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Research Article

Constitutive Elements of Professional Development Programs for Primary School Mathematics Teachers: Evidence of Proposals from Multiple Countries

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Abstract: A comparative study is presented, developed with the purpose of identify and characterize proposals for the professional development of mathematics teachers in primary schools around the world from situated approaches, which involve collaborative learning amongst teachers that results in the improvement of their professional practice. The countries whose proposals were analysed are Argentina, Australia, Brazil, Canada, Colombia, Cuba, Spain, France, England, Japan, Mexico, and the Dominican Republic. The conclusions identify and characterize some privileged conditions for designing training processes that contribute to the professional development of mathematics teachers: they must be creative, situated, proactive, collective, based in evidence, and balanced. The professional development of mathematics teachers requires the articulation between two components: an institutional training component and a specific training component subject to a map of vacancies and priorities based on evidence taken from national and international evaluations.

***Keywords*:** teacher professional development, continuous teacher training, specialized knowledge of the mathematics teacher, primary school, comparative education

1. Introduction

The present study was developed within the framework of the international program "Teacher Empowerment Initiative: Large-Scale Teacher Professional Development (TPD)", implemented by the TPD@Scale Coalition for the Global South. The coalition is a collaborative initiative of ministries of education and other government agencies; international organisations; development agencies; non-governmental organisations; universities, education centres, and training institutions; research and policy centres; private companies; and other actors in the educational and technological sectors involved in teacher professional development. The program aims to improve teaching practices in the Global South by advancing equity, quality, and efficiency in TPD systems. To achieve this general objective, ten research studies were conducted in: Argentina, India, Mexico, Rwanda, Sierra Leone, Tanzania, Vietnam, Zambia, Lebanon, and Pakistan. In the Argentine case [The research in Argentina is conducted by the Latin American Faculty of Social Sciences (FLACSO)], the interest lies in studying and developing models of professional development for teachers in initial literacy in Argentina. This implied the need to establish an updated state of the art of professional

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development for teachers in reading, writing, and mathematics. In the following lines, we present the results obtained for the case of the state of the art of professional development proposals for teachers in mathematics.

The research on the development of the mathematics teaching profession is varied and addresses multiple challenges: entry and teacher training, curricula and programs for teacher training, continuous training structures and professional paths according to the contexts and schools, teacher evaluation, and transfer of results in teacher training (Rosli & Aliwee, 2021; García et al., 2010). Conceptions of these topics have varied along with the modification of the nature and role of mathematical knowledge in society, transitioning from a relationship of dominance-typical of the twentieth century-to one of management and competent use of this knowledge-in the twenty-first century (Villella et al., 2021).

Over time, continuous teacher training has received different names: improvement, permanent training, updating, recycling, in-service training, professional development (Villella et al., 2021; Vezub, 2019; 2010). The choice of each of these terms refers to traits of identity and professional profile, of the conception of teaching work, of the type of knowledge, of the skills that teachers are expected to perform in the educational system and in society. This study is framed in the conception of teacher professional development, as it understands that teacher learning is expanded, articulated and integrated throughout its trajectory, encompassing both initial and continuous training (Ávalos, 2011; FLACSO, 2019). The diversity of meanings in relation to continuous training highlights the need to characterize actions that focus on teachers and that have as their ultimate purpose to achieve improvements in both teaching and learning for their students. In the case of this study, we present a review of continuous training offerings for primary school mathematics teachers. This study is part of the program "Empowering Teachers Initiative: Teacher Professional Development (TPD) at Scale" which aims to improve teacher practices in the Global South by advancing the equity, quality, and efficiency of TPD systems. Scaling quality TPD to facilitate active participation for all teachers is a complex undertaking. TPD systems are multi-layered, comprising numerous components and actors within a broader educational ecosystem. In the Global South, specific groups of teachers face multiple challenges, with limited quality TPD opportunities that may not cater to their specific contexts. Overcoming these challenges requires a holistic approach to scaling that goes beyond mere numbers and addresses the contextual problems of practice faced by different groups of teachers. The objective of this research is to contribute to the development of continuous mathematics training systems that meet the professional needs of primary school teachers so that TPD systems are more equitable, effective, and efficient. Given this framework, we ask ourselves: What characteristics should a professional development proposal for mathematics teachers have to be equitable, effective, and efficient?

The process of globalization and the information society are factors that promote access to readings that allow us to discover, study, and compare the complex framework that represents the educational process in each society, and in our case, that of TPD systems in mathematics. According to Carnoy (2006) and Caballero et al. (2016), we present a comparative study on how various countries implement teacher training procedures for those who teach in the early years of primary education. The purpose of this comparative work is to identify and characterize proposals for the professional development of mathematics teachers in primary schools around the world from situated approaches, which involve collaborative learning amongst teachers that results in the improvement of their professional practice.

2. Theoretical framework of reference

The study is developed based on the assumption that the teacher's knowledge, the teaching of mathematics, it is: personal, as it is specific to each subject and depends on their conceptions, values, attitudes and professional experience; contextualized because it is generated, linked to professional contexts; integrated and complex because it encompasses knowledge, of various kinds (scientific, didactic, psychological, sociological ...); practical, since it is nourished by professional development; partially tacit, because it acquires meaning in action accompanied by a theoretical reflection that allows knowledge from action to be transformed into knowledge or theory from action; dynamic, changing, evolving, given that it grows through interactions with students, professional and personal experiences; social, because it is built in working communities; and critical, insofar as it allows the production of knowledge-based on teaching (Climent, 2005).

