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Research Article

# A Conceptual-Methodological Framework to Investigate the Mathematical Practices with DGS in Secondary Students

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**Abstract:** This contribution presents a conceptual-methodological framework to study mathematical practices from the instrumental aspect inherent to the activity of learning instrumented with Dynamic Geometry Systems (DGS) of a student at a secondary school [In Argentina, secondary education is compulsory training for young people from 12 to 17 years old]. This framework has a conceptualization of the mathematical practice with Dynamic Geometry Systems and a method to favour its operationalization. The framework has been built from diverse theoretical perspectives, in the environment of ergonomic approach of instrumental approximation in a didactical perspective and a qualitative methodology. The method to define the practice indicators that allow to identify it in the field is illustrated with parts of the results of the research in which it originated. One important implication of the framework is that it values, from a didactical perspective, the instrumental character of the activity of using Dynamic Geometry Systems in students that learn geometry, through a concrete definition of mathematical practices with Dynamic Geometry Systems and the operationalization of its analysis for a geometry content. This framework is an original contribution that refers to the knowledge about the existence of mathematical practices with Dynamic Geometry Systems about the content of school geometry.

***Keywords*:** situation of learning of an instrumented activity, mathematical practice with DGS, ergonomic approach, instrumental genesis

## 1. Introduction

The revolution that the Digital Technologies of Information and Communication has caused in the educational systems of many countries since several years ago, has been channeled through projects of implementation of digital technologies on a big scale (Pérez, 2019). These have caused the more and more frequent use of Dynamic Geometry Systems (DGS) for school geometry teaching, a situation that has created an increase in the studies which care about the use of digital technologies in Mathematical Education (Sinclair *et al.*, 2010). According to Drijvers *et al.* (2010) we affirm that in the contexts of mathematical learning that results from the integration of digital technologies, a new ecology of learning emerges and, as such, new mathematical practices are produced due to the potential of recent developments in dynamic technologies. These practices are called new because correspond to one different manner to


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do the mathematical activity, specifically that one is done with executable representations. The mathematical practices have been studied from different theoretical perspectives such as didactical suitability, theory of activity, and social constructive, which are described below.

The perspective of didactical suitability is proposed by the Onto-Semiotic Approach of mathematical cognition and instruction (OSA). This theory assumes the situation-problem as a primitive notion and defines the theoretical concepts object (personal and institutional), meaning and mathematical practice, this last one as "every performance or expression (verbal, graphic, etc.) done by someone to solve mathematical problems, communicate the obtained solution to others, prove it or generalize it to other contexts and problems" (Godino & Batanero, 1994, as cited in Godino *et al.*, 2008, p. 4, free Spanish translation). Some studies about the notions of mathematical practice and systems of mathematical practice from the OSA are Etchegaray *et al.* (2019), Giacomone *et al.* (2016), Godino *et al.* (2009; 2017), Mateus Nieves (2017) and Gutiérrez (2018).

Another perspective is proposed from a historical-cultural approach based on the contributions of the Theory of Activity. Obando (2015) seeks to understand the mathematical activity of the student in the classroom and defines the mathematical practice as the group of actions that the individuals (in their relations with each other and the environment) which, in the course of their activities (about diverse kinds of events or phenomena), guide their objectification and subjectivation processes about quantity and shape (for example, to measure, count, buy, sell, exchange, build, create, estimate, describe, localize, etc.), as the variation of one or another (movement, change, comparison, transformation, etc.) (p. 55, free Spanish translation).

Some studies about the mathematical practice from a historical-cultural approach are Jiménez *et al.* (2017), Marín and Valencia (2018), Obando (2019), Obando *et al.* (2014) and Parra-Zapata *et al.* (2021).

From the social constructive perspective, Bowers *et al.* (1999) conceive the mathematical activity as from an inherently social and cultural nature. The authors, interested in collective mathematical learning of the community of the classroom, assume the mathematical practices of the classroom as one of three aspects of its own micro-culture, that belong to the ways of acting and reasoning mathematically that are taken as shared and get institutionalized, and contain the ways of interpreting and solving specific institutional activities from the students as their individual correlates. Another theory is the Moschkovich (2002, as cited in Uygun, 2016), which explains the mathematical practice by dividing it into two groups: daily mathematical practices that are expressed by the day-to-day experiences of the students which are related to mathematics (such as buying, classifying and organizing), and the second group is academic mathematical practices, which are the ones in which the students handle their responsibilities (such as creating and testing conjectures, creating mathematical arguments and discuss them in the way that mathematicians do).

Other investigation works, in accordance with Drijvers *et al.* (2010) consider new practices that emerge in the new ecology of geometry learning. This new ecology is the result of the integration of Dynamic Geometry Environments (DGE) and it's characterized by the kind of interactions that happen between students, teachers, tasks and technologies. In this sense, it can also be affirmed that they repower themselves or modify other practices: the visualization, as a cognitive process that allows one to get conclusions of a geometrical object from its representation and heuristic exploration, is repowered because it receives a bigger impulse from the dynamic aspect introduced by the DGE; Olive and Makar (2010) established that the introduction of this elements in the classroom makes the preponderance of the demonstrative practice change, welcoming others such as exploring, conjecturing, validating, modelling, deducing and constructing. Besides, more than dragging, maybe the most obvious and newest practice that the DGE made possible, is related to the cognitive aspects of learning geometry as in other practices; Arzarello (2001) affirms that the measure in a DGE is also a mathematical practice of a physical kind, such as dragging, own of dynamic geometry. This aspect together with the dragging modalities (Olivero, 1999), seems to give place to other kinds of mathematical practices such as justifying and arguing.

