can be immediately linked to their inverse.
- Complexity can be constrained, while still engaging with a mathematical integral whole environment.
- Novel symbolic moves can be made (Venenciano et al., 2021, p. 425).

After the first cohort was three years into the program, Dougherty and Slovin (2004) described the schematic (representation) diagrams from their research on the Measure Up project. Students used a variety of means of representing problems, which Dougherty and Slovin described as a "progression" of "interiorization, condensation and reification" (p. 298; paraphrasing Sfard, 1995). In grade three, the students in this study represented their word problems along a range of representational abilities. According to the author's definitions, interiorized representations occurred as problem-specific, whereas a condensation representation began to link the specific to a general approach. A reification representation was comprised of multiple types of representation and were more generalized and abstract than the other two representations. The generalization of the (structure of the) representation increased across the progression described above toward further abstraction. Regarding this third-year point in the research project, Dougherty and Slovin (2004) state, "Student solution methods strongly suggest that young children are capable of using algebraic symbols and generalized diagrams to solve problems" (p. 301).

Another implementation of this model occurred in Sweden where Eriksson and Eriksson (2021) conducted a study implementing Davydov-inspired learning activities in multilingual classrooms. They found that engagement in this math curriculum led children to increased generalization and awareness of relational structures due to the way symbols arise from activity within this approach. The researchers state, "The main results of this study indicate that young, newly arrived students, through tool-mediated joint reflective actions as suggested in the ED curriculum, succeeded in analysing arithmetical structures of positive integers and rational numbers" (2021, p. 363).

In another recent implementation of this curricular approach, Sidneva (2020) presented that all of the children in her study showed significant improvement in all general math abilities. Perhaps due to the developmental nature of the Elkonin-Davydov curricular approach, it was found that progress in math abilities was not dependent on the children's different developmental levels of regulation and motivational readiness to learn relational concepts. This curriculum was designed so that children can enter the program at differing levels of development and still show gains in math abilities.

As noted earlier, instruction through the Elkonin-Davydov curriculum includes opportunities for reflective practice, with this component built in as an intentional step in the process of problem solving. Progression through the curriculum has been found to foster the expansion of a student's mental representations from initially concrete to include abstraction, most likely due to children's interactions with the curricular activities being mediated from the beginning through representing relationships with objects/physical materials and symbols (Venenciano et al., 2021).

Implementations of the Elkonin-Davydov curriculum across the globe have led researchers to the same conclusions. Studies have demonstrated that children exposed to the Elkonin-Davydov curricular approach had a better ability to comprehend abstract relationships and symbolic representations, as well as compute problems generally considered beyond their years (Schmittau, 2004).

5. Discussion

This conceptual paper focuses on the development of foundational cognitive abilities of relational thinking that are emerging in the early years of a child's life (abstractions and pattern generalizations, and symbol systems and representations) and proposes an instructional model that could foster these abilities. It also seeks to demonstrate how curricular and pedagogical design grounded in Vygotskian theories that position education (i.e. teaching and learning) as a driver of development, achieves successful outcomes across implementations, (Vygotsky, 1978; 1991). A plausible explanation for the findings from the implementations of the Elkonin-Davydov curricular approach has shown how the learning design works through the concurrent introduction and repeated exposure to concrete and abstract concepts from the commencement of instruction, with the early emphasis on representing relations symbolically. The successes reported from implementations of the approach exemplify some of Vygotsky's strongest assertions regarding developmental education. The instruction aspect of the Elkonin-Davydov curricular approach relies on the concept of ZPD through social interaction with others, notably the classmates and the teacher who function as the more knowledgeable other/s. The acts of reflection and articulation inherent in the curricular approach also account for the successful findings across studies, because these interactions with one's self and others through inner dialogue and spoken language are how human children intramentalize their learning. The significance of this manuscript lies in the identification of a cultural tool that promotes and drives children's cognitive development and in explaining the theoretical basis of the tool's design that aligns with Vygotskian principles, features of cognitive science and child development. The findings from the numerous implementations of the Elkonin-Davydov curricular approach are significant and support the idea that this is an important pedagogy for fostering mathematical, relational and computational thinking.

The findings suggest that due to its interactional process orientation, the Elkonin-Davydov curricular approach should be used to foster young children's cognitive development, particularly to improve young children's mathematical ability and computational skills. More importantly, applying the Elkonin-Davydov curricular approach to early childhood education ensures educators perceive children's development from multiple angles. Rather than understanding child development from a single standpoint, the Elkonin-Davydov curriculum offers insightful comprehension of child development from both general cognitive science principles and children's social-linguistic-cultural background influences. The Elkonin-Davydov curriculum provides a balance between reexamining standardized developmental sequences described by developmental psychology and reconsidering a child's individual needs as described in sociocultural theory. As a means for further improvements, researchers should also continue to follow implementations of the Elkonin-Davydov curricular approach in young children's development. The curriculum constantly needs to be updated to meet societal requirements and children's needs, particularly since technology is now societally pervasive with relational and computational thinking skills at a premium. The Elkonin-Davydov curricular approach can be implemented to meet the needs of the current digital age as a way to foster higher mental functions. The findings from the implementations suggest that the investment in professional development for classroom teachers in profoundly understanding the approach would benefit children's learning and their teaching. This may require a school to restructure its math program for at least the first five grades, as the curriculum is based on a multi-year design that aligns with child development to achieve maximum outcomes.