Authors such as Podolsky et al. (2017) mention the importance of professional development being nurtured by collective spaces that encourage teachers to share their experiences and knowledge. It is of interest to characterize

the form that TPD takes in relation to the interactions between teachers in training. In this sense, it is necessary to methodologically differentiate the proposals according to the modalities of work. In the first instance, individual or collective professional development proposals can be recognized. Additionally, it is possible to characterize collective work according to the following taxonomy of modalities or meanings (Fiorentini, 2008):

• Cooperative work: carrying out joint actions by mutual agreement, where part of the group does not have autonomy or decision over them. There are unequal and hierarchical relationships, but some help others (co-operate) by performing tasks whose purpose does not result from the joint negotiation of the group.

• Collaborative work: the members of the group work together (co-collaborate) and support each other, trying to achieve common objectives negotiated by the group collective. Therefore, relationships tend to be non-hierarchical, having shared direction and co-responsibilities. It is characterized by being open and flexible in the review of agreements.

• Artificial collegiality: collaboration that is neither spontaneous nor voluntary, implies obligation, bureaucracy and administrative regulation (times and spaces are fixed), oriented towards the achievement of pre-established objectives in instances of power.

• Balkanization: collaboration that is fragmented into subgroups, which may be at odds with each other. This situation does not prevent collaborative teams from being within subgroups.

This taxonomy does not intend to indicate that one work methodology is better than another but rather provides aspects to differentiate them and allows us to choose the one that allows interpreting the methodological aspect of each TPD.

With regard exclusively to the TPD of mathematics, the latest trends in research are based on the conception of teachers as professionals in the teaching of mathematics who possess a knowledge of use (Rowland & Turner, 2009), which is their own and distinctive, and therefore is associated with the idea of specialized teaching knowledge (Villella et al., 2018a). [Specialized character is understood in the sense attributed in the Specialized Knowledge of the Mathematics Teacher (MTSK) model]. Specialization attends to the task of teaching, which differentiates the teacher from other expert users of mathematics. Professional teacher knowledge makes it possible to design teaching proposals in such a way that the contents prescribed by the curriculum are transposed to students (Chevallard, 1985); and is composed of:

• The conceptual framework of mathematics that teaches: the integration of mathematical contents to be taught in an intra-and extra-mathematical conceptual network, establishing relationships between concepts and types of problems that allow solving;

• The way in which mathematical knowledge is produced and how the teacher can produce it in relation to the topics developed in the classroom;

• Knowledge about the didactics of mathematics: personal knowledge and beliefs about how mathematics is learned and how it is taught, which supports the planning of their classes.

The TPD perspective seeks the design of initial and continuous training paths that look at the situated action of the teacher who teaches mathematics in the classroom, considering him or her as a reflective professional. The professionalization of the teaching role assumes that teaching does not legitimize itself by the implementation of routines focused on the presentation of knowledge but rather makes sense in the awareness of why and how students learn. Within the framework of the TPD, it is necessary to offer mathematics teachers study spaces in which they can rethink their practices to enrich them and adapt them to the needs of the groups of students with whom they work on a daily basis.

Reflection on teaching as a professional practice should refer both to studying what teachers do in the classroom when they manage a class, but also to the moments in which they interact with the mathematical content that they select and prioritize to support teaching sequences (Villella et al., 2018b). These interactions are contemplated in the relationships between knowledge; the relationships between experiential knowledge and theory; the discourse; knowledge about what to teach, for what and how; knowledge of teaching strategies, among others. This professional knowledge allows teachers to "look with sense" (Mason, 2002), being able to identify practical problems related to the teaching of mathematics.

There is ample evidence that disciplinary training and general psychopedagogical training are not enough to teach a discipline (Niss, 2004), but that knowledge and skills related to both mathematical content and its teaching and learning

processes are required. The Specialized Knowledge of the Mathematics Teacher (MTSK) (Montes et al., 2021; Carrillo et al., 2018; Carrillo et al., 2013) is a theoretical model that allows interpreting the knowledge that teachers evidence in mathematics teaching practices. This model preserves, for analytical purposes, the division between the domains of content knowledge and didactic content knowledge proposed by Shulman (1986), as well as contributions made by Ball et al. (2008). It works with two domains that, crossed by the conceptions and beliefs that the teacher has about mathematics, are subdivided into subdomains that are referenced by acronyms (Figure 1).

The mathematical knowledge comprising:

• Knowledge of Topics (KOT): reference to mathematical knowledge as an object of teaching, encompassing its phenomenological perspective and its applications, the various registers of representation, the definitions, properties, foundations and procedures to construct them.

• Knowledge of the Structure of Mathematics (KSM): refers to the conceptual networks that make up school mathematical content.

• Knowledge of Mathematical Practices (KPM): it contemplates elements of mathematical work, how it is defined, how it is justified, how it is validated, how it is represented, among others.

The didactic knowledge of the content that comprises:

• Knowledge of Mathematics Teaching (KMT): focuses on the teacher, contemplating knowledge about theories (personal or formal) of teaching, of the material and virtual resources to develop it, and knowledge of strategies and resources and how to orchestrate them in the mathematics classroom.

• Knowledge of Mathematics Learning Features (KFLM): focuses on the learner, contemplating the knowledge of personal or formal theories about learning, the ways in which students interact with mathematical content and their interests and expectations.

• Knowledge of Mathematics Learning Standards (KMLS) refers to a holistic view of the curriculum, including the learning expectations at a given stage of the education system, the level of conceptual development, or the level of development of mathematics.