The referred studies in the previous paragraph do not care about defining the mathematical practice produced when a tool such as DGE is used. That is the reason why we focus on conceptualizing that kind of practice to identify and classify systematically which are the practices that secondary students effectively develop in the resolution of open problems in specific topics of geometry using a DGS. In that context, this contribution presents a conceptual-methodological framework developed and used in an investigation of the mathematical practices that Argentinian secondary school students develop to resolve problems about the congruence of triangles with GeoGebra (Pérez, 2019). The framework defines the mathematical practice with DGS with the idea of valorizing the instrumental aspects which are typical from the activity of using a DGS by a student from the dimension of instrumentalization of instrumental genesis, and the framework operationalizes it through an analytical process that allows to create the reduction, treatment and analysis of data simultaneously from a qualitative perspective.

The notion of the conceptual framework is understood in a constructive sense, as a weave of theoretical relations of different didactical concepts that allow to define a concept of investigation, and the methodological framework is understood as the operationalization of the concept to approach it empirically. In particular, the methodological framework, with a qualitative orientation, is built by a process of six steps that analyses the instrumented activity of learning with a DGS of a secondary student, which progressively takes to identifying mathematical practices with DGS that this student develops. Said process is what we call the operationalization of mathematical practice with DGS. From a didactical perspective, the conceptual-methodological framework values the instrumental character of the activity of using a DGS that the students carry through a concrete definition of the mathematical practices with DGS, and operationalizes its analysis when the content that is studied is a geometrical concept. From an investigative perspective, it offers resources for wider theoretical arguments about: mathematical practices with DGS that secondary students develop when they use a DGE in their learning activities; the potential of digital tools like DGE to produce said practices; and new teaching methodologies for geometry with the use of DGE as a resource.

In the first section, an original conceptualization of the mathematical practice with DGS is presented and in the next its operationalization from the methodological point of view. In this, the analytical process is illustrated with parts of the results of the research by Pérez (2019), from which it originated. In the final part of this contribution, we will provide some conclusions regarding recommendations and possible implications of the conceptual-methodological framework.

## 2. Conceptualization of mathematical practice with DGS

The conceptual-methodological framework works with an instrumental perspective (Pérez, 2014) and it's situated in the Ergonomic Approach of Instrumental Approximation (Monaghan, 2007). We define the activity of the use of DGS that the students do in the classroom as a learning situation of an instrumented activity (Pérez, 2019), which is an adaptation to the SAI model of Rabardel (1995). In this sense, when the student, cognitive subject, uses de DGS as a device to develop a task in the classroom, the relationship student-DGS is established through an instrumental genesis [According to Drijvers *et al.* (2013) "(…) nontrivial and time-consuming process of an artefact becoming part of an instrument in the hands of a user is called instrumental genesis" (p. 26)] and two types of issues are implied. On the one hand, actions from the student about the DGS as processes that correspond to the instrumentalization dimension of the genesis, which involve the processes that go from the subject to the device and allows to recognize mathematical practices that underlie the activity of use (an example is the manipulation of geometric objects that the student does through DGS skills according to his or her reasoning). On the other hand, the actions that imply conditionings for the student's performance are derived from software that corresponds to the instrumentation dimension of the genesis. In that order of ideas, there are two concepts that are key to tracking the practice: the second activities are "relative to the management of characteristics and properties own by the artefact" (Rabardel, 1995, p. 171, free Spanish translation) and to the dimension of the activity that determines the schemes of use, and the first activities that "are oriented towards the object of the activity, and for the ones which the artefact is a realization medium" (Rabardel, 1995, p. 171, free Spanish translation) and it's the dimension of the activity that determines the schemes of the instrumented action. In that way, a geometry content would be addressed conceptually as a mathematical content for the learning situation of instrumented activity. Figure 1 synthesizes the conceptual framework we will present.

It could be affirmed that there are two lines of work (Pérez, 2019) in Mathematical Education in which the proposals that study or relate mathematical practices with the use of devices are placed. A line that investigates and develops aspects of what learning implies, doing and using mathematics, and seeks to develop ways to help all of the students to learn the mathematical practices (Ball, 2002), and another line that is based on a closer relationship between mathematical knowledge and mathematical practice, encouraged by the use of technologies at schools (Olive & Makar, 2010; Arzarello, 2001; Olivero, 1999). We place our conceptualization of the mathematical practice in this last line of work.

