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Research Article

An Integrated Production and Inspection Model With a Heuristic Product Inspection Policy When Inspection Errors Exist

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Abstract: This study proposes a heuristic inspection policy to address the existence of type I and II inspection errors. The deterioration process often shifts from an in-control state to an out-of-control state toward the end of a production lot, producing more nonconforming products. As a result, if the initial (late-produced) products in a production lot are chosen for inspection, the products identified as nonconforming (conforming) may need to be inspected repeatedly. The decision regarding the demarcation point between initial and late-produced products, their respective inspection rounds, and the production lot size must be made simultaneously to minimize the expected total cost per conforming product. This paper provides numerical examples to explore the effect of inspection errors on the optimal production lot size, inspection policy, and the associated cost.

Keywords: economical production quantity (EPQ), inspection errors, deteriorating process, quality control

1. Introduction

Scholars have extensively studied the problem of economically controlling product quality in an imperfect manufacturing system. For example, higher production rates accelerate the deterioration of the production system, leading to machine failures and defective items (see Malik & Kim, 2020). Smaller batch sizes can reduce the number of nonconforming products when a deteriorating production system is used (see Djamaludin et al., 1994; Porteus, 1986; Rosenblatt & Lee, 1986; Salameh & Jaber, 2000; Wang & Sheu, 2003; Yeh & Lo, 1998). In contrast to using small batch sizes to control the number of defective products, Raz et al. (2000) proposed a dynamic inspection method to control quality and minimize the quality control cost. Wang (2007) further studied the impact of inspection errors on inspection plans.

Imperfections in the production process have led to the inability to meet the demand for high-quality products at the production stage. However, after defective products are eliminated or reworked at the end of the inspection process, customers will be satisfied with product quality (see Tuan et al., 2020). When product inspection is considered for a completed lot at the end of production, performing a 100% inspection (see Jaber et al., 2008) can eliminate all nonconforming products when the inspection is perfect. However, 100% inspection may be expensive (see Muhammad, 2011) and prone to inspectors making inspection errors. Consequently, production and inspection systems may need further improvement (e.g., Muhammad, 2016; Pal & Mahapatra, 2017; Yoo et al., 2009). As such, regarding cost, using

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a partial inspection policy is an alternate, more economical approach to controlling product quality. For example, using the lot sizing model proposed by Porteus (1986) and Yeh et al. (2000) (see Hu & Zong, 2009; Shih & Wang, 2016; Wang, 2004; Wang & Meng, 2009; Wang & Sheu, 2001; Wang et al., 2004; Yeh & Chen, 2006), it was considered that only the products produced toward the end of the production lot should be inspected since more nonconforming products are likely to be produced during this period in a deteriorating production system. Wang et al. (2004) further extended Wang and Sheu's (2001) production and inspection model to involve type I and II inspection errors, with shortage being neglected. Hsu and Hsu (2013) investigated the effect of inspection errors on shortage. However, for the initial part of the production run, which comprises uninspected products, nonconforming products will result in external quality costs, e.g., warranty/return costs and loss of goodwill (see Sarkar & Saren, 2016). The present study suggests that initial and late-produced products in a production lot undergo different inspection strategies. That is, the focus should be on inspecting products that have been inspected to be nonconforming (conforming) when they are part of the front (rear) segment of the production lot to economically diminish the number of wrongly accepted (rejected) products.

The remainder of this paper is organized as follows: our proposed production and inspection model with type I and II errors is outlined in Section 2. In Section 3, a no inspection policy is investigated. In Section 4, an illustrative numerical example is given to explore the effect of inspection errors on the optimal solution. In Section 5, a conclusion is drawn.

2. Production and inspection model

The following notations are used to develop our proposed production and inspection model with two types of inspection errors.

c_{I} : inspection cost for identifying the quality status of a product (\$/product)
c_{R} : unit restoration cost for the process (\$)
c_{m} : manufacturing cost per product (\$/product)
c_{s} : joint cost of the process setup and maintenance (\$)
D : market demand of the products per unit time
c_{h} : inventory holding cost for a product per unit time
α : the probability that inspecting a conforming product to be nonconforming (i.e., type I error)
β : the probability that inspecting a nonconforming product to be conforming (i.e., type II error)
r : revenue from accepting a conforming product (\$/product)
c_{a} : cost of accepting a nonconforming product (\$/product)
c_{r} : cost of rejecting a conforming product (\$/product)

