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Teachers Who, While Using Technological Devices to Teach Mathematics, (Re) Construct Their Specialised Knowledge

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Abstract: In this article, we share the design and implementation of the preparatory phase of a didactical experiment with teachers of mathematics, using informatics technologies, in classrooms intended for young teenagers from 12 to 17 years old from the province of Buenos Aires, Argentina. We based this design on the contributions of different theoretical frameworks (MTSK, Mathematical Working Space, Collaborative Research, Teaching experiment methodology) and analysed the data obtained from the administration of an instrument created ad hoc, to generate information that will allow us to build the didactical experiment. We conclude that the design of the experiment requires a joint reflection between the groups of teachers and researchers, to make the teacher's specialised knowledge explicit, guarantee its performance and goal achieving.

***Keywords*:** specialised knowledge, teaching experiment, mathematic working space, technologic resources

1. Introduction

We assume that, to study the relationship between teaching and learning mathematics, it is necessary to start from a scientific approach to the problems generated from the communication of mathematical knowledge: professional problems from the people who teach mathematics. This approach considers the class in a wider way, thinking of it as an object of study, in which we can analyse the interaction and interdependence that exist among teachers, students and the mathematical contents intended to be taught (Stylianides & Stylianides, 2013). During the classes, the construction of meaning doesn't necessarily imply the appropriation of the mathematical knowledge that circulates. As Vargas Vargas et al. (2020) affirm, the design of problem-based learning situations that use technological resources is adequate to achieve mathematical learning. Therefore, teachers should master certain professional skills (both conceptual and methodological) to properly integrate technological resources in their classrooms and thus, produce mathematical learning (Arévalo et al., 2019). Under some circumstances, that construction favours a structure that allows the memorization of such knowledge: all of the teacher's work in the classroom is focused on reaching that goal (Hernández, 2020; Soares & Soares, 2020).

In this article, we centre on an aspect of the relation described: the one that considers the moments of the

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mathematician's class, in which the one who teaches, professionally assumes the decision of using technological resources such as devices, to manage the teaching sequences designed (Hoyles, 2004). Research associates professional development with coherence between classroom conceptions and practice (Kaiser & Li, 2011) and professional knowledge (Bell et al., 2010). In this study, we focus on the cognitive aspect to deepen the reflection on practice, and assume that mathematical and technological knowledge allow teachers to plan, manage and reflect on teaching. We interpret this professional decision from the framework, which assumes the class as a mathematical working space brings (Kuzniak, 2011), in which the teacher shows evidence of having specialised knowledge for such a task (Carrillo-Yañez et al., 2018). We formulate the study of this teaching performance, in the management of a collaborative work (Bednarz, 2004) between researchers and teachers through the design and management of a didactical experiment (Reigeluth, 2000). We understand this experiment as a controlled intervention in the planning processes, class management and learning evaluation that worries about the issue of the reproducibility of its results (Winslow, 2009). We share the previous process to the construction of the didactical experiment, making explicit how we designed instruments and analysed the results of their application that will be used for the design of the experiment.

We show the previous stage that we qualify as necessary for the design of a mathematics teaching experiment that, as an extension of didactical engineering, will be developed with teachers who work in a secondary school in the province of Buenos Aires, Argentina. The novelty of our study is the process of building the instruments used in that previous stage mentioned. With the experiment, we intend to answer the following questions: How to improve mathematical learning in real school contexts? Which technological resources for teaching can be used for this improvement? How to explain such improvement? How to substantiate it?

2. Theoretical framework

Different authors consider the Didactic of Mathematics as a science of design (Wittmann, 1995; Collins et al., 2004; Hjalmarson & Lesh, 2008; Kelly et al., 2008; Lesh & Sriramn, 2010), from a question that structures their production:

> Should mathematics teachers consider themselves […] as engineers or other scientists oriented toward design, whose research is supported over multiple perspectives -practical and disciplinary- and whose work is guided by the necessity to solve real problems, from the need to elaborate relevant theories? (Lesh & Sriramn, 2010, 124).

From this vision, the Didactic of Mathematics is oriented toward the design of processes and resources thought to improve mathematics teaching and learning and, in the same sense as Reigeluth (2000), is understood as a theory of educational design that offers an explicit guide about the best way to help learning and improving. In that way, the research based on design, design research, or research experiment [We could trace a parallelism between design experiment and didactical engineering, while the intention of didactical engineering is sharing a way of conceiving didactical work (Artigue, 2011). From its origins, didactical engineering is linked to didactical interventions (experimentations) in classes, becoming sequences of activities that put the theoretical work to a test. But both procedures differ in that the experiments assume different interpretative theoretical frameworks, while didactical engineering is based in an explicit framework: the Theory of Didactical Situations of Brousseau. Besides, through teaching experiments, the intention is to design and look for resources for teaching that in didactical engineering are analysed as everything that builds the space in which students are faced] (Brown, 1992; Kelly et al., 2008) is a set of methodological approaches in the study of learning in context. In this methodology, the design and systematic analysis of strategies and teaching resources are used in close relationship with research, so that they will become interdependent. Starting from the premise that research -that has a particular duration- includes the phase of designing innovative curricular tasks and experimentation in classroom contexts. We could trace a parallelism between design experiment and didactical engineering, while the intention of didactical engineering is sharing a way of conceiving didactical work (Artigue, 2011). From its origins, didactical engineering is linked to didactical interventions (experimentations) in classes, becoming sequences of activities that put the theoretical work to a test. But both procedures differ in that the experiments assume different interpretative theoretical frameworks, while didactical engineering is based in an explicit framework: the Theory of Didactical Situations of Brousseau. Besides, through teaching experiments, the intention is to design and look for resources for teaching that in didactical engineering are analysed as everything that builds the space in which

students are faced. The evaluation of their results is done by considering the influence of the context, identifying the restrictions and conditioning factors (The Design-Based Research Collective, 2003). What is designed is a complete learning environment with tasks and resources to sequence and support learning, with the aim of producing explanatory descriptions instead of descriptive ones (Reimann, 2011): the research is inserted in the teaching world, generating a propitious space for collaborative work among groups of teachers and researchers. Most of the experiments are conceptualised as case studies, oriented to support the learning of groups of students in a particular content domain. The theoretical intention is to identify and describe how the different groups of students learn during a period of time (Battista, 2011), adapting the curricular goals of the research and using theoretical models to justify the design of the tasks (Steffe & Thompson, 2000).