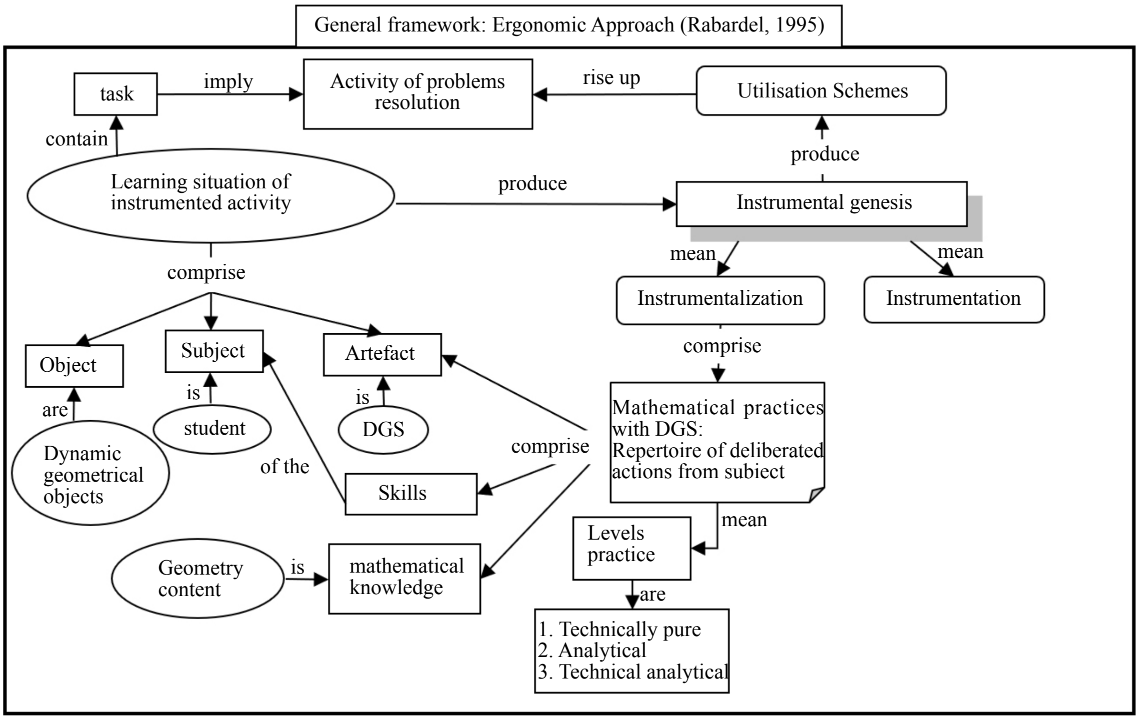



**Figure 1.** Conceptual framework to investigate mathematical practices with DGS

As said by Ball (2002), we allude to the mathematical practice in relation to the actions of a student in the framework of the realization of an activity in which mathematics is used. In this way we are talking about an instrumented activity, that is, the activity developed by the student is mediated by technological devices. These are related directly to the mathematical knowledge implied in the activity's development, which is framed in the different areas of the curriculum of school mathematics, and with the mathematical reasoning that emerges in this knowledge. The relation established is a come-and-go, which means, on the one hand, the device brings a particular mathematical knowledge to be used and to determine the mode in which it is constructed, on the other hand, the mathematical knowledge and reasoning that influence the use and design of the device. As a consequence, mathematical practice is presented and developed in the actions of a student that does an activity with artefacts in which mathematics needs to be used. In this way, we assume three elements involved in the learning situation of instrumented activity that constitute the mathematical practice: instrumented subject, used device and involved mathematical knowledge. In our case mathematical practice with DGS, and we conceive it as the one that is developed in the specific context of the use of dynamic geometry, in relation to what the students do during the development of activities in the classroom that are related to the resolution to geometry problems.

Since the mathematical activity developed by the student is doing geometry in a DGE, the practice can be understood as a synonym of the actions or the performance of the student that faces a geometrical problem using a DGS. As such, the practice has a pragmatic dimension that recognizes that the student is a cognitive subject and his or her activity is not neutral nor mechanical, and for that, the intention behind the action is a relevant element in the practice. In this sense, we agree with Arzarello *et al.* (2002) that the practice is more than actions and we admit as one of its constitutive elements to the cognitive aspects of the student's performance, which is, the cognitive dimension of the mathematical practice with DGS. As a consequence, the practice is a concept that involves more elements that exceed the actions of the subject with the software themselves, as has been argued and documented by authors like Arzarello (2002) and Moreno-Armella and Santos-Trigo (2008). We consider practices not only to those actions of the student that have a counterpart of a technical character, this is, that are visible because they carry an instrumented action (such as dragging and measuring), but to other kinds of actions that are not visible like this (such as conjecturing and arguing), which are cognitive actions.

We propose a categorization of mathematical practice with DGS into two types, according to the two dimensions that it has, pragmatic and cognitive:

Those that the student develops properly with the technological environment of the DGS, as such, proved through an instrumented action, we call them instrumented.

Those that emerge in the instrumented activity of the student, but are developed outside digital technology, without a visible counterpart through an instrumented action, we call the analytical.

In this way and according to Santos-Trigo and Moreno-Armella (2006) and Moreno-Armella and Santos-Trigo (2008), we call mathematical practice with DGS (Pérez, 2019): the repertoire of deliberate actions that a student develops in the framework of his or her instrumented activity of learning, to solve a task with the use of DGS as a technology of dynamic type and dynamic geometry as a particular system of knowledge. The practices, recurrent actions directed by the intentions of the student to solve the task, consist of three components that are inherent to the instrumented activity of the student:

a) Artefact that corresponds to the DGS,

b) Geometrical knowledge [It is about the knowledge used by the one who is doing the practice], and

c) Abilities, ways in which the student uses his or her visual, manual and cognitive capacities to apply the knowledge that is put into play in problem-solving.

According to the intentionality of the student in the moment of doing the mathematical practice with DGS and the presence or absence of each of its three components, we distinguish in it three levels and inside each level different types, that determine the indicators that will allow to recognize in the instrumented activity of a student, when it's happening one or other practice of each level. The levels and types of practices are described through a generic characterization for geometry, that considers the basic tools for geometry without coordinates and doesn't include the use of sophisticated tools such as sliders.

**Level 1** Technically pure practice only relates to the artefact component and doesn't include any kind of reflection about the action. Table 1 describes its types.