In this study, we adopt the imperfect process considered by Wang and Sheu (2003), which can be addressed as follows. Assume that the process is initially setup and maintained in an as-good-as-new and in-control state. Each time when a product is produced, it may randomly shift from an IN state to an OUT state. Once the process shifts into an OUT state, it stays there until the end of the production lot. Let $\overline{P}_j$ be the probability that the process is still in an IN state after producing j products since the last setup. Then, the probability that the process is in an OUT state at the end of a production lot with size of N is given by $1-\overline{P}_N$ (e.g., see Wang, 2005). Thus, there is an expected restoration cost of $c_{\mathrm{R}}(1-\overline{P}_N)$ to bring the process from an OUT state back to an IN state. The conforming rate for a production lot with size of N (denoted by $\rho(N)$) is given by (see Wang & Sheu, 2003)

$$\rho(N)=\frac{1}{N}\left\{\sum_{j=1}^{N}\left[(j-1)\theta_1+(N-j+1)\theta_2\right]\left(\overline{P}_{j-1}-\overline{P}_j\right)\right\}+\theta_1\overline{P}_N, \tag{1}$$

where θ_1 and θ_2 are the probability that a product will be conforming if it is produced in an IN state and OUT state, respectively.

This study proposes a heuristic inspection policy to address the situation created by inspection errors for a production lot based on the following process. First, note that most conforming (nonconforming) products are produced during a production lot's initial (late-produced) stage. Given a production lot with a size of $N > 0$ for the first n (the last $N - n$) products, if a full inspection is performed in the first inspection round, then products identified as nonconforming (conforming) will be inspected again in the next inspection round, i.e., the second inspection round. The above inspection procedure will be repeatedly performed and terminated as soon as the next inspection round has been evaluated. We found that the number of conforming (nonconforming) products that are incorrectly rejected (accepted) cannot be significantly reduced (refer to Figure 1 for the inspection process).

This paper also explores conditions for no inspection to avoid the enormous costs incurred by a full inspection. Note the two extreme cases: (i) when $n = 0$, there is a strict inspection strategy. That is, when a product has been identified to conform, it would be inspected again. However, if it has been identified as nonconforming, it should be disposed of directly. On the other hand, (ii) when $n = N$, there is a loose inspection strategy. That is, a conforming product identified by inspection should be directly accepted. Otherwise, it would be inspected again. Our proposed inspection model balances these two extreme inspection policies.