To conduct this type of research, three phases are taken into account (Cobb & Gravemeijer, 2008): the preparation of the experiment, the experimentation or practice and conducting retrospective analysis. Its implementation gives rise to the possibility of generating registers of the practices that help exemplify, starting from different representations about teaching mathematics, different theoretical ideas (Villella & Steiman, 2021). This way of studying the educative moment, in which mathematics is taught, overcomes the idea of action research -in terms of innovation- to focus on rational action, based on pre-established didactical knowledge about the educational fact.

The studies of design allow to relate educative practice and theoretical analysis, through situated reports (Skovsmose, 1999) about student's learning, relating directly the learning process and how it has been promoted:

- they test theories into practise;
- work with teachers in the construction of learning;
- recognise the limits of theory;
- capture the specificity of practise and the potential advantages of adapting the theory to its context in an interactive and refined way;
- study the daily problems of the classroom, of schools and communities that influence teaching and learning, adapting teaching to such conditions (Kelly et al., 2008; Shavelson et al., 2003).

There are also difficulties in its implementation, which emerge from the complexity of the situations in the real world (many variables are not there or they can't be controlled):

- the coordination problems in the collection of data derived from the participation of various researchers;
- the difficulties of managing and analysing the great amount of data that comes out;
- the complexity of the comparison between designs and the problem of delimiting the origin of knowledge that the researchers acquire through the investigation process, due to the continuing dialog produced between the theory and the practice (Collins et al., 2004; Dede, 2004).

The practice of teaching is uncertain, unpredictable (Cols, 2000) and impossible to analyse without including its moral dimension. It responds to needs that are far beyond the intentions of the direct subjects. When we consider mathematics teaching as a social practice, we focus on the transformations of the physical, social and individual background that it produces.

The teachers who work in math, assume themselves as teaching professionals once. Among other qualities, they consider that the problem resolution process that they face their students with, produces as a result, the construction of locally adapted mathematical knowledge. This knowledge stresses the subjectivity -of teachers and students- that are reformulated starting from diverse scopes, linked to the social scene. The specialised knowledge of the one who teaches, stands as a way of understanding the curricular prescribed mathematical knowledge, linked to the understanding of how they are learnt and how they can be taught (Villella et al., 2018). In this way, this knowledge acquires a specialised character (in the sense attributed in the model of the Mathematical Teacher Specialised Knowledge (MTSK)) that considers:

- the knowledge of mathematics' subjects (Isnawan et al., 2022) structure and doing (knowledge of and about maths);
- knowledge of mathematics teaching, of the characteristics of its learning and curriculum (pedagogical and didactic knowledge).

And they intersect with the beliefs and conceptions about mathematics and its teaching (Carrillo-Yañez et al., 2018). There is a difference between the mathematical knowledge that a person who teaches needs and the one needed by other mathematics users (common knowledge advanced or superficial that doesn't require to be taught).

Therefore, the specialisation refers to the help of the design in teaching situations (considering the relations among the own contents, and the relations with the contents of other subjects) and the analysis of the learning results; not to the advancement of the needed mathematical knowledge. This specialised knowledge is personal, for it is characteristic of the teach and different from others and it depends on his or her conceptions, values, attitudes and experiences (vital and/or professional). It is practical knowledge, destined to intervene in the classroom situation and it nurtures from it the theoretical reflection that allows to transform the knowledge from the action or the theory starting from action. Likewise, it is dynamic, changing and evolving knowledge that grows through the interactions with students, teachers, and professional and personal experiences. It is built in working communities collaboratively and allows us to produce knowledge based on teaching and learning. (Villella et al., 2021).

The reflection over teaching means to analyse the student situation fostered by the teacher's management in the mathematics class, the role of resources in the transformation of the working spaces created ad hoc, the study of the meaning of knowledge provided by the different resources chosen, and the social and institutional aspects that stress this mathematical working space. These spaces (Kuzniak, 2011) are organised environments, which allow to solve mathematical tasks and are based upon two levels: a cognitive one and an epistemological one. The cognitive level includes the visualising, building and testing processes, which are necessary for the resolution of the problems that can emerge from any chapter that are part of school mathematics. In the epistemological field, we locate the representative (icons, figures, symbols that allude to the mathematical objects), artefacts (resources with the potential to be used by each student as an instrument for learning) and referential (mathematical subjects alluded to in each activity).

3. Methodology

To answer some questions presented in the introduction, we collected certain data that will allow us to get precise information about the group of teachers, with whom we'll be working. In this way, we generated, together with the teachers we could work with, a mathematics classroom as an experimental scenario for the resolution of problems with the use of informatics technology. Like this, the one who solves the problem is allowed to experiment to create conjectures, explore the steps and solutions found many times to improve or adjust them to what was required, discuss the findings with the group and teachers in charge (Villella et al., 2021). It is possible that the student's action, mostly at the beginning of the activities in which technological resources are used, might become kindred with processes of trial and error. We discuss then with the teachers whether this quality of resolution lasts through time or, if after an exploratory period, the resolutions leave that venturesome character, to transform into testing and argumenting processes. Besides, we try to determine if this shift of working qualities is spontaneous or it's due to a thoughtful or intended intervention of the one who teaches. We affirm, upon the basis of previous investigations (Bifano et al., 2012; Ferragina, 2012; Villella, 2002; Villella et al., 2021), that the feedback that proceeds from the computer through the dialog with the software used, with the mathematical knowledge and the elaboration of logical arguments, is the elements that sustain what has been surmised, and in this way, allow to justify its validity.