**Table 1.** Types of practice of level 1

| Type of practice | Action |
|---|---|
| Drag | Activates the tool *Move* and with the cursor moves a geometrical object of the graphical area changing its position there. |
| Measure | Activates the tool *Distance or Length*, clicking a segment or polygon, or consecutively in two different points. |
| | Activates the tool *Angle*, clicking inside a polygon, or consecutively in three points that determine an angle whose vertex is the second of them, or in two segments or concurrent lines. |
| | Activates the option *Show Label*, in the "Basic" tab of the Dialogue of Settings Box, with the categories *Name & Value* or *Value*, for a segment, polygon or angle. |
| Activate trace | Activates the option *Show trace* of the contextual menu or *Show trace* in the tag "Basic" of the Dialogue of Settings Box, for a point, a segment or a line, and drags the object directly in the graphic area or modifies the construction in a way in which it implies a displacement of said object with an activated trace, and its path appears traced. |
| Hide/Show | For a geometrical object visible in the graphical area, from its contextual menu can deactivate the option *Show Object*, or from the Dialogue of Settings Box activates or deactivates the option *Show Object* from the verifying box that corresponds to the tag "Basic" or the circular icon that appears next to the name of the object in the list on the left, making the geometrical object show or hide in the graphical area, respectively. |
| Tracing | Uses an available tool (in the toolbar) to create in the graphical area a geometrical object with particular properties, such as point, segment, line, ray, circle, midpoint or center, parallel or perpendicular line, perpendicular bisector, angle bisector, regular polygon, circle according to the radio, segment according to the longitude and angle according to the amplitude. In the three last cases, the longitude that is used as given information does not correspond to an existing object in the construction. |
| Zoom | Zooms in or out the graphical area from its center, through the use of the tools *Zoom In* and *Zoom Out*, the touchpad or the scroll wheel of the mouse. |
| Transfer measures | Uses the tools *Circle: Centre & Radius, Compasses, Segment with Given Length or Angle with Given Size* to transfer, in the first three cases, the distance between two points or the longitude of a segment, and in the last case, the amplitude of an angle, always of existing objects in the construction. |

**Level 2** Analytical practice only relates to the component of geometrical knowledge, there is a glimpse of reflection and its use has a doubtful intentionality. Table 2 describes its types.

**Table 2.** Types of the practice of level 2

| Type | Action |
|---|---|
| Conjecture | Form enunciates of a conditional character, a hypothesis of work or a supposition, about a particular fact of a geometrical situation, based on observation or the analysis of hints or empirical evidence, done through an exploration that gives a high grade of certainty about what is being affirmed. |
| Arguing [We assume that an argument is created by one or more arguments expressed informally and coherently connected, but not necessarily in a deductive way.] | Using reasons or points of view [We admit as reasons or points of view to the verbal, visual, numbered or any kind of manifestations.] to support or reject a statement with the goal of finding ideas that establish its grade of certainty. As it is a communicative act, we need to consider the characteristics of the social group in which it is expressed, regarding the acceptance that makes of information such as data, guarantees, the way in which arguments are articulated in the argumentation (for example the use of analogies, schemes of logical reasoning, similarities or contrasts), and the ways in which the arguments are expressed. |
| Visualizing | Get geometrical information of a figure through the visualization (Duval, 1998) of a dynamic construction, identifying the elements that create it and some configurations that might be created with them, with the aim of finding underlying geometric relations. |
| Systematizing information | Effectuate a process of register of information that comes from the problem in the resolution process, through statements given as truth referred to figurative and conceptual aspects of the construction used. |
| Justify | Elaborate an argumentation (of a deductive character), that supports as true a conjecture created inside the knowledge system of dynamic geometry, through a process that consists of chaining arguments in a way that a proposition concluded in a specific argument can be given as information for another one. As a product of said process, you get the validation explanation [It's a justification whose guarantees come from non-theoretical sources (for example empirical, of authority, rituals, or personal conviction) (Camargo, 2010)]. |
| Explore | Carry an activity of an investigative character in the world of theory and statements that create individual knowledge, looking for statements that allow to justify an affirmation or making decisions about where to direct the project (empirical exploration) of the resolution, based on regularities (properties or geometrical relations) that might be generalized or of properties that had not been identified yet. |

**Level 3** Technical-analytic practice relates to the three components, which is why they are constituted through a combination of types of practice of level 1 and 2 that develop themselves together. Some examples can be Drag-Conjecture, Measure-Visualising and Zoom-Explore.

## 3. Operationalization of mathematical practice with DGS

The sight will be particularly placed in the student's experience in relation to the use that he or she does of the DGS for the resolution of problems about a specific school geometry topic, which demands methodology to interpret, comprehend and deepen in the subject's actions and what they imply such as decisions, intentions and reasoning that back them up. It's that way that we would reach the goal of identifying, describing and classifying mathematical practices (of any level) developed in the resolution activity of geometrical problems with DGS in the classroom.

### 3.1 *Instruments of data collection of information*

Its design answers two key questions: how is the concept of mathematical practice with DGS operationalized according to the defined dimensions? And, how to infer the intentionality of practice through inquiry? These questions show two factors to consider. The first one is that it is core to make pertinent questions to the student to infer from their answers elements that allow to identify the components of the practices that they developed, in particular, the geometrical knowledge can be observed *in situ* through the retelling that the students do about their resolution processes.

The second factor is that we must count on the analysis *a priori* of the resolution of the problems designed and the characterization of the answers from the perspective of the resolver, to get elements to contrast with the proposals for the students.