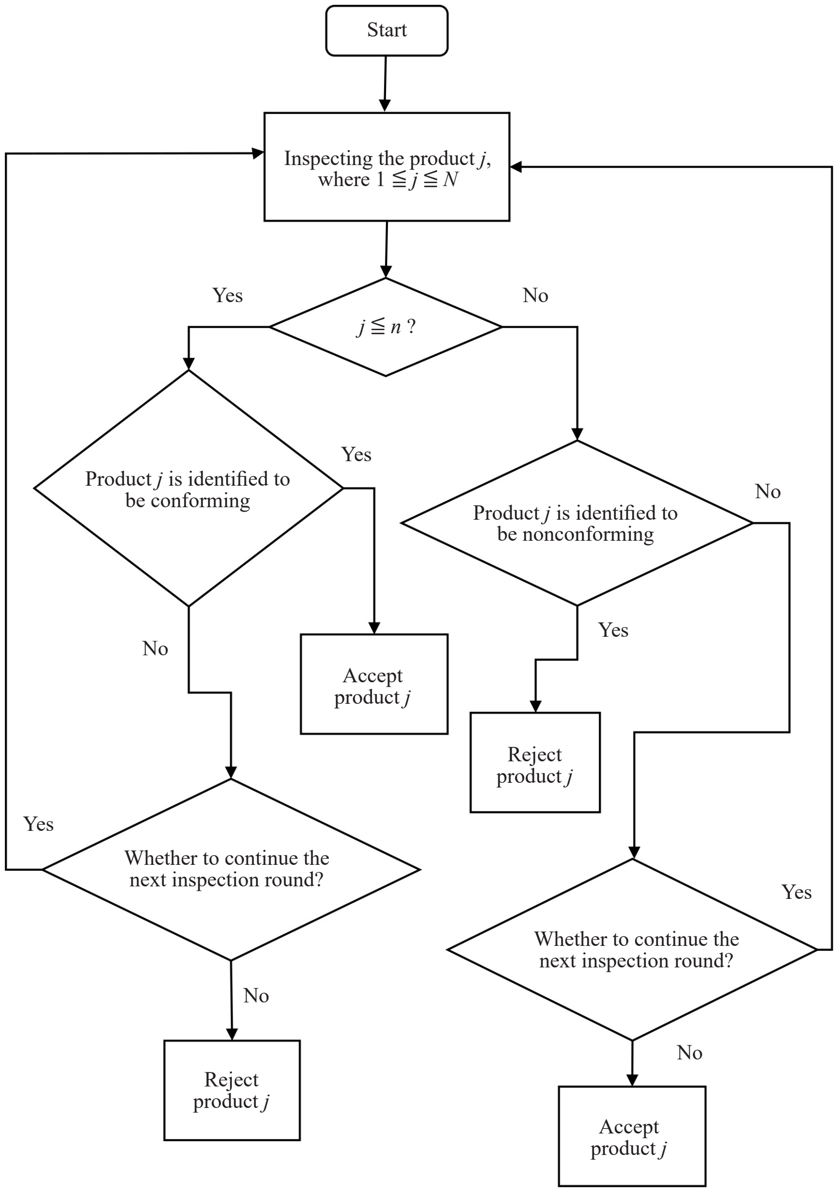

Figure 1. The proposed inspection decisions and outcomes

Table 1. The effect of our proposed inspection policy on the first n products

		After the k-th inspection round, the expected number of the inspected products identified to be	
	Before the k-th inspection round Conforming/Nonconforming	Nonconforming	Conforming
1st	$n\rho(n)$ $n(1-\rho(n))$	$\alpha n\rho(n)$ $n(1-\rho(n))(1-\beta)$	$(1-\alpha)n\rho(n)$ $n(1-\rho(n))\beta$
2nd	$\alpha n\rho(n)$ $n(1-\rho(n))(1-\beta)$	$\alpha^2 n\rho(n)$ $n(1-\rho(n))(1-\beta)^2$	$(1-\alpha)\alpha n\rho(n)$ $n(1-\rho(n))\beta(1-\beta)$
3rd			
k-th	$\alpha^{k-1}n\rho(n)$ $n(1-\rho(n))(1-\beta)^{k-1}$	$\alpha^k n\rho(n)$ $n(1-\rho(n))(1-\beta)^k$	$(1-\alpha)\alpha^{k-1}n\rho(n)$ $n(1-\rho(n))\beta(1-\beta)^{k-1}$

For the first n products in a production lot with size of N, the expected number of conforming and nonconforming products are given by $n\rho(n)$ and $n(1-\rho(n))$, respectively. If the first n products are chosen to be inspected, then a product should be inspected at the k-th round of inspection if it has been found to be nonconforming at the $k-1$-th inspection round for $k = 2, 3, \cdots$.

After k inspection rounds, the number of conforming products that will be correctly accepted is given by (refer to Table 1)

$$\vartheta(n,k)=\begin{cases} n\rho(n),\, k=0, \\ n\rho(n)(1-\alpha)\Sigma_{j=1}^{k}\alpha^{j-1}, k=1,\ 2,\cdots, \end{cases} \tag{2}$$

where $k = 0$ is the case of no inspection. When no inspection is considered, the cost of incorrect acceptance and rejection for the first n products are given by $c_a n(1-\rho(n))$ and zero, respectively. On the other hand, if inspection is considered, then after the k-th inspection round, the resulting expected total related quality control cost, including the costs of inspection, incorrect reject, and acceptance, becomes (refer to Table 1):

$$nc_I + \left\{ n\rho(n)\Sigma_{j=1}^{k-1}\alpha^j + n(1-\rho(n))\Sigma_{j=1}^{k-1}(1-\beta)^j \right\} c_I + n\rho(n)c_r\alpha^k + c_a n(1-\rho(n))\beta\ \Sigma_{j=1}^{k}(1-\beta)^{j-1}. \tag{3}$$

After the k-th inspection round is performed for the first, if performing the $k+1$-th inspection round cannot significantly reduce the number of conforming products incorrectly rejected, then one should cease inspection at the k-th inspection round. Thus, an upper boundary for k is regarded as the smallest k that satisfies the following inequality: $n\rho(n)\alpha^k - n\rho(n)\alpha^{k+1} \le \varepsilon_1$, where ε_1 is a pre-determined small positive value, or equivalently,

$$\bar{k}(n;\alpha,\varepsilon_1) = \left\lceil \text{In}\left(\frac{\varepsilon_1}{(1-\alpha)n\rho(n)}\right) / \text{In}(\alpha) \right\rceil. \tag{4}$$

On the other hand, Table 2 shows that after the i-th inspection round, the number of conforming products that would be correctly accepted is given by

$$(1-\alpha)^i(N\rho(N)-n\rho(n)) \text{ for } i=0,\ 1,\ 2,\ \ldots, \tag{5}$$

where $i = 0$ represents the case of no inspection. When i inspection round is performed, the resulting expected total quality related control cost for products $n+1$ to $N-n$ is given by $c_a[N-n-(N\rho(N)-n\rho(n))]$ for $i = 0$ and

$$(N-n)c_{\mathrm{I}}+c_{\mathrm{I}}\left\{\left(N\rho(N)-n\rho(n)\right)\Sigma_{j=1}^{i-1}(1-\alpha)^{j}+\left[N-n-\left(N\rho(N)-n\rho(n)\right)\right]\Sigma_{j=1}^{i-1}\beta^{j}\right\}+c_{\mathrm{r}}\alpha\left(N\rho(N)-n\rho(n)\right)\Sigma_{j=0}^{i-1}(1-\alpha)^{j}+ c_{\mathrm{a}}\left[N-n-\left(N\rho(N)-n\rho(n)\right)\right]\beta^{i} \text{ for } i=1,\ 2,\ \ldots, \tag{6}$$

respectively.