To develop this preparatory stage of the experiment, we designed various instruments. First, we designed an inquiry destined to teachers who use technology in their mathematics classrooms, to go into their pedagogical knowledge. In accordance with Shulman (1986) and the contributions of Carrillo-Yañez et al. (2018), we define the Pedagogical Content Knowledge (PCK) as the fusion of the mathematical content intended to be taught and its didactic, to make it accessible to the students that will work with it. We considered that it is necessary to recognise the specialised character of the mathematics teacher, specialisation that is understood as the exclusivity of some of the knowledge that this professional has. In this proposal, we characterise that specialised knowledge made by two different domains: Mathematical Knowledge (MK) and the PCK that include several subdomains, imbued by the beliefs and conceptions that teachers have about mathematics teaching and learning. We take into account, starting from the studies of Mishra and Koehler (2006), the understanding of teachers over the interaction of mathematical content, mathematical didactics and use of technological resources. In this way, our inquiry tried gathering information to determine, in the teachers that were part of the test, the:

- Content Knowledge (CK): what the teacher knows about the mathematical content that he or she provides to the class.

- Pedagogical Knowledge (PK): which knowledge do teachers have about teaching practices, strategies and methods to promote the student's learning.

- Technological Knowledge (TK): which knowledge do teachers have about technology can be integrated into the planning and management of their classes to achieve learning from their students.

The interaction of these types of knowledge generated the need to analyse as well:

- Technological Content Knowledge (TCK): knowledge about how the mathematical content can be represented through technology: the ability to use the technology available to the contents of the subject.

- Pedagogical Content Knowledge (PCK): how the subjects are organised in problems in order to represent and adapt to the diverse interests and abilities of the students.

- Technological Pedagogical Knowledge (TPK): knowledge on how to use the technology in the teaching practices, strategies and methods to promote the learning of the mathematical content that the students must study.

- The Technological Knowledge of Pedagogic Content (TPACK): relates technology, didactic and mathematical content that allows the teachers to develop teaching strategies, appropriate and specific to the context, in which they work.

The survey that we prepared to use all the assumptions explained, was composed of the following group of questions:

• Group 1: (Technological Knowledge [TK])

1- How did you learn to work with technologies in mathematics?

2- How would you qualify your learning process about it? Why?

3- Do you consider that you are updated regarding the technologies that can be used to teach mathematics? Why?

• Group 2: (Content Knowledge [CK])

1- Do you take courses or lectures regarding the mathematical content that you teach to be up to date with the new advancements? Which are the ones that interest you the most? Why?

2- Do you have access to books or magazines about the didactic of mathematics to be updated about which resources to use and how to use them in your lessons? What do you care about reading in them?

• Group 3: (Pedagogical Knowledge [PK])

1- Do you make a diagnosis of your groups before you develop each content? How do you do it?

2- What relation does the result of your diagnosis have with the class's development? Do you adapt the classes to those results?

3- Which learning styles can you identify in your students? How would you describe them?

4- Do you adapt your class's script to those styles? Why?

5- With which instruments do you evaluate the mathematical production of the students? Is it always the same type?

• Group 4: (Pedagogical Content Knowledge [PCK])

1- Do you adapt the level of difficulty of the contents you have to teach to the level of mathematical knowledge of the students? How? Why?

2- Do you adapt your class's script to the level of complexity of the content? Do you use any special resource?

3- What kind of teaching strategy do you use with more frequency? (Cooperative work, projects…).

• Group 5: (Technological Content Knowledge [TCK])

1- Which resources are more useful to work in your lessons? Why?

2- How do you use the technological resources in your training: only as tools for teaching, to understand what you will teach…?

3- What kind of resources are the ones you like the most? Are they the ones you use?

4- How do you use technologies in a classroom?

• Group 6: (Technological Pedagogical Knowledge [TPK])

1- When you use technology, can you anticipate the obstacles that the students might have to face?

2- Do you have the resources to choose the best tools for each content?

3- Can you adapt the tools to each content?

4- Do you help the students relate to the tools?

• Group 7: (Technological Pedagogical Content Knowledge [TPACK])

1- Do you guide the students in the use of technologies to evaluate the learning of the contents?

2- Do you look for methodologies and tools regarding the mathematical content you need to work in the classroom?

3- Do you incorporate technological tools in the design of the teaching sequences?

In this way, we seek to establish a method of collaborative work, in which we tried producing relevant knowledge about the professional practice and find dialog channels between what is known as the investigation world and the practice world. In this way, we favour a process of knowledge production about the teaching and learning of mathematics that is created together with researchers and teachers who interact to contribute from their specificity to knowledge construction. This knowledge emerges from the reflection about the practice of teaching mathematics and comes back to it to provide a meaning to the decisions that teachers make daily in the teaching process. The implementation of this collaborative method, supposed interventions in mathematics that is taught that not only are fertile in terms of student's learning but also for the reflexion of teachers about how they teach. From this collaborative research we questioned, in the framework of school practice, the viability of certain pedagogical devices designed by investigators for the improvement of the quality of the student's mathematical learning: we retrieved the voice of the teacher through the understanding of their practice that is produced when he or she became the co-author of the teaching situations that will be the object and instrument of the research and that will shape the second phase of the experiment. From this perspective, researchers dialogue with teachers within the context in which they work, to understand how they substantiate and apply their decisions (Bednarz, 2004). The activity was organised so that it would promote and hold a conversation between teaching practice and reflection, between teaching and didactic. In the process, periodic meetings with the participants, allowed us to form an interpretation zone around situations that are an object of exploration. This reciprocal space for argumentation developed a series of discussions about how each person produces meanings in the teaching context; giving meaning to the co-construction of knowledge and turning into a place of arbitration between the research culture and the teaching culture. During the interactions, the teachers analysed the dimensions of their experiences and questioned their knowledge when they crossed them with those ones given by researchers.