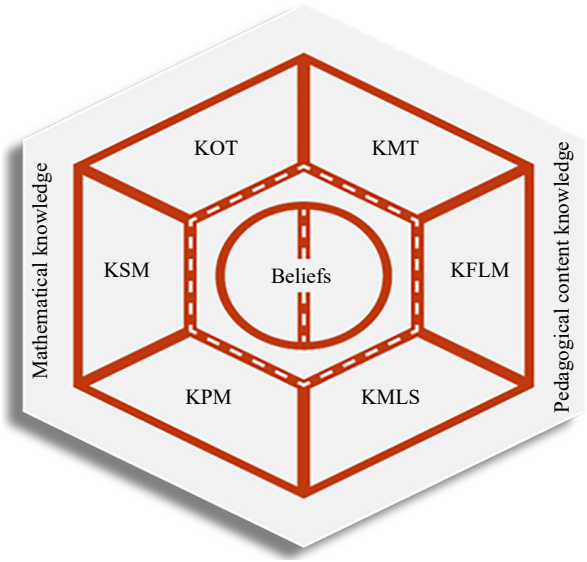

Figure 1. MTSK model, adaptation of the original presented in Carrillo et al. (2013, p. 2989)

In this research, this interpretative model is considered to analyse the main contents of the TPD systems, observing whether teacher training includes in its topics the knowledge: of the topics, of the structure of mathematics and of doing mathematics (knowledge about and of mathematics), and of the teaching of mathematics, the characteristics of mathematics learning and the mathematics curriculum (pedagogical and didactic knowledge); intertwined with beliefs and conceptions about mathematics and its teaching (Carrillo et al., 2018; Carrillo et al., 2013).

3. Methodology

The present study was carried out using a qualitative and interpretative comparative research methodology (Vasilachis de Gialdino, 2006). In agreement with Carnoy (2006), this comparative study presents research on how various countries, selected based on specific criteria that we will present later, implement teacher training procedures for those who teach in the early years of primary education. For this article, results of interest were selected.

The process of globalization and the information society are factors that promote access to, readings that allow us to discover, study and compare the complex framework that represents the educational process in each society, and in our case, that of the TPD systems in Mathematics. The comparative approach is used as a science that studies educational phenomena or facts in different places around the world or at different historical moments to establish global educational improvements (Dale, 2007; Caballero et al., 2016). Consequently, this research does not intend to establish comparisons of quality, effectiveness, or value judgments about the TPD programs studied. We describe the main characteristics that allowed their development, point out the actions they included, and the results they achieved. In short, the comparative view seeks to examine the similarities and differences of the training proposals, always with the aim of contributing to the improvement of practices.

For the selection of the proposals for the professional development of mathematics teachers included in the research, an exploratory search was carried out that considered exclusive and non-exclusive selection criteria (EC_n and NEC_n, respectively where n represents the order):

• EC_1: these are governmental and/or non-governmental initiatives for the continuous training of teachers in Mathematics aimed at teachers in the first grades of primary education;

• EC_2: has a situated approach. This means that the construction of knowledge that promotes training considers the understanding that each teacher has of the situations of practice within which he or she evolves, that is, those training offers focused on the school are of interest (Vezub, 2019; 2010);

• EC_3: its implementation period does not exceed the last 10 years, since previous studies covered older experiences [Experiences of mathematics teacher training prior to this period, developed between 1999-2011, can be seen in: https://www.preal.online/innovacion2];

• NEC_4: incorporate technology into TPD systems.

The exploratory mapping included research in international databases, including ERIC (Education Resources Information Center) and JSTOR (Journal Storage), using keywords, identified in the theoretical framework of reference [The keywords used in the exploratory mapping were: teacher improvement in mathematics, ongoing training in mathematics, teacher updating in mathematics, teacher retraining in mathematics, in-service training in mathematics, professional development for teachers in mathematics. These words were associated with the phrases: “program” and “primary education”, and the temporal filter “last ten years” was used], and in the websites of governmental and non-governmental organizations, with the subsequent review of specific bibliography and the analysis of proposals, as well as the consultation of key informants belonging to national and international research networks dedicated to the continuous teacher training of Mathematics. TPD experiences were selected considering the *best available evidence* (Reimers, 2020) among primary and secondary sources: regulations (in the case of government programs), program documents, impact evaluations, research related to experiences.

Finally, the exploratory search, whose main characteristics and recurring points are presented in the following section, consolidated 13 TPD programs in Mathematics from the following 12 countries, listed in alphabetical order:

(1) Argentina: National Permanent Training Program Our school.
(2) Australia I: Australian Government Quality Teacher Programme.
(3) Australia II: Quality Teaching Rounds.
(4) Brazil: Interpretative and Specialized Knowledge of the teacher of and who teaches Mathematics.
(5) Canada: Teachers Learning and Leadership Program for Experienced Teachers.
(6) Colombia: Program Everyone to Learn.
(7) Cuba: Postgraduate Professional Development Program.
(8) Dominican Republic: System of training in context.
(9) Spain: Teaching experiment based on the model of the Specialized Knowledge of the Mathematics Teacher.
(10) France: Mathematics Network for All.

(11) England: Mathematics Teacher Exchange.

(12) Japan: Lesson Study.

(13) Mexico: Online community Teachers of Mathematics of Mexico.

We designed one instrument for the collection and systematization of the information. This instrument facilitated the organization of the unstructured data referring to each training experience, and the identification of invariant and distinctive characteristics among the selected teacher professional development proposals. The collection instrument is a table that incorporates the dimensions of analysis in its first column and the narrative description resulting from the research in the second (Table 1). The description of the TPD systems consists of dense notes that gather the information obtained from the consulted sources.

Table 1. Model of data collection and systematisation table (own production)

Table n°: Country: Name of the TPD system	
Name of the initiative:	
Organization/Framework in which it is included:	
Country/countries:	
Effective date:	
General description:	
Comment:	
Consultation website:	
Documents/research on the initiative:	

In the interpretative phase of the study, the data and information collected in the descriptive stage were analysed in detail. To examine the information, parameters and indicators were established. Indicators are the smallest comparative unit of measurement, while parameters encompass indicators of a similar nature (Caballero et al., 2016). The parameters and indicators developed considering the conceptual framework were:

• Structure of the TPD system: i.e., its scope, extension, objectives, pedagogical design, financing modality, trainers' profiles, use of technological resources.