Mathematical practice with DGS consists of the deliberated actions from the student that are directed by their intentions to solve the task, the components of the practice are the geometrical knowledge and the student's skills. For these reasons, it is necessary to track the instrumented activity of the students during their development through different means and registers, and also afterwards to regain the intentionality behind the actions done. For this, the information data collection instruments considered in the methodological framework are of different types, used in two different moments of the fieldwork, during and after the work in the classroom, and they complement each other.

To keep a detailed record of the procedure of the students for the execution of their actions in the development of their resolution processes in their instrumented activity is a priority, it is the key resource to identify mathematical practices. For this reason we use: screen video recording of the students that get their activities with the software, with which we can identify the practices of level 1 and find elements for the other two types, that might be complemented with the collected information through other kind of registers; audio recordings in the moment in which students work individually in their computers, that collect the student's speech during their instrumented activity of resolution with a better audio quality *in situ*, and it turns into a supportive register for the video recordings; the narratives that consist in the oral formulation in the first person from the students, about how was, according to them, the development of their activities of resolution of the problems presented in the activities, and as such they collect the explanation *in situ* of the student's experience with the software in the activity's resolution, with which we pretend to know among other things, the path they followed, the tests they did, the aspects they considered, the decisions taken, their comprehension about what the activity requested, and how they performed the actions for the solution.

As intentionality is one of the main dimensions of mathematical practice with DGS that can be recognized through attitudes or DGE expressions in the student, non-participant observation from the investigator during the problem's resolution, registered in a field diary, turns into a key element because it allows to perceive the characteristics from the classroom context in which the practices are produced. In this way, the researcher can capture details about the behaviour of the students that are not reflected entirely in the video or the audio, their ways of performing with the software and their declarations about the actions that they performed for the resolution of the task (narratives). It's unreplaceable information that becomes a primary source of information, complemented by audio and video recordings.

When we use GeoGebra as a DGS for the resolution of problems, as a product of the student's activity we obtain the files that GeoGebra produces and saves, that contain the construction protocols that result from it. These products also provide necessary information for the research because they allow to track the way of working from the students with the DGS, as it has the evidence of what was done with the software to obtain the performed construction.

The explanation interview (Vermersch, 2010) after the work in the classroom, as a dialogue between the investigator and the student, supported by the resource of the video of its instrumented activity, is useful because it allows inquiry into the intentionality of the actions and decisions taken during the resolution of the problems, and searching for elements that help to find those aspects seen in the student's activity in relation with the analytical practices that might not be clear.

The elaboration of the questions of the protocol of the explanation interview uses mainly the information collected in the video and audio recordings, specifically in the parts of the video in which the existence of indicators of a mathematical practice was identified, and if it is necessary can be complemented with the audio, turning into observation units that will help to create some questions. As a support for the elaboration of questions, we use constructions and construction protocols that result from the files created by the students in the software, and the narratives that allow us to glimpse the general reasoning followed during the resolution and extract elements that might allow to define in a more precise way the practices of level 2 and 3, which are some of the complementary registers to the video with some kind of reflective character of why and what for each action was done.

The questions of the interview search to clarify those aspects or matters that are not very clear in the instrumented activity performed, inquiry on the reasons and decisions taken, ask why they used certain tools and how were they employed, as well as what took the student to do that procedure, unraveling the reasoning and intentionality to discern over the practices of level 2 and 3. The interviews would be done after the implementation of the task, in a place outside the classroom and showing the fragments of the selected videos as observation units of the interview's protocol. During the interview it is convenient that each student has the device and the files they used for the resolution of the task, to have the possibility of showing something of their work if they want. The interview is recorded in audio and video, of the screen in which the videos are shown and the students' screen.

The video and audio recordings, the non-participant observation and the explanation interview (Vermersch, 2010) are used as the main techniques of information data collection. For each participant student two (2) information data collection devices will be used in the classroom, a video camera focused on his or her screen and an audio recorder placed on their working table.

Figure 2 shows the complementarity of the data collection instruments through the direction of the arrows, the information obtained from the instrument that the arrow points at is complemented by the one that is in its origin, when the arrow has a double direction, it is assumed that there is a mutual complementarity.