Table 2. The effect of our proposed inspection policy on the last $N-n$ products

		After the i-th inspection round, the expected number of the inspected products identified to be	
	Before the i-th inspection round	Nonconforming	Conforming
	Conforming/Nonconforming		
1st	$N\rho(N)-n\rho(n)$ $N-n-(N\rho(N)-n\rho(n))$	$\alpha(N\rho(N)-n\rho(n))$ $[N-n-(N\rho(N)-n\rho(n))](1-\beta)$	$(1-\alpha)(N\rho(N)-n\rho(n))$ $[N-n-(N\rho(N)-n\rho(n))]\beta$
2nd	$(1-\alpha)(N\rho(N)-n\rho(n))$ $[N-n-(N\rho(N)-n\rho(n))]\beta$	$\alpha(1-\alpha)(N\rho(N)-n\rho(n))$ $[N-n-(N\rho(N)-n\rho(n))]\beta(1-\beta)$	$(1-\alpha)^2(N\rho(N)-n\rho(n))$ $[N-n-(N\rho(N)-n\rho(n))]\beta^2$
3rd			
i-th	$(1-\alpha)^{i-1}(N\rho(N)-n\rho(n))$ $[N-n-(N\rho(N)-n\rho(n))]\beta^{i-1}$	$\alpha(1-\alpha)^{i-1}(N\rho(N)-n\rho(n))$ $[N-n-(N\rho(N)-n\rho(n))]\beta^{i-1}(1-\beta)$	$(1-\alpha)^{i}(N\rho(N)-n\rho(n))$ $[N-n-(N\rho(N)-n\rho(n))]\beta^{i}$

After the i-th inspection round is performed for the last $N-n$ products, if performing the $i+1$-th inspection round cannot significantly reduce the number of nonconforming products incorrectly accepted, then one should cease inspection at the i-th inspection round. Having used a similar approach to determine $\bar{k}$, one can set an upper bound of i as $\bar{i}$, where $\bar{i}$ is the smallest integer i that satisfies the following inequality: $[N-n-(N\rho(N)-n\rho(n))]\beta^{i}-[N-n-(N\rho(N)-n\rho(n))]\beta^{i+1}\le\varepsilon_2$, where ε_2 is a small positive value. Alternatively,

$$\bar{i}\,(N,n;\beta,\varepsilon_2)=\left\lceil \mathrm{In}\left(\frac{\varepsilon_2}{\left[N-n-\left(N\rho(N)-n\rho(n)\right)\right](1-\beta)}\right)/\,\mathrm{In}(\beta)\right\rceil. \tag{7}$$

Consider a production lot with size of N with an inspection policy, (n, k, i), where $N\ge n$. Then, the expected number of conforming products correctly accepted (denoted by $\phi(N, n, k, i)$) can be obtained by adding equation (2) and equation (5), which gives

$$\phi(N,n,k,i)=\vartheta(n,k)+(1-\alpha)^{i}\left(N\rho(N)-n\rho(n)\right), \tag{8}$$

for $k = 0, 1, 2, \ldots, \bar{k}$ and $i = 0, 1, 2, \ldots, \bar{i}$, where $\vartheta(n, k)$ is given in equation (2). The expected total revenue is given by $r\phi(N, n, k, i)$.

In addition, adding equation (3) and equation (6) gives the expected total quality control related cost (denoted by $\psi(N, n, k, i)$), which is comprised of the costs of inspection, incorrect reject and acceptance:

$$\psi(N,n,k,i)=Nc_{\mathrm{I}}+\left\{n\rho(n)\Sigma_{j=1}^{k-1}\alpha^{j}+n(1-\rho(n))\Sigma_{j=1}^{k-1}(1-\beta)^{j}\right\}c_{\mathrm{I}}+n\rho(n)c_{\mathrm{r}}\alpha^{k}+c_{\mathrm{a}}n(1-\rho(n))\beta\Sigma_{j=1}^{k}(1-\beta)^{j-1}+c_{\mathrm{I}} \left\{\left(N\rho(N)-n\rho(n)\right)\Sigma_{j=1}^{i-1}(1-\alpha)^{j}+\left[N-n-\left(N\rho(N)-n\rho(n)\right)\right]\Sigma_{j=1}^{i-1}\beta^{j}\right\}+c_{\mathrm{r}}\alpha\left(N\rho(N)-n\rho(n)\right)\Sigma_{j=0}^{i-1}(1-\alpha)^{j}+c_{\mathrm{a}} \left[N-n-\left(N\rho(N)-n\rho(n)\right)\right]\beta^{i}, \tag{9}$$