• Profiles of the target teaching population: considering their socio-cultural characteristics, the institutions in which they are developed, seniority in the teaching profession, etc.

• The main contents of the training: disciplinary and didactic.

• Policy diagnoses to which the initiatives respond.

• Problems in the field of mathematics teaching that the initiative seeks to address.

• The main didactic approaches to the teaching of Mathematics on which the design of the TPD is based.

• Reports on the evaluation of their results.

The analytical strategy deployed in the interpretative phase involved the identification of indicators and parameters in the data collection tables. For this methodological operation, the following coding system was used, which organised the indicators with subindices I_n (where n represents the order) and for this methodological operation, the following coding system was used, which organised the indicators with subindices In forming parameters Pn (where n represents the order): to group indicators of the same parameter (P_n):

• P_1: Structure of the TPD system: that is, its scope ($I_{1.1}$), extent ($I_{1.2}$), objectives ($I_{1.3}$), pedagogical design ($I_{1.4}$), financing modality ($I_{1.5}$), profiles of the trainers ($I_{1.6}$), the use of technological resources ($I_{1.7}$).

• P_2: Profiles of the target teaching population: considering their sociocultural characteristics ($I_{2.1}$), the institutions in which they work ($I_{2.2}$), and their seniority in teaching ($I_{2.3}$).

• P_3: Main contents of the training: disciplinary ($I_{3.1}$) and didactic ($I_{3.2}$).

- P_4: Policy diagnoses that the initiatives respond to.
- P_5: Problems in the field of mathematics education that the initiative seeks to address.
- P_6: Main didactic approaches to the teaching of Mathematics on which the design of the TPD is based.
- P_7: Reports on the evaluation of their results.

4. Results

Comparative education provides valuable information about the similarities or divergences of educational systems around the world. In the case of this study, which presents a temporal update of others that have been conducted, the potential lies in its specificity: improving the continuous training of primary school mathematics teachers. With this intention, the most outstanding results of the conducted research are presented. We organise the presentation of the results according to the established parameters and indicators.

In relation to the structure of the TPD systems, an analysis is carried out of their scope, extension, objectives, pedagogical design, financing modality, profiles of trainers, and the use of technological resources.

The scope of the experiences analysed varies according to their format: five have universal but voluntary coverage (Argentina, Australia I, Australia II, Cuba, Japan), while the rest are aimed at on the diagnoses made, an aspect we will delve into in the following paragraphs. Regarding the duration of the training provided to teachers in mathematics, it varies widely between partial and annual offerings, or combinations of both. Most of them are proposals that refer to continuous training extending up to a year (Argentina, Australia I and II, Canada, Colombia, Cuba, and Japan); followed by training facilities ranging from one month to less than a year (Argentina, Brazil, Cuba, Spain, and the Dominican Republic); and, to a lesser extent, shorter training programs, from a few days to less than a month (Argentina, Brazil, England). Only one program was found with a proposed duration longer than a year (Brazil).

The objectives of the TPD systems included in this study are framed in the conception of teacher professional development. Although its formulation is varied (Table 2), they all understand that teacher learning is expanded, articulated and integrated throughout their trajectory, and they recognize the need to develop actions focused on teachers whose ultimate purpose is to achieve improvements in teaching and in the learning of their students.

Table 2. Statement of objectives of the TPD systems studied

Country	Name of the TPD system	Objective of the TPD systems
Argentina	National permanent training program our school	To give centrality to teaching and strengthen the teaching task in post-pandemic scenarios through the diversification of didactic strategies appropriate to the context of heterogeneity and inequality of educational trajectories.
Australia	Australian government quality teacher programme	Improving Australian education, focusing on the need to strengthen all schools by: supporting the professional position of teachers; attracting the best people to become teachers; ensuring national coherence in schooling; empower schools to meet community needs; ensuring that all schools perform well; providing meaningful information to families; create safe school environments; accelerate the results of indigenous education; ensure smooth transitions from school to work or higher education.
Australia	Quality teaching rounds	Improve the morale, confidence, job satisfaction and retention of early-career teachers across Australia.
Brazil	Interpretative and specialized knowledge of the teacher of and who teaches mathematics	To develop the specificities of the teacher's knowledge that contribute to students understanding what they do and why they do it, always, and thus develop the pleasure of learning mathematics.
Canada	Teachers learning and leadership program for experienced teachers	Share and disseminate innovative practices developed by teachers, with the purpose of benefiting Ontario students.
Colombia	Program everyone to learn	Transform the pedagogical practices of teachers to improve learning in Early Childhood and Basic Primary Education, emphasizing the latter in mathematics and language in schools with insufficient performance in standardized tests.

Table 2. (cont.)

Country	Name of the TPD system	Objective of the TPD systems
Cuba	Postgraduate professional development program	Provide professional development opportunities; the acquisition of skills or the assimilation of new work techniques and procedures; the deepening, updating or complementation of knowledge, and even requalification if necessary.
Dominican Republic	System of training in context	Improve the preparation of teachers who teach the subject of Mathematics in Basic Education.
Spain	Teaching experiment based on the model of the specialized knowledge of the mathematics teacher.	To offer instances of reflection to teachers on the different aspects linked to the teaching and learning of mathematics, that they must know when considering taking a problem to the classroom.
France	Mathematics network for All	Improving the teaching of mathematics in schools located in disadvantaged areas of France, which involves: reducing educational inequalities in mathematics among students from different socioeconomic backgrounds; improving the quality of mathematics teaching in participating schools by providing teachers with high-quality training and resources; and promoting inclusion and success for all students.
England	Mathematics teacher exchange	Raise the level achieved by students through a school-led system to raise the mathematics proficiency of their teachers.
Japan	Lesson study	To improve the pedagogical practices of primary schools in Japan through the collaborative work of the teaching teams.
Mexico	Online community reachers of mathematics of Mexico	To disseminate mathematical knowledge from the various perspectives of the currents of mathematical thought and to transmit educational innovations through shared scenarios for the teaching and learning of mathematics.