For the application of the inquiry -core instrument of this previous work- we used a criterion sampling to select participants who gathered certain predetermined characteristics. The inquiry was created as a questionnaire that was supplied online using Google Forms in August 2021. For our study, the sample of teachers was built on the basis of the following qualities that each of them fulfilled: being active at the time of being surveyed, teaching using technological resources, having trained as mathematics teachers. Some of them, who expressed their consent to be part of the ongoing research, were invited to continue working with the research team. At the beginning of the study, the aims of the inquiry were discussed with these selected teachers, with the purpose of accomplishing a shared understanding of its meaning. It was previously asked each teacher to choose, according to their dispositions, one, two or three groups of students, which they were working with technologies in the mathematics classroom and to qualify their behaviour with the working proposal. To do so, they were asked to use the scale of Figure 1.

1: when the reference to the use of technology wasn't introduced;

2: when the reference to the use of technology was introduced with little frequency;

3: when the reference to the use of technology was introduced with a lot of frequency:

4: when the reference to the use of technology was introduced constantly in the classroom.

Figure 1. Qualification of student's behaviour criteria

The statistics test applied for the first analysis of the student´s data was the Kruskal-Wallis as an alternative method to an analysis of variance for ordinal data. This is a non-parametric test that doesn't imply a distribution of probability for the data and their results are obtained from procedures of ordering and counting. The test of William Kruskal and W. Allen Wallis (named after their last names) is useful to test if a group of data comes from the same population. It is often used to determine if there is or not a statistically meaningful difference between the averages of three or more independent groups. It was useful to examine the significance of the difference between groups of qualifications or scores coming from various groups or independent tests taken simultaneously.

4. Results

In this section, we share the results obtained during the development of the described methodology. It is important to highlight that since we choose to follow a collaborative model such as the one, we describe, the results generate synergies within the research that force researchers to assume certain unforeseen decisions. In the presentation that follows, we hope to be clear about the description of those incidents that appeared and that led us to generate or complete instruments to continue analyzing the responses obtained and thus, characterize this first phase of our study.

The questionnaire was sent to 917 teachers. A total of 293 responses were collected from twenty different schools, which represented a general response rate of 31.9%. From those answers, some of their authors were invited to participate in the study, resulting in 24 teachers as volunteers. Their ages oscillated between 25 and 58 years old ($\mu = 41.5$); their seniority as mathematics teachers varied between 5 and 31 years ($\mu = 15.4$). Regarding their experience of working with technologies in the mathematics classroom, it was characterised in trimesters of annual use, in the following way:

- 1 trimester a year, 2 teachers, which matches with the geometry theme section.
- 2 trimesters a year, 16 teachers, which matches with the geometry and calculation theme sections.
- 3 trimesters a year, 6 teachers, which matches with the geometry, calculation and algebra theme sections.

The aspects evaluated in the students, which emerged due to informal dialogues with them in the school, were the ones shown in Figure 2.

a. I use the calculator when I have to solve activities.
b. I use my PC's software to do the activities that I'm given to solve.
c. I use the internet to find out everything that I don't understand about the activity that I'm given to solve.
d. When I work with a PC, I rather do it in a group for discussion.
e. When I work with a PC, I rather do it individually to focus on what I'm doing.
f. I do everything that I need to solve with a piece of paper and a pencil.
g. I use the calculator or the PC only to check what I did is correct.
h. I search in the PC's software what I can do before I try solutions for the activities.
i. I play with the PC's software and so I try possible options for the activities I need to solve.

Figure 2. Aspects evaluated in students regarding their use of technologies

To get information about how the student's attitude changed in the mathematics classrooms, in relation to the time that they had used the technologies, the teachers were asked to qualify the students who had been working one, two and three trimesters in classrooms mediated by technological resources.

Following these instructions, the teachers valued the student's behaviour toward the use of technologies (referring to what is described in Figure 2) out of 2,114 students whose ages oscillated between the 12 and 15 years old from seven schools located in the same region (4 of public management and 3 of private management). They were in 1st, 2nd, or 3rd year of secondary and were distributed in the following way: 1,015 (48%) worked with technological resources during one trimester; 528 (25%) during two trimesters; and 5,721 (27%) had three trimesters of working with technological resources. All of these students (100%) worked with calculators; 697 of them (33%) worked with Spreadsheets as well, and 916 of them (43.3%) worked with GeoGebra. Of the ones who used GeoGebra, 754 (82.3%) also used Spreadsheets.

In this study, the independence condition required for the applied statistical test was achieved, and the data was ordinal and grouped according to the time of use of technological resources in the mathematics class (one, two or three trimesters).

With the aim of qualitatively analysing the results provided from the statistical analysis of the data, we proceeded to create a second group of data collecting the information from the teachers through an interview via the Zoom platform. Four teachers from the study were interviewed: their senioritis was from 2, 5, 15, and 20 years of service. It wasn't a structured interview, in which they were asked if the conclusions they aimed at in the global results were similar to those ones of their personal experiences; and if they could, as from this experience, explain those results. This last thing aims to know the arguments they used to explain the differences observed in the students regarding the use that they give to technological resources in the classroom.

The ratings that the teachers assigned to the students, after interviewing them in the nine evaluated aspects, created a first group of statistically analysed data (Figure 3). The goal was to detect if said ratings suggested changes in the attitude of the students in relation to the time, which they spent using the technologies to solve the problems presented in the mathematics classroom.

