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Research Article

Pure-Cubic Optical Soliton Perturbation Having Cubic-Quintic Law of Self-Phase Modulation via the New Kudryashov Integration Scheme

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Abstract: This paper secures dark and bright pure-cubic optical soliton solution to the perturbed third-order nonlinear Schrödinger's equation having the cubic-quintic law nonlinearity of self-phase modulation with a couple of self-frequency shift and nonlinear dispersion perturbation terms which related to the Hamiltonian. The recently introduced new Kudryashov integration scheme is used to retrieve these soliton solutions. The suitable parameter selections for the existence of such solitons are also presented. In the second stage, the effects of these terms on the soliton forms are investigated by the cubic-quintic form of the problem under study. The findings obtained are supported by 3D and 2D figures and necessary explanations are presented.

***Keywords*:** pure-cubic soliton, absence of chromatic dispersion, cubic-quintic law, the new Kudryashov scheme

1. Introduction

The use of nonlinear partial differential equations (NLPDE) for describing many physical phenomena today is an undeniable fact. Although the "soliton", introduced into the literature by J. S. Russell based on a chance observation and later named by Zabusky and Kruskal, holds a unique significance in such investigations, the solutions of these equations are equally important. Nowadays, different models related to NLPDE are available in many fields. For example, in [1], the CBS-BK model, formulated as a combination of the Calogero-Bogoyavlenskii Schiff (CBS) equation important in plasma physics and the Bogoyavlensky Konopelchenko (BK) equation significant for Riemann wave propagation was studied using both the multiple Exp-function and the Hirota bilinear technique, yielding multiple and stripe soliton solutions. In [2], the study examined the cubic and quintic forms of the Helmholtz equation an important heat diffusion model using the $\tan(\phi/2)$-expansion and the $\exp(-\Omega(\eta))$-expansion methods, and obtained kink solitary wave solutions. Many studies can be cited as examples [3–10]: beginning with investigations of water waves, later gaining momentum through quantum mechanics, and currently including unique models and solutions in the field of nonlinear optics.

The basic physics of soliton propagation is the existence and sustainment of the delicate balance between chromatic dispersion (CD) and self-phase modulation (SPM). In the context of nonlinear optics, the soliton is a fundamental phenomenon that emerges as a result of the interaction between the (group velocity dispersion term (GVD) or CD) and SPM. Initially, based on the form of the Schrödinger equation introduced into the literature, this phenomenon could be

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briefly described as an interaction between "CD and Kerr law nonlinearity". However, since the early 2,000 s, different forms of SPM have also been introduced into the literature. These include power, parabolic, dual power, anti-cubic, polynomial, triple power, and Kudryashov law forms. Naturally, models developed based on these different SPM forms and the studies associated with these models also hold a significant place in the literature. It may so happen that during the propagation of such solitons, a CD could be running low. In such a situation, it is advisable to discard the CD and replace it with third-order dispersion (3OD). This would provide the necessary balance that would enable the solitons to sustain during its propagation. One of the inherent drawbacks of including 3OD is the effect of soliton radiation and the slow-down of solitons during its propagation. This paper overlooks this pair of issues and focuses on the integrability of the model in the presence of perturbation terms. The governing nonlinear Schrödinger's equation (NLSE) with 3OD and without perturbation terms is not integrable analytically. Therefore, the inclusion of self-frequency shifts would make the model integrable. The new Kudryashov approach is the integration algorithm that has been implemented in the work to carry out the integration of the model. The recovered soliton solutions appear with the parametric constraints that are also presented.

1.1 *Governing model*

This paper focuses on the following pure-cubic NLSE, in its dimensionless form in the absence of chromatic dispersion having the third order dispersion [11] by adopting the parabolic law nonlinearity as:

$$i\frac{\partial q}{\partial t}+ia\frac{\partial^3 q}{\partial x^3}+\left(b_1|q|^2+b_2|q|^4\right)q=i\left(c_1|q|^2\frac{\partial q}{\partial x}+c_2\frac{\partial |q|^2}{\partial x}q\right), \tag{1}$$

where $q=q(x,\ t),\ i=\sqrt{-1}$ unit complex number. The terms, $iq_i,\ iaq_{xxx},\ b_1|q|^2q,\ b_2|q|^4q,\ ic_1|q|^2q_x$ and $ic_2\left(|q|^2\right)_x q$ are the terms that represent temporal evolution, the third order dispersion, parabolic law of self-phase modulation, and soliton self-frequency shift, respectively. All terms are with real-valued coefficients.