In the context of continuous professional development, it is necessary to offer teachers study spaces where they can rethink their mathematics teaching practices to enrich them and adapt them to the needs of the student groups they work with daily (Villella et al., 2021). Like all teacher training tasks, each TPD system has a function, a form, and a focus (Grevholm et al., 2009). The formative tasks proposed to teachers who teach mathematics in the early years of primary school have the "function" of building specialized knowledge (Carrillo et al., 2018). To fulfil this function, the form and focus can vary, resulting in different pedagogical designs for teacher training. The comparative study shows that the study spaces intended for teachers take on the following designs:

• Academic updates with university rank: in the case of the experiences developed in Argentina, Cuba, and Spain, their format is identified in the completion of seminars, the approval of which grants a university-level degree.

• Short courses and courses: in the proposals from Argentina, Brazil, Colombia, Cuba, France, Mexico, and the Dominican Republic, there is the possibility for each teacher to take short-term training courses focused on the training priorities for the updating and deepening of the curriculum, which may not be linked to each other.

• Annual projects focused on institutional problems (promoted in the training systems of Australia I, Canada, Colombia and France): in this type of offer, the proposed approach is the design and implementation of an institutional pedagogical project in mathematics with the participation of the school's teaching staff. In these TPD systems, the educational community carries out a systematic action of analysis and discussion focused on the pedagogical task itself and in relation to jurisdictional and national policy. It involves formative actions for the entire teaching team as part of participatory institutional evaluation for the improvement of teaching processes and conditions. The preparation and analysis of information as a privileged input for improvement actions and the search for effective strategies to achieve it position teachers as producers of situated pedagogical knowledge.

• Exchange programs: this type of training design takes the form of a teacher exchange program between countries or regions of the same nation. On exchange trips, teachers observe and participate in class planning meetings and visit classrooms to watch their colleagues teach math. This is the case with the proposals from Australia I and England.

• Design, development, observation and analysis of classes (promoted in the cases of Australia II, Brazil, Colombia, England and Japan): this type of formative design stimulates teamwork among teachers from the same institution or different schools to progressively improve their pedagogical practices through mutual evaluation and critique of teaching practices. The form that this type of TPD system takes is that a group of teachers and specialists in

mathematics didactics meet with a common problem regarding their students' learning, design a class for the students to learn a specific content, the teacher carries it out, and finally, they examine and discuss what they observe in the implementation of the proposal. Through multiple interactions in this process, teachers have many opportunities to discuss student learning and how teaching impacts that learning.

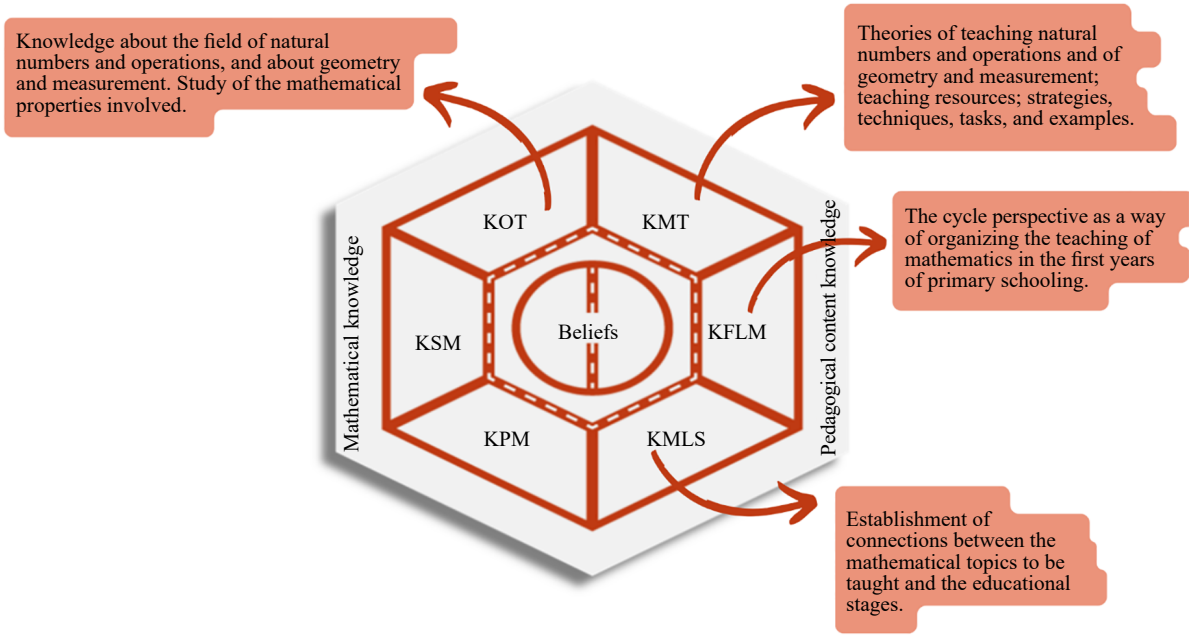

Figure 2. Knowledge (in terms of the MTSK model) shared by the studied TPD systems