Evaluated criteria	Use of technologies during 1 trimester				Use of technologies during 2 trimesters				Use of technologies during 3 trimesters			
Occurrence frequency	1	2	3	4	1	2	3	4	1	2	3	4
I use the calculator when I have to solve activities.	29	12	15	90	6	6	4	51	9	7	9	42
I use my PC's software to solve the activities I'm given.	18	29	30	10	11	6	6	42	10	9	10	47
I use the internet to find out everything that I don't understand about the activity I'm given to solve.	17	9	12	56	5	5	6	25	12	9	8	20
When I work with the PC, I rather do it in a group to discuss.	16	18	4	45	7	6	5	59	14	7	12	80
When I work with the PC, I rather do it individually to focus on what I have to do.	25	17	30	76	6	6	5	55	8	6	7	43
I do everything that I need to solve with a piece of paper and a pencil.	69	15	29	32	15	3	4	4	12	7	9	10
I only use the calculator or the PC to check if what I did is correct.	4	14	27	56	7	4	4	33	6	6	8	12
I investigate in the software of the PC what I can do before I try solutions to the activities.	31	18	10	32	6	5	8	46	8	9	9	77
I play with the software of the PC and I try the possible alternatives to the activities I have to solve.	3	9	19	89	5	12	5	45	6	5	6	12

Figure 3. Chart with results of teacher's considerations to the evaluated criteria

The interview, to which we refer was conducted via Zoom with each selected teacher. The first activity we started with was asking them to answer the following question with three words: what is generated in your classroom, then you apply technologies to solve the activities? The words that appeared are presented in the frequency chart in Figure 4. Among the words that refer to the class climate, the ones that stand out are: astonishment, fight and investigation. Astonishment appears related to the use of resources, which are not frequent in the classroom; the word fight appears as an analogy referring to the climate, in which discussion, argumentation and the need to convince the other appear, sometimes in vehement climates; the word investigation shows the type of activity given as a way of working when the technologies appear in the classroom.

Anxiety	1	Apathy	1
Argumentation	1	Astonishment	4
Bustle	1	Dialogue	1
Discussion	1	Fun	1
Rehearsal	1	Enthusiasm	1
Investigation	2	Negotiation	1
Fight	3	Concern	1
Test	1	Reflexion	1

Figure 4. Frequency in the use of words to describe the class's climate

The questions from the interview done to the teachers were grouped according to which aspect of their professional profile we are analysing. The answers are presented identifying each teacher with letter T and a number: 2, 5, 15, 20, that correspond to their seniority in secondary teaching; the presentation of results doesn't necessarily correspond with the sequence, in which the questions and answers happened during the interview. Here they are presented to make their reading easier:

• Group 1: (Technological Knowledge [TK])

T2: In the ISFD (Instituto Superior de Formación Docente -ISFD- is the institution of tertiary level, not academic, that, in Argentinian Educational System, is in charge of teacher's initial training) where I studied, we worked with calculators and software of dynamic geometry. We were not allowed to use them to find solutions for the activities they proposed to us. Some of our teachers (I especially remember the Methodology professor) made us argue about how we used the resource before we discussed the result. That was interesting because they put us inside the resolution issue. Now that I work, I have less time than before to check what is available, so what I do is I go to what I'm sure I know: Spreadsheets and GeoGebra. I don't look for many other offers because I'm not sure of how to use them.

T5: The kids in the classroom ask you to use the devices each time they can. I don't know how to do it completely, so I ask for help from my partners from informatics. If they are in the lab, I dare to go with the kids and there I let them explore and solve. I discussed with them the result they achieved and nothing else. In fact, I don't care much about it and the little I know is because my son asks me every time, he uses the computer and with him I find out what it is all about.

T15: In my training, we didn't have access to the machines. Some things with my scientific calculator, but only to solve. I did a few courses at the INFoD (Instituto Nacional de Formación Docente -INFoD- is the institution, dependent on the Education Ministry of the Argentine Nation, which is in charge of the permanent training of teacher's institutions across the country, through virtual and face-to-face offers that develop different topics referred to teaching and institutional management) and through the trade union, but they seemed very boring to me. I want to know what each command is for (the little buttons that appear on the screen) so I could ask the kids to use them. It makes me dizzy not knowing it and that they know more than me. So, I try to ignore them. I don't forbid them, but I don't encourage them either.

T20: In my teaching training, I wasn't trained in technologies. We saw them as something new, very potent, but not within the reach of our possibilities (economical as regarding our training). The Commodore 64 brought some interesting things, but you had to connect the TV, program, read… too much effort and I never saw big results. When they put the programmes on the computers that the kids were given at school, I thought I would go insane. I wasn't aware of anything and they were asking all the time. I put on the batteries and started some courses. Some of them were failure and other opened up my mind, they calmed me because I realised that I didn't need to know everything about the software beforehand and that the most important thing is not in the machine or the program, but in what allows you to do and what it gives as a result. I understood that it is not about skills, but properties and of those… I have all the answers!

• Group 2: (Content Knowledge [CK])

T2: I finished my teaching training not very long ago, so the contents are very fresh for me, at my reach. I think that in the future I will take courses, but rather than about content, they would be about resources, materials. I like calculation and I don't think there is much newer to learn about, so not for now.

T5: I didn't take any course since I graduated. The raise to get hours, doesn't leave you with much time for that. Anyway, whenever I can, I will take a course in Geometry. For now, I don't need to. I'm not studying now, I have focus on getting hours. Maybe in the future, I'll do something.

T15: I took several courses and realised that the best thing is to sign yourself into something more organised. I don't know, a Diploma or a Bachelor's Degree. I'd like to start now that I'm incumbent and I have all the hours I can and want. I think that doing something related to teaching mathematics would be nice. I know that in the University of my area (the University of San Martín), there are offers, so I will read them. Through all this time, I only read things that arrive at the school and some things that my partners share.

T20: If you don't train and update permanently, you lose the track and are excluded. I have taken many courses to understand teenagers and how to organise the classes, but I like the ones that discuss mathematics best. In those, I have fun because I think of alternatives to the answers of the exercises again, and that fascinates me. I read what comes from the editorials and some loose things that the Ministries send, but when they don't dig into the mathematical stuff, they bore me a little (well, a lot).