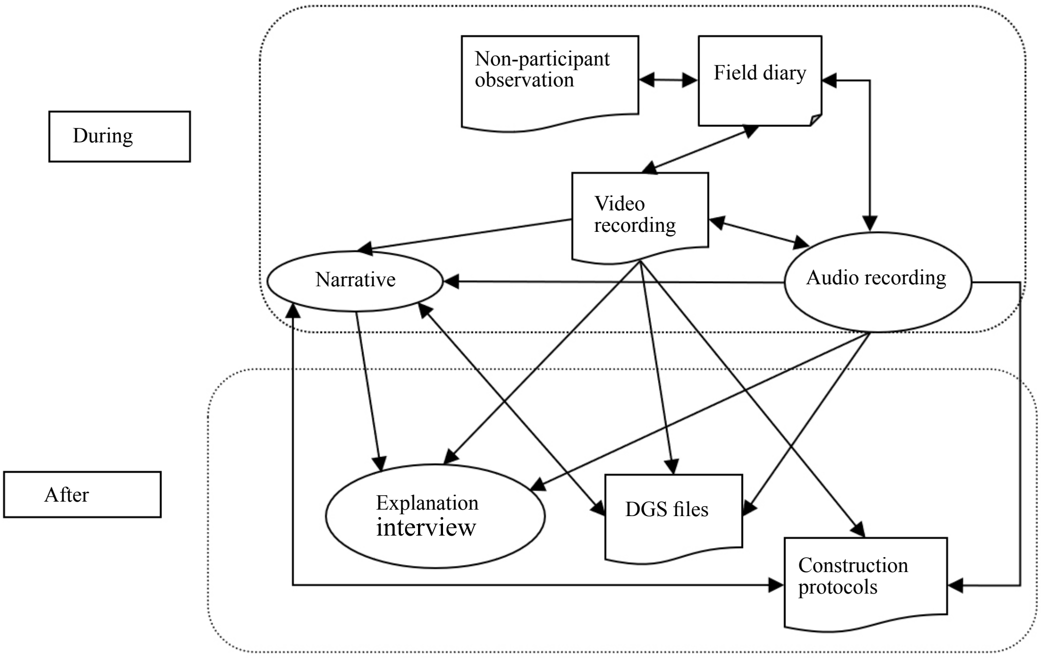



**Figure 2.** Scheme of triangulation and complementarity of investigation sources

## 3.2 *Analysis of the instrumented activity*

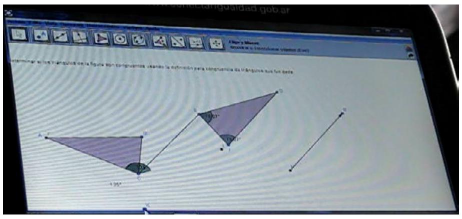
The process of construction of the main body of data of our methodological framework allows us to simultaneously do the reduction and depuration of the collected information and the analysis of data for each student. The process consists in 6 steps that progressively lead to the identification of mathematical practices with DGS, through the determination of their constitutive elements for the instrumented activity of the student's learning. To illustrate with examples some steps of the analysis process we will use fragments of the report of the research (Pérez, 2019) in which it was developed:

1) Identifying the instrumented actions through the second activities done by the student through the video recording. We register how in Table 3, in the order that it happens, detailing the elapsed time and the tool used, and identifying them as moments of the development of the instrumented activity of resolution of problems for the student. Some ranks of the table are left blank and are not designated as a moment, because they belong to times when the student did not do any instrumented activity but that they are worth recording because they suggest some indication of student reflection. The table also transcribes the dialogues of the instrumented activity (some of them are narratives) and it is useful to include screenshots of the construction during them.

**Table 3.** Extract from the table of identification of the second activities of Romina for video 1 of day 2

Video 1-Day 2

The student has two tabs of GeoGebra open on the same file of the construction, uses one first, which she closes soon, and then continues her work on the other. This is the screenshot with which this class's session starts.



| M | Time | Tool | Instrumented Action |
|---|---|---|---|
| 1 | 00:02 a 00:23 | | Eliminates the *K* point from the option erases of the dropdown menu.<br>Eliminates point $B'$ from the option erase of the dropdown menu (eliminating as a consequence the $\Delta DEF$, the $\overline{CE}$, point *I*, point *H*, the *HG* vector, and the measure of the $\angle ICA$. |
| 2 | 00:34 a 1:05 | Compasses<br>Displace graphical views<br>Parallel line<br>New Point | Builds a $\odot_{F,vector\,HG}$<br>Drags the plane in a diagonal up the superior part of the graphical zone.<br>Traces the parallel line to the $\overline{EC}$ that goes through point *F*.<br>Determines the point *K* in $(\odot_F \cap l) \in S_{\overleftrightarrow{EF},\neg D}$, being $l \parallel \overline{EC}$. |

2) Describing the instrumented activity of the student from the video and audio recordings. It detailed how the student performed each of the instrumented actions referred to in the moments of the identification table of the second activity, as well as what happened at the time of the blank rows, in the ones that correspond to the type of practice marked. Besides, we include screenshots of the construction and the dialogues of the instrumented activity are transcribed. Figure 3 shows this step.

/ (2:21) *Then she did like a turn with the cursor and took it to point E, moved it over the* $\overline{EF}$ *up to F, and did a turning move similar to the previous one and took the cursor to point C, then to point E*. (2:35) (situation 2 of the visualization practice)/(execution of what was decided in the visualization practice). Moved the cursor randomly over the screen and took it to the line creation button, down-dropped the menu and activated the tool (2:45) Vector between two points (situation 1 of practice of level 3 conjecture-measure-conjecture), and right away she moved it to the graphical area, in $S_{\overleftrightarrow{FD},\neg E}$, she clicked obtaining point *G* and a vector in its origin, took the cursor towards the inferior part of the screen and then clicked obtaining point *H*, final point of the vector. With the cursor "go across" the recently defined vector and then takes it to the Undo button and clicks once, making the recently obtained vector disappear. [The tool Vector between two points is still active] Took the cursor to point *G* and clicked, zoomed in two times, then took the cursor to point *H* and clicked, zoomed out and left the figures in a smaller size than the one she was using, finally obtaining vector *GH*. She moved the cursor randomly and then took it to point *F*, clicked and took it to *C*, left it for a moment and then took it to *B*, and then after a moment she clicked, obtaining vector *FB*. /

**Figure 3.** Fragment of the description of the instrumented activity of Romina for the video 1 of day 1

3) Transcribe the interview detailing: clarifications about the type of situations that happened in the same interview; explicit a geometrical object pointed or referred to by the student or the investigator that is not mentioned in their speech; include the gestures done with their hands or arms that might be registered in the video, because they complement what the student wants to communicate with their speech; describing the instrumented activity that the student develops during the interview, if it is the case; cuts during a phrase development; images of a screenshot of the respective video when the idea is to show the description performed.