To study the focus (Grevholm et al., 2009) of the TPD proposals, it is necessary to review the specialised knowledge of the mathematics teacher that the training activities aim to mobilise (Montes et al., 2021). To analyse the parameter related to the content on which the training proposals are focused, we start by recognising that there is a difference between the mathematical knowledge that the teacher needs (specialised content knowledge) and that which another user of mathematics needs (advanced or superficial common knowledge that does not need to be taught) (Carrillo et al., 2018; Carrillo et al., 2013). Specialization refers to the task of teaching, so another aspect that allows us to describe TDP systems is the main disciplinary and didactic aspects of training. Teachers' mathematical knowledge includes the mathematical knowledge (KOT) they possess, since it allows them to plan, manage and reflect on teaching situations. In the training offered, work is primarily on natural numbers and operations, and geometry and measurement; there is less predominantly over: fractions, statistics, algebra (early and equations), and probability. The didactic knowledge of the content encompasses the knowledge that the teacher has about teaching, the characteristics of learning, and the learning standards of a particular content. Regarding knowledge about the teaching of mathematics (KMT), proposals are highlighted around theories or didactic models for teaching natural numbers, arithmetic operations, and plane geometry, including teaching resources, teacher intervention strategies, and example analysis. Regarding the Knowledge of Mathematics Learning Features (KFLM), the treatment of the cycle perspective is reiterated to organize the teaching of mathematics in the early years of primary education and the planning of the subject's teaching. These topics appear in connection with the teaching standards (KMLS) by proposing links between the mathematical topics to be taught, the educational stages, and the results of standardized assessments. In the analysed TPD systems, there is no mention of proposals for activities that would allow for a deeper understanding of the structure of mathematics (KSM), which, for example, would enable the establishment of content networks between the number and operations axis and the geometric one. There is also no mention of proposals that engage primary school teachers in the study of Knowledge of Mathematical Practices (KPM), for example, regarding the ways to produce and proceed with solving geometric

problems using software, or regarding the argumentation and validation of conjectures, and the communication of mathematical ideas. In Figure 2, we summarise the knowledge (in terms of the MTSK model) that we recognise as common in the studied TPD systems. This allows us to affirm that it is necessary in training programs aimed at primary school teachers to consider a balance between didactic knowledge and mathematical knowledge.

It is relevant to consider that the selection of specialized content on which TDP systems focus is defined based on evidence gathered from national and international standardized assessments. Some training programs express the link between the selection of topics and developments in research in the field of Mathematics Education, while others rely on consultations with key actors in the educational system, such as supervision and school management teams.

It coincides that TPDs that take professional teaching actions (their own and others) as the basis of the discussions of the training actions are those that have indicators of the impact of improvement in the teaching and learning processes of mathematics. In these experiments, the emphasis is placed on the reflective activity, seen as an activity where, by questioning the teaching practice of a certain mathematical content, the teachers make explicit the code and the rules of operation. These methodologies seek to generate situations of subtle rupture with the familiarity that teachers have with the elements that make up the teaching and learning processes. Observing with surprise the elements that make up both the teaching proposals and the management of the class, allows their explicitness and study.

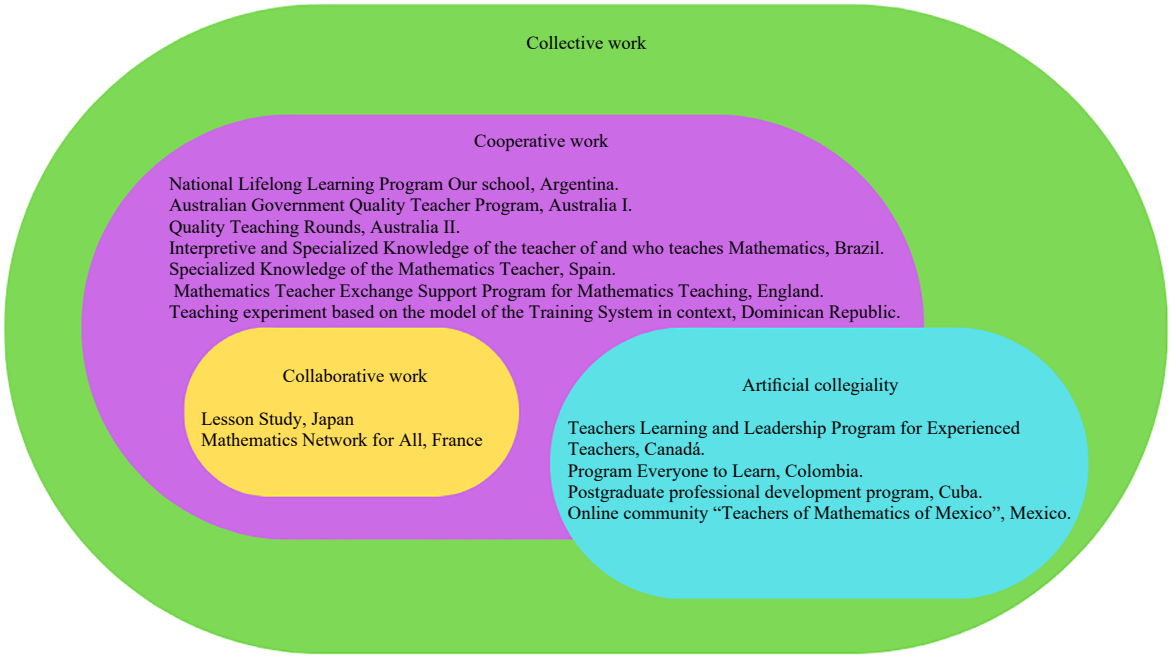

Figure 3. Type of collective work of the teaching population participating in the TPD systems studied

The mathematics teacher training analysis proposes shared actions of co-construction of teaching problems with others. Where these others arise as a questioning externality that they present themselves as neighbour in an experience of responsibility in understanding the word heard. In the TPD systems analysed, proposals for collective work are recognized (Fiorentini, 2008) with three different levels of involvement and relationship among the participating teaching population: cooperative work, artificial collegiality, and collaborative work (Figure 3). Designing training actions from the perspective of teacher professional development requires transcending the conception of permanent training as an individual fact subject to the requirements or needs of a professional career conceived and undertaken exclusively in solitude. Beyond the conceptual and procedural limits of the clearly defined and characterized terms by Fiorentini (2008), the analysis of the indicator related to modalities or meanings of collective work invites us to consider how morphemes provide clues about the type of methodology that guides the educational practice. In this way, for a teacher, it is not the same to be trained through collective work as it is through a collaborative proposal. Collaborative

work is the action of working where there is the ability to co-labour, for which personal commitment is necessarily required to achieve common goals in the shared and co-responsible action of teaching, meaning that the act of teaching is conceived and carried out jointly. It should be clarified that the indicator does not intend to show that one training methodology is better than another, but rather it provides aspects to differentiate them and allows those responsible for designing training proposals to choose the one that enables understanding and comprehending the other.