for k = 0, 1, 2, …, $\overline{k}$ and i = 0, 1, 2, …, $\overline{i}$. Consequently, the expected total cost (TC) per conforming product (denoted by TC(N, n, k, i)), which includes the costs of process setup/maintenance, restoration, quality control, manufacturing, revenue, and inventory holding:

$$\mathrm{TC}(N,n,k,i)=\frac{c_{\mathrm{S}}+c_{\mathrm{R}}(1-\overline{P}_N)+\psi(N,n,k,i)+Nc_{\mathrm{m}}}{\phi(N,n,k,i)}+r+c_{\mathrm{h}}\phi(N,n,k,i)/(2D), \tag{10}$$

where $\phi(N, n, k, i)$ and $\psi(N, n, k, i)$, are represented by equation (8) and equation (9), respectively.

The main objective of this paper is to find an optimal production lot size N^* and its associated optimal inspection policy (n^*, k^*, i^*) to minimize the cost function given in equation (10). Since it is not easy to obtain a closed-form solution for (N^*, n^*, k^*, i^*), a searching procedure was conducted as follows:

A procedure for (N^*, n^*, k^*, i^*):

Step 1. For a given (N, n), where $n \le N$.

Step 2. Compute TC(N, n, k, i) for each (k, i), where k = 0, 1, 2, …, $\overline{k}$ and i = 0, 1, 2,…, $\overline{i}$.

Step 3. Varying (N, n) and go to Step 2 until an optimal (N, n, k, i) that minimizes TC(N, n, k, i) is obtained.

3. No inspection policy

In this section, the conditions for a commonly used inspection policy, no inspection, are investigated. It is obvious to see that the first n (last $N-n$) products are not worthy of performing k (i) inspection round if the following inequality holds (refer to equation (3)):

$$\begin{aligned} & c_{\mathrm{a}}n\big(1-\rho(n)\big)-rn\rho(n) \\ & \quad \le nc_{\mathrm{I}}+\left\{n\rho(n)\frac{\alpha-\alpha^k}{1-\alpha}+n\big(1-\rho(n)\big)\frac{1-\beta-(1-\beta)^k}{\beta}\right\}c_{\mathrm{I}}+n\rho(n)c_{\mathrm{r}}\alpha^k \\ & \quad +\ c_{\mathrm{a}}n\big(1-\rho(n)\big)(1-(1-\beta)^k)-rn\rho(n)(1-\alpha^k). \end{aligned}$$

Arranging the last equation gives

$$\frac{c_{\mathrm{a}}(1-\beta)^k-c_{\mathrm{I}}\left(1+\dfrac{1-\beta-(1-\beta)^k}{\beta}\right)}{c_{\mathrm{r}}\alpha^k+c_{\mathrm{a}}(1-\beta)^k+r\alpha^k+c_{\mathrm{I}}\left[\dfrac{\alpha-\alpha^k}{1-\alpha}-\dfrac{1-\beta-(1-\beta)^k}{\beta}\right]}\le\rho(n). \tag{11}$$

On the other hand, for the last $N-n$ products, let $\lambda(N,n)=\dfrac{N\rho(N)-n\rho(n)}{N-n}$, then, no inspection is more attractive than performing i inspection round if the following inequality holds (refer to equation (6)):

$$\begin{aligned} & c_{\mathrm{a}}\left[1-\lambda(N,n)\right]-r\lambda(N,n) \\ & \quad \le c_{\mathrm{I}}+c_{\mathrm{I}}\left\{\lambda(N,n)\frac{1-\alpha-(1-\alpha)^i}{\alpha}+\left[1-\lambda(N,n)\right]\frac{\beta-\beta^i}{1-\beta}\right\}+c_{\mathrm{r}}\lambda(N,n)\left(1-(1-\alpha)^i\right) \\ & \quad +c_{\mathrm{a}}\left[1-\lambda(N,n)\right]\beta^i-r(1-\alpha)^i\lambda(N,n). \end{aligned}$$