• Group 3: (Pedagogical Knowledge [PK])

T2: Before I start with each topic, I ask the kids what they remember about it or what they think I will introduce to them. I listen to them and try to give them answers to their comments, but being honest: I have my class prepared and I give it in the way I think will be better. In general, I base on a text book that I use as a guide and propose activities with photocopies. I take a test when I finish with each topic, and they have to solve it individually. During the class, they can argue and work in groups and if they want, use the calculator or the computer. The kids often apply what they read in the photocopies (I manage that they keep the definitions and concepts) and have them handy to solve the activities. I don't have time to wait too long for them, so I rush to them and when a team or one of them gives me the solution to the problem, I share it with the rest and make them move forward. I try to take many make-up tests so they won't have the subject pending.

T5: I always make a diagnosis at the beginning of the year, but after that, I get by my intuition and present the topics as I think will be better. What I'm realising now is, the fewer restrictions I present to work (I mean, apply the method, look for such a definition) and the more I let them be, they have a better time and they commit more to the activity. I don't take tests: poor them! They live being evaluated with questions I ask them all the time, with that and the results of the activities is enough. Besides, each one marks him or herself and if I don't agree, I argue with them (even if it is to make it higher or lower).

T15: I make a diagnosis of the situation with questions from each class. I give them activities, in which they can spread all of their creativity to solve problems and use what they have in hand: even the internet, so they feel comfortable. The truth is that the classes become more fun for them and for me because there is always something to talk about. At the beginning of the year, I give them a chart with what they have to accomplish each trimester and they have to complete it with yes, no, a few times, many times. With that chart, it is clear to them if they passed the subject or not: we argue the note, but the important thing is if they passed or not. Those charts say things like: I could finish the task on time; I respected the agreements; I used varied resources… For a while now, since they have a computer at hand, they want to try everything with the computer. I let them, and I must admit that sometimes they surprise me. I don't take written tests. I register what they do on the screen, what they save and share with me or what they show on posters that they hang on the classroom walls.

T20: I don't make diagnoses anymore. Well, yes, I do, but not as I'm asked to. I present each topic with a tale, a reference to mathematics history, and an anecdote. In those dialogues, they expose what they know, what they don't, what they need to learn to refute or check what I'm telling them. We present problems: they have to elaborate solutions, justify what they did, and convince everyone that it was the best procedure. I give them some time to solve it. In the meantime, they can ask me as a "consultant". In that query, I use the opportunity to ask them questions or give them directions that make them study the content, develop it or look for other sources. I have a repertoire of problems for each topic (after all these years, ha!) and as they don't have all the same difficulties, I give them out as they move forward. They all accomplish the minimum and some of them outgrow it. For the one who is left behind, there's extra homework. What's good about this method is that they are always busy and worried about doing the tasks, and I'm satisfied that they learn on the way. Besides, except for disrespect and shouting, anything goes: cell phones, computers, tablets, pencils and paper.

• Group 4: (Pedagogical Content Knowledge [PCK])

T2: I try to make the contents seem "easy", even though sometimes I don't make it. In fact, I look for many examples so that they will understand what it's all about. I really like to refer to reality, that's why I use videos and movies so the content will come out of there. Sometimes, this becomes more complicated and I decide to explain from the board and with questions make sure that they understood.

T5: I usually propose team work so that they can argue among themselves and choose a spokesman to share it with the general group. With the speaker that I chose and changed for each activity; I make sure that I will listen to all of them at some point of the year. When the spokesman gives his or her version, I ask questions and together we reach a conclusion. I propose situations that can catch their attention but sometimes I don't know where to get them from and I see myself teaching as my teachers used to teach me.

T15: They work in groups, pairs, and the whole class. My classrooms are a big "conversation": everyone talks at the same time. I reserve "the use of the final word", with which I close any topic and make sure that they keep a written record of what I want them to learn. They can use whatever they want, I feel safer with the whiteboard and marker.

T20: The classes are not always identical: we work in pairs, in bigger teams, individually. I give the initial story; I situate them and they have to design the solution. They can use whatever they want. For me, it's useful to use slides and the projector with charts and pictures to catch their attention.

• Group 5: (Technological Content Knowledge [TCK])

T2: I use the computer to investigate the internet and get material. In class, what we use the most is the calculator and, in some occasions, the spreadsheet. I want to know how to use software, but I'm not sure and don't even know a lot about how to apply it. The kids decide when and how they use the resources.

T5: My information is on the Web. I get everything from there. Now, I have become friends with WhatsApp so I create groups and we share the material there. I ask the kids to use everything they have at hand. I'm afraid that maybe I won't know how to answer a technical question, so when I propose an activity with a calculator or computer, I always do it with my computing peers to be sure, or I ask one of them to be close.

T15: I propose to the kids to look in websites that I previously searched, information about every topic. Careful: I only search with them through sites that I have searched before, I don't take the risk of maybe bumping into websites that might make me uncomfortable. The calculator is always handy. I'm more acquainted with GeoGebra only on the graphical screen: I still have trouble understanding CAS and I don't use it. Still, it's something.

T20: The kids bring the internet, their cell phones and anything you can think of to the classroom. I don't decide, they bring it. That's why I negotiate their use. On some occasions, I ask them to tell me how they found such an alternative and I question it from mathematics (what I know). In others, I show them what I did and ask them to complete it. I show myself open to the technology and be honest with them because I don't know everything I should, but I know as far as I can (even better, as far as I want).

• Group 6: (Technological Pedagogical Knowledge [TPK])

T2: It's hard for me to anticipate what the kids will do when they use the computer, but not what will happen with a calculator. That doesn't mean that I don't motivate them to look for information about how and what to use it, even though it might just be for that, to motivate them. I don't use it in every class.

T5: Before I take anything to the classroom, I first attempt it and try to solve what I will propose to them with that resource. I make sure that, in that way, I'll swipe at every possibility, even though I surprise myself with alternatives each day.