An aspect to consider in comparative studies is the social and economic contexts in which educational offerings are developed. That is why it becomes relevant to analyse the financing modality of TPD systems. The modality of financing the teacher training proposals varies in three formats: those financed by the government in a centralized manner without the time spent on professional development affecting the salary, the employment relationship, or the paid vacation period of the participating teachers (this is the case of: Argentina, Australia I, Australia II, Colombia, Cuba, France, England, Japan, and the Dominican Republic); those financed by the government in a decentralized manner, delegating the administration of the funds to teachers (Canada); those paid for by teachers (Brazil, Spain and Mexico). In this last group, the Spanish case stands out, since the value of the training proposal is a function of the price of the credit in the European Education Area.

Information and communication technologies have been gradually incorporated into the areas of teacher professional development. This generates the need to reflect on the insertion of these technologies in the training processes related to the teaching of mathematics. The experiences surveyed in Argentina, Australia II, Brazil, Canada and Colombia in this study refer to the incorporation of technological resources for Mathematics teacher training. The use of technological resources in these TDP systems has dissimilar intentions, tools and scope. Sometimes technological tools are proposed as a means for teacher training. In this group, there are platforms with virtual classrooms that incorporate digitized bibliography, audiovisual material, discussion forums; YouTube channels for seminar broadcasting; platforms for synchronous virtual meetings. In other cases, technology is used as a didactic tool for the teaching of mathematical knowledge: those proposals that incorporate software for the teaching and learning of mathematics (for example, GeoGebra). Regarding the scope of the use of technology in continuous teacher training, there are totally virtual proposals and others that articulate face-to-face proposals with virtual instances, differentiating, in turn, between synchronous and asynchronous. The relationship between access to technology and the possibility of designing and developing continuing teacher training proposals that incorporate technological resources either as a means or as a didactic tool cannot be separated from this analysis.

Another possible approach to the experiences is by considering the selection of the target teaching population. We identified four selection criteria for participating in the training proposals: voluntary without restrictions, voluntary with requirements related to job performance, voluntary with affiliation, and mandatory based on the performance of schools in standardized tests (Figure 4). In the case of continuous professional development proposals with unrestricted voluntary participation, the following countries are included: Argentina, Brazil, Spain, England, Japan, and the Dominican Republic. The TPD systems of Australia I, Australia II, and France allow voluntary participation with requirements related to job performance. These requirements consider: the type of management and/or modality of the schools where the target teacher works, the socioeconomic level of the student population served, the seniority in teaching, and/or the re-entry into teaching work. The Mexican case requires paid membership in the National Association of Mathematics Teachers. While the training systems in Canada, Colombia, and Cuba provide for mandatory participation of teachers based on the poor performance of schools in standardized tests. Both the fragmentation of voluntary professional development proposals for mathematics teachers and the criteria by which some programs mandate coursework for teachers in schools with low standardised test scores demonstrate the protection and expansion of children's right to education and highlight the relevance of teachers' right to ongoing training. That is to say, the right of teachers to ongoing professional development throughout their careers is recognised, gains meaning and intentionality in the right of new generations to receive a quality education.

The management of the characterized TPD systems requires profiles of trainers specialized in the didactics of mathematics. The presence of specialists from universities dedicated to training and research in the field of professional development of mathematics teachers is mostly observed, as well as the identification of experienced volunteer teachers, who, after a period of training, assume training proposals.

The way in which the disciplinary and pedagogical content is studied and analysed in the training proposals allows us to recognize the main didactic approaches to the teaching of Mathematics on which the design of the TPD is based.

The proposals analysed are mostly based on constructivist approaches to the teaching and learning of mathematics with an emphasis on teaching practice, considering the specialized interpretative knowledge of teachers in the first years of primary school. In some cases, the system of beliefs and motivations of the teaching staff for the exercise of their profession is incorporated.

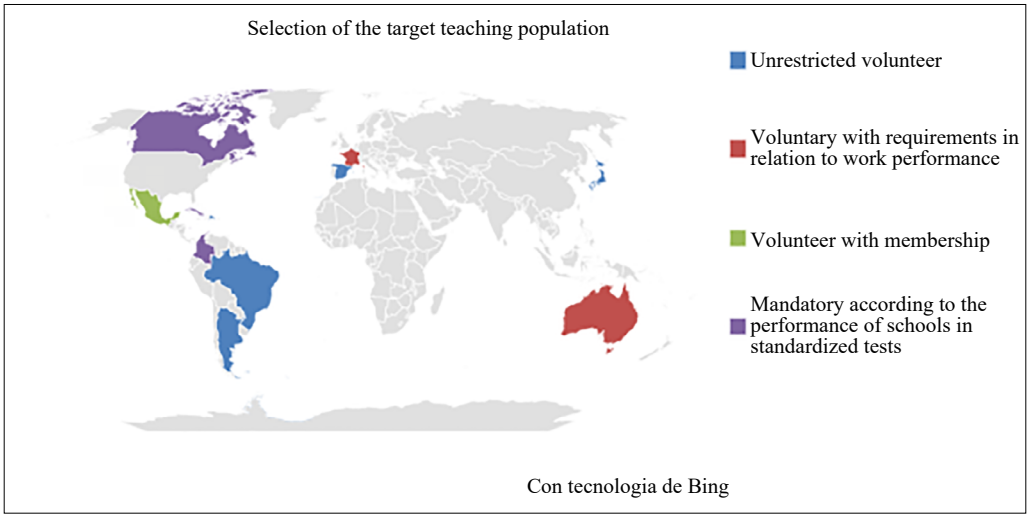

Figure 4. Criteria for selecting teachers to participate in the studied training proposals