4) Determine the practices of level 1. Each one of the moments in the identification table of the second activities (step 1), corresponds to a situation susceptible [The adjective susceptible is understood in the sense of the recurrence of the actions that conform to the practices, that according to the definition, as long as the recurrence of a determined type

is not verified, we don't assure that this type of practice was developed.] of being a mathematical practice of level 1, are called instrumented actions situations. Each one is associated with the corresponding type of practice and determined as mathematical practices with DGS of level 1 to those that verify the recurrence of actions in their use. The situations of instrumented actions that contain elements of cognitive nature, are called situations of instrumented and cognitive actions and they are considered as susceptible to being a mathematical practice of level 3, their analysis is done in step 6. Table 4 shows this step.

**Table 4.** Extract of the table used in the determination of the practices of level 1 of Guillermo

| INFORMATION | | | | Practice |
|---|---|---|---|---|
| M | V | D | Instrumented Situation | |
| 12 | 2 | 1 | Uses the tool Compasses to generate a circle and drag it over two points of the construction with the intention of doing the translation of the $\Delta B\,A'\,C$ | Drag |
| 31 | 1 | 2 | Uses the tool Move to drag a vector created as a free object, over a segment created by him, with the intention of making the vector parallel to the segment and its magnitude being the longitude of the segment. | |
| 47 | 1 | 2 | Uses the tool Distance or Length to measure the distance from $B'$ ($A'$) a $C'$ with the intention of measuring the $\angle B'\,(A')\,C'\,A''$ | Measure |
| 48 | 1 | 2 | Uses the tool Distance or Length to measure the longitude of the segment $\overline{C\acute{A}''}$ with the intention of measuring the $\angle B'\,(A')\,C'\,A''$ | |

5) Characterize the first activities based on what was obtained in points 1, 2 and 3. The aforementioned actions are conformed by groups of instrumented actions depending on whether they suggest any step in the resolution process of the problem, and in its register we distinguish the moments of the instrumented actions related, explaining what the student did and the result obtained or the aim he had. The list to which the next fragment belongs to contains 24 items and it is divided between the two classes in which the implementation was performed. Figure 4 shows this step.

VIDEO 3

14-In moments 4 and 5 of the video 3 of the class of day 2, the student tried to find the translation vector for the $\Delta A'\,B'\,C'$ to reach the $\Delta D\,E\,F$, as it is expressed in the interview: "With that new vector I could stand in a point and the distance that was there, the distance between, the distance between that vector translate it to an external way to the figure in which I could allow to find the next one, and continue finding the next figure $E\,D\,F$". To do so, she traced the vector $B'\,E$ and measured its longitude.

15-In moment 8 apparently the student continues with the idea of doing the translation of the $\Delta A'\,B'\,C'$ because she traces a circle of radio with the magnitude of vector $B'\,E$ and centre in the point $B'$.

16-In moments 10 and 11 the student tried to find the points of the transformed figure of the $\Delta A'\,B'\,C'$, by generating and dragging circles with different radio, as she says in the interview: "we had to have, there was a point, that in this case it was point $B$, that I had to stand on that point and find the points of the other figure".

17-In moment 15, the student kept on looking for the points of the transformed figure of the $\Delta A'\,B'\,C'$ through translation. She assures it in the interview saying: "there we kept on with the plan of the parallels and the vector", so she traced a parallel line to the $\overline{B'C'}$ that goes through $B'$ and a parallel line to the $\overline{A'B'}$ that goes through $C'$.

**Figure 4.** Fragment of the list of first activities from Gaby

6) Determine the practices of levels 2 and 3. The blank ranks of the identification table for the second activities (step 1), constitute the fundamental input for the determination of the practices of level 2. Said ranks correspond to cognitive situations which are classified into two types according to the presence of a low level of the technical component: one type is the situations in which there is an instrumented action which is light but that the cognitive component has a predominant presence, which is considered as instrumented and cognitive actions situations and its analysis is postponed to the practices of level 3; and a second type that corresponds to other situations in which the isn't any instrumented

actions, so they are assumed as situations of cognitive actions. From this we chose those who are susceptible to being a mathematical practice of type 2 when they accomplish that their realization had the aim of solving a problem, without any kind of instrumented action, it has a glimpse of reflection and its use has a doubtful intentionality. Figure 5 shows this part.

| |
|---|
| In the situation between the minute 17:52 and 17:55 of video 1 of the class of day 2, after tracing a circle of radio $A'B$ with a centre in $A'$, Guillermo took the cursor to the intersection of the parallel line to the vector $BA'$ that goes through $B$, and the circle of centre in $A'$, and immediately to the button of the tool New Point. This situation allowed him to determine which was the image of point $A'$ according to the translation that he wanted to apply to the $\Delta BA'C$ according to the $BA'$ vector. This situation corresponds to a visualization practice because in it we identify the intersection that was mentioned and the configuration that might be created with it, determining the point on it as an image from $A'$. This situation is reported in the description of the instrumented activity in the part identified as situation 5 of the visualization practice. |

**Figure 5.** Fragment of the description of a cognitive action situation from Guillermo to the type of practice visualization

The situations of instrumented and cognitive actions identified are analyzed to characterize the practices from level 3 in three steps: 1) we verify the effective existence of instrumented and cognitive actions and we determine what is the type of each of them and how they are associated; 2) based on said association we determine the type of practice of level 3 it might be about, obtaining a list of instrumented and cognitive situations susceptible of being practices from level 3; and 3) we evaluate the attribute of the recurrence on each of the situations from the list obtained to determine which ones are mathematical practices with DGS and in that way create the repertoire of level 3 practices that the student uses. Figure 6 shows the description of a situation of this type.