6. Conclusion

With the above literature review, the analysis of theories and implementations, and the investigation of the Elkonin-Davydov curricular approach, we present our conclusions as follows.

First, relational, and thus, computational thinking is underpinned by cognitive development in the areas of abstract thinking and symbolic representation. The Elkonin-Davydov curricular approach to teaching math, especially the "prenumerical" phase in early childhood, promotes relational thinking and introduces abstract representations at an earlier age than most instructional systems in the United States (Schmittau & Morris, 2004). This curricular approach is offered here as an example of developmental education that through its design fosters the development of what Vygotsky called higher mental functions (Vygotsky, 1991). This curricular approach serves as an example of a cultural tool that through interaction can be seen to drive cognitive development in the humans who engage with it.

Second, whether we see the Elkonin-Davydov curriculum through a cognitive science lens of experiential exposure which activates neural growth and connection, or through a sociocultural lens of the inevitable influence, indeed co-constitution, of the environment, this curricular approach to teaching math can be seen as a driver of development. If certain cultural tools can be seen as driving development, then the tools that we choose to use or, even place in front of children for their use, have significance in their inherent (to the tool's design) ability to shape and guide the way a child thinks. The internalization of psychological tools is a transformational process that changes and expands cognitive development and abilities.

Last, engagement in (or exposure to) the Elkonin-Davydov curriculum, starting with the "prenumerical" period, asks children to begin to think in a relational, or abstract, way. As this is structured over time with compounding complexity, this curricular approach shapes a child's thinking to recognize and comprehend relations and the dynamic nature of problem solving, starting with incipient conditions and recursively cycling through a process of computation.

As the use of computational devices, artificial intelligence, and other digital technologies becomes ever more ubiquitous in our society, exposing children to cultural tools that will foster their cognitive skills becomes increasingly important.

Conflict of interest

There is no conflict of interest.

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Appendix A

The “Prenumerical” Period of Mathematics Instruction.

“The Experimental Introduction of the Concept of Quantity in the First Grade” (p. 134-138).

Topic I: Comparing and assembling objects (according to length, volume, weight, composition and other parameters).

1. Practical problems of comparing and assembling.
2. Isolating attributes (criteria) by which the same objects may be compared or assembled.
3. Verbal designation of these attributes (‘by length’, ‘by weight’, and so forth).

Topic II: Comparing objects and designating the results in a formula of equality or inequality.

1. Problems of comparing objects and designating the results symbolically.
2. Verbal designation of the results of a comparison (the terms ‘more than’, ‘less than’ and ‘equal to’). The written symbols of ‘<’, ‘>’ and ‘=’.
3. Making a drawing to designate the results of a comparison (first a ‘copy’ and then an ‘ abstraction’, using lines).
4. Using letters to designate the objects being compared. Writing down the results of a comparison using the formula: A B, A < B, A > B.

Topic III: The properties of equality and inequality.

1. The reversibility and reflexivity of equality (if A = B, then B = A; A = A).
2. The connection between the relationships “more than” and “ less than” in inequalities when the sides being compared are ‘transposed’ (if A > B, then B < A, and so forth).
3. Transitivity as a property of equality and inequality; if A = B, and B = C, then A = C; if A > B, and B > C, then A > C; if A < B, and B < C, then A < C.
4. The shift from evaluating and the properties of equality and inequality, using physical objects to having only letter formulas available. The solution of varied problems which require a knowledge of these properties (for instance, problems involving the connection between relationships, such as Given A > B, and B = C; find the relationship between A and C).

Topic IV: The operation of addition (and subtraction).

1. Observations of changes in objects in one or another parameter (such as volume, weight, length, or time). Representation of increase or decrease with the symbols ‘+’ and ‘-’ (plus and minus).
2. Upsetting a previously established equality by changing one or the other of its sides. The shift from equality to inequality. Writing formulas of the type: is A - B, then A + K > B; if A = B, A - K < B.
3. Methods of shifting to a new equality (‘reconstructing’ it according to the principle: Adding an ‘equal’ to ‘equals’ yields ‘equals’). Working with formulas of the type: if A = B, then A + K > B, but A + K = B + K.
4. The solution of varied problems requiring that addition (and subtraction) be used in shifting from equality to inequality and back.

Topic V: The shift from an inequality of the type A < B to equality through addition (or subtraction).

1. Problems which require this shift. The necessity of determining the value of the difference between the objects being compared. The possibility of writing an equality when the specific value of this difference is unknown. The method of using x. Writing formulas of the type: if A < B, then A + x = B; if A > B, then A - x = B.
2. Determining the value of x. Substituting this value in a formula (introduction to parentheses). Formulas of the type: A < B, A + x = B, x = B - A, A + (B - A) = B.
3. Solving problems (including ‘word problems’) which require the indicated operations.

Topic VI: Addition and subtraction of equalities and inequalities. Substitution.

1. Addition and subtraction of equalities and inequalities: if A = B and M = D, then A + M = B + D; if A > V and K > E, then A + K = V + E; if A < V and B = G, then A +/- B > V +/- G.
2. The possibility of representing the value of an equality as the sum of several values. Substitutions of the type: A = B, B = E + K = M, A = E + K + M.
3. The solution of various problems involving the properties of relationships to which the children have already been introduced (many of the problems require simultaneous consideration of several properties and adeptness at evaluating the meaning of the formulas; the problems and their solution are described below).

Derived from:

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