T15: If we are going to use software because I say so, I make sure before that I've explored it enough so that the question which they might ask won't surprise me. The calculator and the spreadsheet are easier for me, and they are more frequent in my class.

T20: Calculators, spreadsheets and GeoGebra come and go through the tables as if they were in their houses. I always play with those resources before I give the tasks to them and I try to solve them to anticipate possible questions or recommend alternative paths. There are new things in the cell phone that I'm discovering and I'm fascinated about: the photo, something that I don't remember its name is fabulous: it gives you the values, gives the graphic… How can I turn that into a problem? I don't know, I haven't taken it to the classroom yet.

• Group 7: (Technological Pedagogical Content Knowledge [TPACK])

T2: I use technologies for the kids to solve problems and situations that I give them. I don't use them during the tests and evaluations because I use short problems and they don't need them. Before I take the sequences to the class, I think about how I can use them.

T5: I don't use technology to evaluate. It seems more interesting to me to read what the students write as a solution. Yes, I let them use them as a resource if they need to.

T15: When I create the activities, I think before if technology could be used or not. When I introduce them, it takes me a long time to think about how to use them. In the tests, I don't allow them or forbid them: if they need them, they can use them.

T20: I have to think about how to use them in my activities at more frequency every time. Last year, I took an exercise from GeoGebra as a test and evaluated the protocol. This year I'll do it more often: it tells you a lot, the same as the algebraic view. I have discovered that, if I ask them to explain what they are reading, they can tell more than when writing (laughs).

5. Analysis and interpretation of the results

The execution of this preparatory phase of the experiment allows us to affirm that the increase in the use of technologies in time (medium growth of 0.0026525), as it shows in Figure 3, is not the guarantee of quality and depth in the study of the mathematical content.

The answers from the teachers to the group 1 of the questions, enable us to think that the teacher's activity in the school is not conceived as a linear process of transmission of scientific-cultural knowledge organised in academic modules prepared for the sequential learning. The teachers prove through their arguments that they have been trained in different ways to develop an activity that appears as mediational between their students and the mathematical content that they need to acquire. In the idea that a model of permanent training is a theoretical construction that allows us to differentiate several ways of encouraging attitudes and aptitudes in teachers, we can infer that in the four cases described, have gone through different training models: a model based on activities funded in the communication of mathematical information, operated through topical courses of vertical communication (T15; T20); a model that introduces on its design activities that work on the teacher's reflection about their practice, demanding the use of the innovations acquired and their evaluation (T15; 15); a model based on self-training, whether because of individual initiative or because of their participation in work groups (T2). We conclude that, in this group of interviewed teachers, the term profession acquires different meanings: it is a significant that supposes a fixed definition that is placed in the margins of every spatial and/or time dimension. These teacher's training has been circumscribed to the acquisition of contents and skills, which are practical, to the detriment of an intellectual dimension that involves the use of technology as a factor of study.

The answers to the group 2 of questions allow to enunciate some qualities that characterise teaching as a profession.

We refer to:

• Knowing the subject, its structure and relations.

• Knowing the peculiarities of the learning of mathematics, the possible mistakes and difficulties of the process.

• Using the research based on the teacher's performance as an instrument for their own professional development.

This group of qualities characterises the problems of the practice of teaching that the interviewees claim that they try to solve in their classrooms, even though sometimes they are not very clear (T2; T5), necessary to be attended (T20); possible obstacles for their jobs (T15).

The answers to the groups 3 and 4 of questions allow us to think about the act of thinking in mathematics class. In that reflection (and not planning, but imagining what can happen in the class, attending to the domain of several variables that might happen during its development) the answers given by the interviewed group, enable us to suppose that there are teaching practices that favour certain ways of acting from the students during their developments. Considering the typology described by Lupinacci (2017) in this repertoire reflected in the interviewees, the ones of theoretical type are favoured (the students work on references interpreting their processes through analogies; T20, T15); rational (the students seem to work through inferences and require the demonstration of their results; T15, T20); automatic (the students develop an investigative attitude based on the copy and paste of procedures, T5, T15); mechanical (the students privilege the experimentation, T2, T5); and experimenting (the students compare and confront; T5, T15 and T20). In the classrooms of these teachers, the students seem to receive sequences of work driven through problems to solve. The management of the class is described as an instrumental orchestration (Trouche, 2005): a didactical management of the devices disposed to the students related to the didactical goals chased by the teacher's intention. The students seem to go through different stages: performing an operation (calculating, ordering data using the criteria established in the statement, using a form, relating the steps of a construction with its possible graphical results…) and finding the answer that solves the given situation: reading the data given for the situations and relate them (drawing, making a figure starting from properties, creating a model -algebraic/functional/geometrical-, graphic, represent, compare data starting from non-explicit criteria, describing concepts…) to get the information needed to find the answer: analyse and/or elaborate mathematical arguments that allow them to establish the rationality of a result or the possible quantity of solutions that might be given for the situation in the context that was established.

The answers to the groups of questions 5 to 7, show that the interviewed teachers show themselves as participants of a mathematical culture that comes to life in their classrooms: they create symbols and mutations that constitute the skills or the language of their communities; they clarify the concepts and make them more precise; they assume that logic and abstraction are not the only models of thinking that should be favoured in the students; and they allow intuition, analogy and visualisation. The incorporation of technologies integrated the cognitive processes of visualisation (having overcome the difficulties associated to the physiological conditions own of visual perception) and the justifying processes, such as informal or formal (having overcome the inherent difficulties to the apparent lack of sense of a deductive organisation of the speech) articulating perception with deduction. The described classroom shows the appearance of direct practices allowed by the software: dragging, measuring, tracing, hiding/exposing, zooming… and those that the teachers encourage with a reflecting and intentional research: conjecturing, validate, verify, corroborate a conjecture, argue, visualise, systematise information and mathematical relations, justify, explore. It is manifested that the professional challenge of teaching is related to the problems that happen in the classrooms as a way of speech of the knowledge that circulates inside them: the problems must generate a cognitive conflict in the students, independently from the device that the teacher proposes as a mediator.