The last equation can be further arranged as follows:

$$\frac{\frac{1-\beta^{i}}{1-\beta}\left[c_{\mathrm{a}}(1-\beta)-c_{\mathrm{I}}\right]}{r\left[1-(1-\alpha)^{i}\right]+\frac{c_{\mathrm{I}}}{\alpha}\left(1-\alpha-(1-\alpha)^{i}\right)+c_{\mathrm{r}}\left(1-(1-\alpha)^{i}\right)+\frac{1-\beta^{i}}{1-\beta}\left[c_{\mathrm{a}}(1-\beta)-c_{\mathrm{I}}\right]} \leq \lambda(N,n). \tag{12}$$

As a result, if equation (11) and equation (12) are satisfied for $k = 0, 1, 2, \ldots, \bar{k}$ and $i = 0, 1, 2, \ldots, \bar{i}$, respectively, then no inspection is optimal. In this case, the cost function of no inspection can be obtained by the following formula (refer to Tables 1 and 2):

$$\mathrm{TC}_{\mathrm{no}}(N)=\frac{c_{\mathrm{s}}+c_{\mathrm{R}}\left(1-\bar{P}_{N}\right)+c_{\mathrm{a}}N\left(1-\rho(N)\right)+Nc_{\mathrm{m}}}{N\rho(N)}-r+c_{\mathrm{h}}N\rho(N)/(2D), \tag{13}$$

where $\rho(N)$ is given in equation (1). Using the inequality of arithmetic and geometric means gives a lower boundary for the no inspection cost:

$$\mathrm{TC}_{\mathrm{no}}(N) \geq -r+2\sqrt{\frac{c_{\mathrm{s}}+c_{\mathrm{R}}\left(1-\bar{P}_{N}\right)+c_{\mathrm{a}}N\left(1-\rho(N)\right)+Nc_{\mathrm{m}}}{2D/c_{\mathrm{h}}}}.$$

4. Numerical examples

In this numerical section, an illustrated example is given by using the process reliability $\bar{P}_j = \bar{q}^{j^{\gamma}}$ (see Nakagawa & Osaki, 1975) to investigate the effect of inspection errors on the optimal production lot size and inspection policy with the following nominal values: $\bar{q} = 0.97$, $\gamma = 1.05$, $\theta_1 = 0.92$, $\theta_2 = 0.07$, $\varepsilon_1 = \varepsilon_2 = 0.05$, $\alpha = 0.06$, $\beta = 0.04$, $c_S = 60$, $c_R = 5.5$, $c_m = 2.5$, $c_I = 1.2$, $r = -12$, $c_a = 2.5$, $c_r = 12$, $c_h = 3.5$ and $D = 195$.

When the production rate goes to infinity, the classical economical production quantity (EPQ) can be obtained as $\sqrt{2c_S D/c_h} = 81.76$. Furthermore, when the imperfect process quality is considered without product inspection, one searches N over $\left[1, 1.83 \times \mathrm{EPQ} \approx 150\right]$ to minimize the expected total cost per conforming product given in equation (13). In this case, one obtains $TC_{no}(N^* = 21) = -0.53549$. When inspection errors are involved, we can apply our proposed solution procedure for (N^*, n^*, k^*, i^*) with the upper bounds of $\bar{k}$ and $\bar{i}$, which are given by equation (4) and equation (7), respectively. This gives $N^* = 26$, $n^* = 26$, $k^* = 2$ and $i^* = 0$, and $\mathrm{TC}(N^*, n^*, k^*, i^*) = -1.67905$ (see Figure 2), where the total quality related control cost takes up 11.4% of the total cost (see Table 3). This suggests that the optimal production lot size is located between the optimal lot size for no inspection (21), and the classical EPQ (81.76). Furthermore, 100% inspection is required, and for those products that have been inspected to be nonconforming, this means they should be inspected one more time. The benefit of adopting our proposed production and inspection model can be evaluated in terms of the cost improvement percentage (Imp.%) by comparing this with the case where there is no inspection, which is computed by $\frac{TC_{\mathrm{no}}\left(N^*=21\right)-\mathrm{TC}\left(N^*, n^*, k^*, i^*\right)}{\left|TC_{\mathrm{no}}\left(N^*=21\right)\right|} \times 100\% = 213.55\%$. On the other hand, for a larger production batch size, e.g., $N^* = 249$, one has $n^* = 41$, $k^* = 2$ and $i^* = 1$ with a resulting cost of 14.70056. This shows that the products in the initial part of the production run (i.e., products 1 to 41) and in the rear part of the production run (i.e., products 42 to 249) requires two rounds and one round of inspection, respectively, in order to diminish the number of conforming (nonconforming) products that would be wrongly rejected (accepted). This is different from the inspection policy proposed by other authors such as Wang et al. (2004) and Wang (2005), where inspection only focuses on the late-produced products of a production lot.