| |
|---|
| In the situation between minutes 8:22 and 8:46 of the video of the class of day 1, which is reported in moment 7 of the list of second activities, the student measures three angles, each one corresponding to an intern angle of each one of the three triangles that appear in the construction in that moment of the solution process. The presence of this instrumented action of measuring would suggest that it is about a situation that corresponds to a practice of level 1 which is to measure because she uses the tool Angle with the aim of measuring some angles. Nevertheless, the way in which that situation was developed reveals a cognitive action that is simultaneously present to the action of measuring. This cognitive action is conjecturing, because the student, after obtaining the measure of the $\angle HGF$, left the cursor over point $F$ for a moment and says "It has to be, this one ($\angle$) right?" and at the same time she points at point $A$. This affirmation of the student might be formulated as the measure of the $\angle HGF$ must be the same that the $\angle CAB$, corresponds to a supposition about the geometrical fact of having obtained the $\Delta HGF$ through the trace of the parallel lines that go through point $F$ and point $B$, which would be the empirical hint that she led during the construction that she did herself, which is what gives her certainty. A fact that confirms that she assumed said position to continue her work is that after doing the instrumented action of measuring the $\angle CAB$ she manifests to be confused, as it is reported in the description of the instrumented activity because she realizes it was not true. That same process of the cognitive action of conjecturing is done for the $\angle EDF$ of the $\Delta EDF$ but in an implicit way because it is not verbally explained that she supposes that the measure of $\angle HGF$ must be the same as in $\angle EDF$; and then, through the action of measuring with the tool angle, she has a confusing action when she obtains the measure of the $\angle DEF$ and realizes that it is different from the $\angle HGF$. For this, the presence of the instrumented action of measuring angles simultaneously with the cognitive action of conjecturing in the considerate situation, leads to conclude that it is about a situation that corresponds to a mathematical practice of level 3. The intentionality of the student corresponds to his aim to prove if the triangle obtained by the tracing of parallel lines that he's been doing is congruent with the triangles given by the task; the instrumented action that he did with the aim of his intentionality if to measure angles $HGF$, $CAB$ $y$ $EDF$; and the cognitive action involved would be conjecturing. We report this situation in the description of the instrumented activity as situation 1 of the practice of level 3 conjecturing-measuring. |

**Figure 6.** Fragment of the description of a situation of instrumented and cognitive actions situation of Gaby for level 3 of the practice

## 4. Conclusions

One of the core points of the conceptual-methodological framework presented is the originality it has as a tool for conceptualization of the mathematical practices that are instrumented with a DGS, with which we have accomplished two important results in a conceptual level. The first one has to do with being able to establish a definition of the concept of mathematical practice of an instrumented learning activity with an artefact of digital technology, in this case, the DGS. The second result is having established the definition of mathematical practice with DGS from a didactical perspective. This allows us to think that we complemented the vision of the mathematical practice of Arzarello (2001) and Olivero (1999), conceiving, also from an instrumental approach from the framework of instrumental genesis, a wider vision of the same that distinguishes its two dimensions, cognitive and technical.

The results of a study that uses the conceptual-methodological framework presented will refer to the participant students and the particular DGS chosen. The results cannot be considered as representative of all the students of the same school grade of secondary. In relation to the software, the repertoire of mathematical practices with DGS that can be found is relative to the experience of the students with a particular DGS. Because of the particularity of the design of the different DGS software that exists, said results are not generalizable to the experience of a DGE of a DGS type. Other tools of dynamic geometry as Cabri-Geometre and *R y C* could be used in a similar way to GeoGebra because those let free exploration, and others as Sketchpad, Cinderella and GEUP could demand another perspective about mathematical practice because work from sketches.

In the methodological framework, the establishment of moments of the fieldwork regarding the work in the classroom and the different instruments designed for each one, allows one to approach the object of study from different perspectives in order to make the collected information complementary, getting a wider vision. The created process for the constitution of the main body of data allows to simultaneously do the reduction and depuration of the information and data analysis, in a systematic, rigorous and deep way. Besides, it allows one to discover the practice progressively through its indicators: second activities, first activities and solution processes.

In the geometry teaching and learning through a DGE field, many research works (Artigue, 1997; 2002; Lagrange, 2000; 2005; Trouche, 2000; 2004; Guin, *et al.*, 2004) go in line with the idea of instrumental genesis and its relation with the construction of knowledge (Hollebrands *et al.*, 2007). In this line, through the conceptual-methodological framework presented, we devoted to studying the mathematical practices from the process of instrumental genesis (Rabardel, 1995). In that way, we consider that the theoretical and methodological contributions of this work to the research of geometry didactic seem important to consider for wider theoretical arguments about mathematical practices with DGS that students from secondary level develop when they use a DGE in their learning activity. Also, in the argument about the potential of the digital tools like the DGE to produce said practices and about new methodologies of geometry teaching with the use of DGE as a resource. Particularly in relation to the mathematical practices with DGS, this work provides elements to identify the type of mathematical activity that the student develops, through the recognizing of the different instrumented and cognitive actions that they might develop in the use of this device. Some publications that used the conceptual-methodological framework presented are Pérez (2023, 2019 November, 2015).

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## Conflict of interest

The author declares that there is no conflict of interest.

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