The results allow us to consider that, before we design the didactical experiment, it is necessary to dialogue, as we did and shared in this article, with the teachers who will participate on it about:

- what we understand as an object: everything that might be indicated, pointed out or referred to when a person does, communicates or apprehends mathematics. In this way we assume the metaphor that consists in moving a characteristic of physical things to mathematics and allows us to fill it with teaching objects -as such, elements that can be analysed during the practice-considering as such to the concepts, properties, representations, arguments…

- the significance of mathematical objects from the action that the students execute relation with said objects through the intervention of their teachers, in particular in classrooms that, as a space of mathematical work, use computing technologies to organise the teaching sequences.

- the idea of the classroom as an institution conceived by students and teachers involved in the same kind of problematic situations: the ones related to learning mathematics through technological resources, that entails to do shared social practices conditioned by the instruments available, the rules and their way of functioning.

- the qualities of a teaching process of a mathematical object for a group of students from the teaching professionals that consciously assume what the mathematical institutions say about the object of study in technological environments.

- the need to analyse the system of practices that occurs in the classroom of mathematics, with the aim of making it worthy for the study of the students and evaluations of their learnings, as well as the improvement of the teaching practices.

- the importance of evidencing the professional role in teaching when the students design situations for their students to learn contents in environments that make the perception dialogue with the deduction and who do not make up with the description, but they invite the resolvers to the analysis and collaborative work.

6. Discussing the results and conclusions

Teaching experiments are conceptualized as case studies, aimed at supporting the learning of groups of students in a particular content domain. The intention is to identify and describe how different groups of students learn over a period of time, adapting curricular objectives to research objectives and using theoretical references to justify the design of tasks (Steffe and Thompson, 2000). In this methodology, the design and systematic analysis of teaching strategies and tools is used in close relation to research, so that they are interdependent. It is based on the premise that the research - which has a certain duration - includes the design phase of usually innovative curricular tasks and experimentation in class contexts. The evaluation of its results is done taking into account the influence of the contexts, identifying the restrictions and conditioning factors. What is designed is a complete learning environment with tasks and resources to sequence and support learning, in order to produce explanatory rather than descriptive descriptions (Reimann, 2011): research is thus inserted in the teaching world, generating a space conducive for collaborative work between teaching and research groups that is what we prioritize in our project.

The first phase of this design requires the analysis of certain conditions prior to the application of the intended sequence. It is the phase that we describe in this article and that meant:

• Identify the objectives of teaching: for this, an interventionist position is adopted from which an attempt is made to problematize the particular problem that is considered from a disciplinary perspective, identifying the ideas that characterize it. This identification was concretized from the application of the survey to teachers and, according to the questions grouped according to the qualities of the teacher's specialized knowledge (Carrillo-Yañez et al., 2018) we complement what Arévaloet al. (2019) consider should be the technological competencies of mathematics teachers and we strengthened the results we shared in Villella et al. (2021).

• Identify the student knowledge that is prior and relevant to the subject to be developed: for this, the relevant previous knowledge is taken into account to build the proposal and the existing teaching conditions to achieve them. In our design, the questions to the students allowed us to identify this knowledge and how it relates to the use of technological resources. We could also complement what Hernández (2020) describes as perceptions about the integration of technologies into mathematics class and its relationship with the ways of teaching as well as student responses, gave us a map of how technological resources are managed in the mathematics classrooms in which their teachers teach. In the same sense, we were able to expand the results shared in Villella et al. (2018).

• Trace a possible learning trajectory: for this, verifiable hypotheses are formulated about the significant changes in the knowledge of the students, foreseeing the way to develop the tasks that are proposed and managing the resources that can be used in the class. Our design highlighted it and enriched it from the inquiry about the resources, the times of use and the contents in which they are applied. In this way, we obtained elements to provide greater clarity and depth to what was stated in the conclusions shared in Villella and Steiman (2021).

The type of instruments we used to structure this first phase of the design of the experiment allowed us to reflect with some teachers on how to select and manage technological resources to develop mathematical content in the classroom, and propose to the students activities that allow them: to interpret what is proposed to them; the information given and establish relationships with the commands provided by the program; formulate and test guesses about the concepts they are learning; design strategies to confirm or refute their guesses; assume the information incorporated;

communicate the results of their findings trying to define what they managed to build, which are the objectives to be achieved in the next phases of the design of the teaching experiment in which we are committed.

The results we arrived in the first phase of a design study, such as the one we shared, allow us to consider that, in a collaborative work such as the one described, components of the teacher's specialized knowledge become evident. The teaching answers to the groups of questions allowed us to disintegrate the teaching knowledge for analytical purposes, and to characterize teaching as a situated activity, in which the implementation of knowledge, processes and teaching skills, provides it a glimpse of a professional performance. These results will be the backbone of the following phases of the work, which we will show in future reports.

In the introduction, we share our research questions: How to improve mathematical learning in real school contexts? What technological resources for teaching can be used for this improvement? How to explain such an improvement? How to substantiate it? In this article, through a survey of teachers and a survey of student attitudes, we reveal what are the elements to modify to achieve the improvement of mathematical learning in the classrooms of secondary schools. These answers will be the necessary inputs to flesh out the second and third phases of the experiment. We did not delve into the technological resources in particular, but we could learn which ones are used in the classroom. We ask ourselves if they will be enough, if they are necessary and thus open the doubts to dialogue in the following phases of the design. The collaborative construction of mathematical work sequences with technological resources for secondary school classrooms, axes of the phases of the teaching experiment not shown in this article, will thus be enriched and professionally challenged.

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Conflicts of interest

The authors declare they have no completing interests.

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