Table 3. The cost distribution for the optimal solution

	Cost/Revenue	Cost	Revenue %
c_S	60	40.84%	
$c_R(1-\bar{q}^N)$	3.3344	2.27%	
Nc_m	65	44.25%	
$r\phi(N, n, k, i)$	-170.8		100%
$\psi(N, n, k, i)$	16.74906	11.40%	
$c_h(\phi(N, n, k, i))^2/(2D)$	1.8181	1.24%	
	-23.8987		

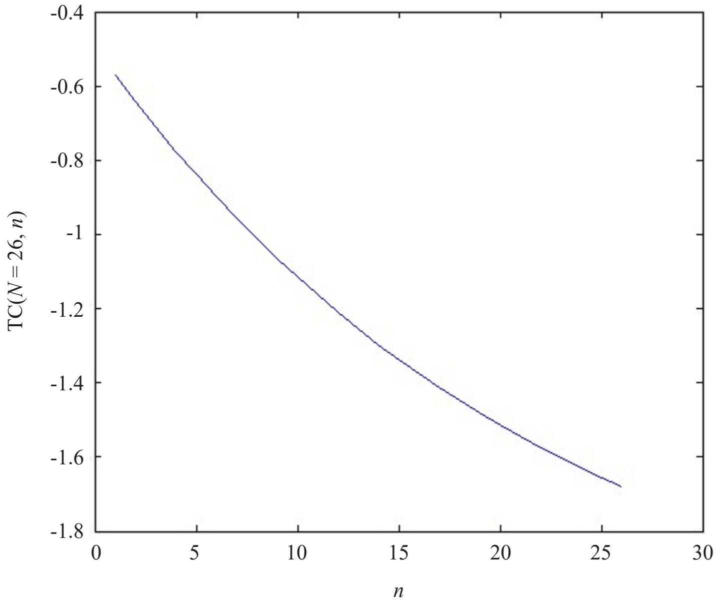

Figure 2. The cost changes for different values of n when $N = 26$

To determine the effect of inspection errors on the optimal solution, the type I and type II inspection errors were varied, α and β, from zero to 0.4 with the other parameter values fixed, and their results are summarized in Table 4 and 5, respectively.

From Table 4, we can make the following observations: when α increases, both the production lot size and Imp.% are non-increasing, and the resulting cost is non-decreasing. Furthermore, if $\alpha = 0$ (with $\beta = 0.4$), then all products ought to be inspected once. For $0.02 \leq \alpha \leq 0.36$, all products should be inspected and once a product has been found to be nonconforming, it should be inspected again. The number of inspection rounds increases with the value of α. Nevertheless, when the type I error is overly large (i.e., $\alpha \geq 0.38$), the no inspection approach is attractive.

From Table 5, as β increases, the Imp.% decreases. In addition, full inspection is always optimal, and both the production batch size and inspection rounds are non-increasing as β increases. Note that no inspection is not attractive even for a larger type II error is up to 0.4.

Table 4. The effect of α on the optimal solution

α	N^*	n^*	k^*	i^*	TC	Imp.%
0	27	0	0	1	-1.89097	253.13%
0.02	26	26	2	0	-1.78868	234.03%
0.04	26	26	2	0	-1.73795	224.55%
0.06	26	26	2	0	-1.67906	213.55%
0.08	26	26	2	0	-1.61180	200.99%
0.1	26	26	3	0	-1.55158	189.75%
0.12	26	26	3	0	-1.49683	179.52%
0.14	26	26	3	0	-1.43748	168.44%
0.16	25	25	3	0	-1.37353	156.50%
0.18	25	25	3	0	-1.30397	143.51%
0.2	25	25	3	0	-1.22806	129.33%
0.22	25	25	4	0	-1.16028	116.68%
0.24	25	25	4	0	-1.08790	103.16%
0.26	25	25	4	0	-1.00975	88.57%
0.28	25	25	4	0	-0.92514	72.77%
0.3	25	25	4	0	-0.83330	55.61%
0.32	24	24	5	0	-0.74432	39.00%
0.34	24	24	5	0	-0.65056	21.49%
0.36	24	24	5	0	-0.54892	2.51%
0.38	21	0	0	0	-0.53549	0.00%
0.4	21	0	0	0	-0.53549	0.00%