With regard to the educational policy diagnoses to which the teacher training initiatives respond, it is observed that they mostly address the difficulties and shortcomings detected through standardized national, regional educational assessment operations, such as the Regional Comparative and Explanatory Study (ERCE), and/or international, such as the study of Trends in International Mathematics and Science Study (TIMSS) or the Program for International Student Assessment (PISA). In addition, some experiences consider the low or non-existent offer of training paths aimed at improving the teaching practices of teachers in practice.

The diagnoses that justify the need for the professional development of mathematics teachers are related to the problems in the field of mathematics teaching that the initiatives seek to improve. The problematization of the quality of mathematical learning achieved by children encounters a variety of concerns that vocational training proposals seek to address:

- Outdated or lack of knowledge and skills that make up initial training;
- Logics of individualistic and isolated teaching work;
- Tension in the management of resources for teaching; and
- Lack of specialized knowledge on the part of teachers in relation to mathematical content and its teaching;

Finally, with respect to the proposals for evaluating the results of the TPD systems collected, two strategies are recognized: those of accompaniment based on considering the degree of satisfaction of teachers with the training received; and monitoring by collecting evidence on their applications in pedagogical practice and their impact on student learning. Only four of the teacher training programs analysed included evaluations with measurable outcomes, such as student performance metrics or teacher retention rates; these are the cases mentioned as Australia II (Gore et al., 2017), Colombia (MEN, 2022), England (Boylan et al., 2019), and Japan (Isoda et al., 2007; Isoda & Baldin, 2023). As mentioned, all the collected experiences are based on collective processes of various kinds. The comparative study of impact evaluations and qualitative monitoring of TPD allows us to affirm that collective training has a favourable impact on the professional development of participating teachers, which, in cases where impact measurements are shown, promotes better learning for their students. This link is an indicator that TPD systems with methodologies that involve learning both to problematize teaching practice and to formulate specific questions about it, enable sustained learning over time (Badillo Jiménez et al., 2023).

5. Conclusions

We began this study by asking ourselves about the characteristics that a professional development proposal for mathematics teachers should have to be equitable, effective, and efficient. We dedicate the article to presenting the results of a comparative study that allows us to reach some conclusions we consider answer the initial question, and others that open new lines of research.

Several studies characterize cases of teacher training in mathematics. Among them, we highlight Climent (2005) and Villella et al. (2021), who identify some privileged conditions for designing training processes that contribute to the professional development of mathematics teachers:

• To be creative, if they create spaces for discussion about the problems that make up the teaching of mathematics, inviting teachers to rethink the meaning of their work, and to legitimize themselves as producers of valuable and necessary pedagogical knowledge to improve and change every day educational work;

• Be situated. Given that these professional problems of mathematics teaching are local and contextualized in the classrooms, TDP systems must include, as a part of their training actions, topics linked to the concrete practice of teaching mathematics. With respect to the proposals for reflection on the teaching of mathematics, they should make it possible to analyse the following student performance: the student performance promoted by teaching management in the mathematics classroom, the role of resources in the transformation of the workspaces created ad-hoc, the study of the meaning of knowledge mediated by the different resources selected, and of the social and institutional aspects that stress this mathematical workspace (Kuzniak et al., 2019); and

• To be proactive, because, based on reflection on practice, decisions related to the management of the class that can be assumed in the future are influenced.

This comparative study adds another characteristic:

• To be collective, since the reflexive exchange with other teachers on how to interpret and intervene in the teaching, teaching and learning processes of mathematics makes it possible to construct curricular and didactic knowledge. This knowledge arises from reflection on the practice of teaching mathematics and returns to it, to give meaning to the teaching decisions that are made daily in the teaching process. Designing, developing, observing and analysing teaching sequences with others as part of the continuous training proposals involves: the re-reading of didactic models, the review of one's own knowledge about the contents, how to plan them to teach them in the classroom; to discuss the management of didactic variables associated with the modification of the environment (Brousseau, 2007) offered to students; the analysis of the level of development of some of the mathematical contents that are used to give significance and meaning to the teaching proposal; the characterization of performance indicators that allow an evaluation of student learning. These aspects favour the development of didactic knowledge of mathematical content by teachers in training.

• To be based in evidence, the professional development of mathematics teachers requires the articulation between two components: an institutional training component and a specific training about teaching maths component subject to a map of vacancies and priorities based on evidence taken from national and international evaluations. These components must articulate with each other so that each training action addresses the main challenge, which is to ensure the right to continuous education to guarantee our society's right to better education.

• To be balanced, training programs aimed at primary school teachers should consider a balance between pedagogical knowledge and mathematical knowledge. We believe that research on the professional development of mathematics teachers should delve deeper into this aspect.

The research indicates the need to investigate the incorporation of technology in the professional development of primary school teachers, both as a means of delivering educational offerings and as a resource for teaching.

The comparative study shared on these pages is not only about maximizing the use of available information on TPD systems, but also about enabling the construction of consensus for improvements in school proposals. The challenge of teacher professional development lies in the possibility of universalizing training offerings with a situated and collaborative approach to achieve concrete impact in schoolwork, in classrooms, and in the educational quality to which all children are entitled, regardless of their nationality.

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Conflict of interest

The article entitled "Constitutive Elements of Professional Development Programs for Primary School Mathematics Teachers: Evidence of Proposals from Multiple Countries" declares that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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