Table 5. The effect of β on the optimal solution

β	N^*	n^*	k^*	i^*	TC	Imp.%
0	27	27	3	0	-1.82758	241.29%
0.02	26	26	2	0	-1.73883	224.72%
0.04	26	26	2	0	-1.67906	213.56%
0.06	26	26	2	0	-1.62051	202.62%
0.08	26	26	2	0	-1.56320	191.92%
0.1	25	25	2	0	-1.50849	181.70%
0.12	25	25	2	0	-1.45561	171.83%
0.14	25	25	2	0	-1.40393	162.18%
0.16	25	25	2	0	-1.35343	152.75%
0.18	24	24	2	0	-1.30417	143.55%
0.2	24	24	2	0	-1.25783	134.89%
0.22	24	24	2	0	-1.21264	126.45%
0.24	24	24	2	0	-1.16859	118.23%
0.26	24	24	2	0	-1.12568	110.21%
0.28	24	24	2	0	-1.08392	102.42%
0.3	24	24	2	0	-1.04330	94.83%
0.32	23	23	2	0	-1.00529	87.73%
0.34	23	23	2	0	-0.96843	80.85%
0.36	23	23	2	0	-0.93267	74.17%
0.38	23	23	2	0	-0.89800	67.70%
0.4	23	23	2	0	-0.86444	61.43%

When the process quality $\overline{q}$ increases from 0.75 to 0.99, then N^* increases with $\overline{q}$, and $n^* = N^*$, $k^* = 2$ and $i^* = 0$. This means that a higher process reliability will allow a larger production lot size since it produces fewer nonconforming products. In addition, the associated cost, TC(N^*, n^*, $k^* = 2$, $i^* = 0$), decreases with the increase in $\overline{q}$ (see Figure 3) as expected. Note that only when the process reliability is good enough (i.e., $\overline{q} \geq 0.96$), one has positive revenue.

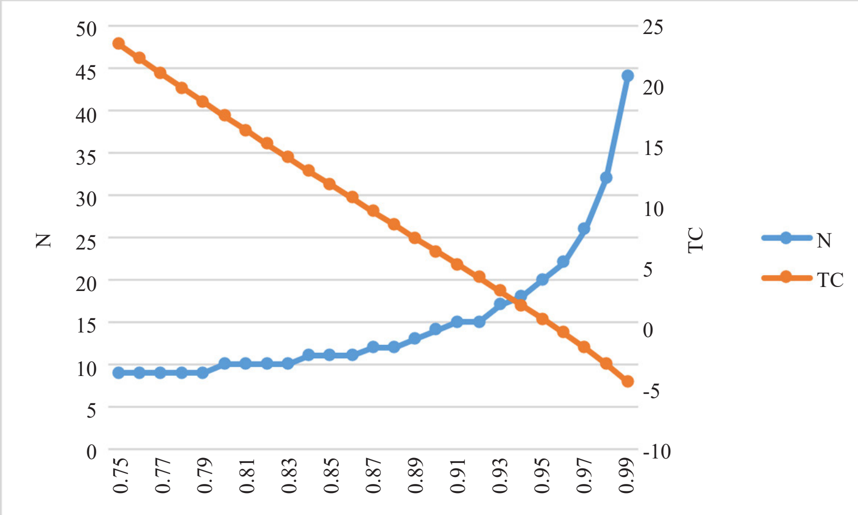

Figure 3. The effect of $\overline{q}$ on the optimal solution

5. Conclusion

When one attempts to determine the optimal production lot size from an imperfect process to achieve a minimization of costs or a maximization of profit, it is necessary to consider how to balance the costs of the various parts of the process, namely, setup/maintenance, production, quality control, and inventory. However, an optimal quality control policy is not easy to establish, especially when the two types of inspection errors exist. This is because type I inspection error will result in the rejection of conforming products, and type II error will result in nonconforming products being released onto the market, which will incur, for example, warranty costs (e.g., see Sarkar & Saren, 2016). In this paper, we propose a heuristic inspection policy by exploring the propensities of a deteriorating process, namely, one that it has a higher chance of shifting from an IN state to an OUT state towards the end of a production lot and thus begin to produce more nonconforming products from that point on. The aim of our inspection policy is to determine the point in the profit of the production lot where products prior to (after) the boundary point should be inspected several times if, on inspection, they are found to be nonconforming (conforming).

A numerical example is used to illustrate how it is possible to coordinate the production lot size and inspection policy to achieve a reduced cost when using our proposed production and inspection policy. Specifically, a production lot size larger than the case for no inspection is suggested. However, if the type I inspection error rate is too large, then no inspection is optimal. In such a case, a decision regarding investment in quality control (e.g., see Yoo et al., 2012) needs to be made with the aim of improving profits, or equivalently, reducing costs.

Future research issues associated with our proposed inspection policy are as follows: firstly, the effect that learning (e.g., see Jaber et al., 2008) has on reducing the probability of an inspection error occurring. Secondly, how the use of information obtained from the inspection of earlier products can facilitate decisions regarding optimal production lot size (e.g., see Shih et al., 2018). Thirdly, how the use of a multistage production system (e.g., Ben-Daya & Rahim, 2003) affects the optimal production lot size at each stage of production.

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Conflict of interest statement

We have no conflict of interest to disclosure.

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