As emphasized above, although the soliton is a form arising from the interaction between CD and SPM and holds particular importance for data transfer and communication in the context of nonlinear optics research on optical fibers has inevitably increased over the last quarter century, given that the internet and intercontinental communication have become an integral part of daily life and the use of social media tools among individuals and communities has surged. Many of these investigations have involved supporting the findings of various mathematically developed models with experimental validation in laboratory settings. One such experimental study concerns soliton transmission in optical fibers in the absence of CD. After the concept of pure-quartic soliton was introduced to the literature with an experimental study in 2016 [12], significant studies were carried out and these studies were followed by studies on pure-cubic soliton. Such as Biswas et al presented the study under the title "Conservation laws for pure-cubic optical solitons with complex Ginzburg-Landau equation having several refractive index structures [13]". Zayed et al. conducted the study titled "Pure-cubic optical soliton perturbation with complex Ginzburg-Landau equation having a dozen nonlinear refractive index structures [14]". Albayrak et al. [15] includes the study titled "Pure-cubic optical solitons and stability analysis with Kerr law nonlinearity". In the study [16], pure-cubic soliton analysis was performed using the tan $(\phi/2)$-expansion technique. Studies on the pure-cubic soliton form have a wide range that is not only limited to the ones mentioned but also may include different forms of the pure-cubic nonlinear Schrödinger equation (fractional, stochastic, etc.).

The article is divided into the following parts: Section 2, includes the mathematical model of Eq. (1) in nonlinear ordinary differential form. Section 3 presents the algorithms of the method and application. Section 4 covers the Result and Discussion and Section 5 is the Conclusion.

2. Mathematical analysis

In order to integrate Eq. (1) to search for its soliton solutions, the following phase-amplitude split-up is considered:

$$q(x,\, t) = U(\xi) e^{i(-\alpha x + \beta t + \varphi_0)},\ \xi = kx - vt, \tag{2}$$

where $U(\xi),\ \beta,\ \alpha,\ v,\ \varphi_0,\ k$ corresponded to real-valued pulse profile, frequency, wave number, velocity, phase constant, and wave width, respectively. Besides, $\beta,\ \alpha,\ v,\ \varphi_0,\ k$ are positioned as non-zero arbitrary values.

By the aid of Eq. (2), Eq. (1) turns into:

$$ak^3 U''' - \left(k(c_1 + 2c_2) U^2 + 3a\alpha^2 k + v\right) U' = 0, \tag{3}$$

$$b_2 U^5 + (2\alpha c_2 + b_1) U^3 - \left(a\alpha^3 + \beta\right) U + 3a\alpha k^2 U'' = 0, \tag{4}$$

where $U = U(\xi)$ and ['] denotes the derivative with respect to "ξ". Eqs. (3), (4) represent the imaginary and real components of the obtained nonlinear ordinary equation. Integrating the Eq. (3) with respect to "ξ" and omitting the integration constant, Eq. (3) shape up as follows:

$$3ak^3 U'' - k(c_1 + 2c_2) U^3 - 3\left(3a\alpha^2 k + v\right) U = 0. \tag{5}$$

Since the Eq. (5) has the solution as:

$$U(\xi) = e^{-\left(\frac{3a\alpha^2 k+v}{ak^3}\right)^{1/2}\xi} \text{ or } U(\xi) = e^{\left(\frac{3a\alpha^2 k+v}{ak^3}\right)^{1/2}\xi}, \tag{6}$$

one can write the following constraints:

$$(3a\alpha^2 k + v) ak^3 > 0, \tag{7}$$

$$c_1 + 2c_2 = 0. \tag{8}$$

In Eq. (4), application of the homogeneous balance principle between the terms U'' and U^5 yields $m+2 = 5m$, results $m = 1/2$. Since m is considered as the positive integer balancing constant, we should construct the following algebraic formula:

$$U(\xi) = \sqrt{V(\xi)}. \tag{9}$$

Inserting the Eq. (8) into Eq. (4) we reach the following ordinary differential equation (ODE):

$$6a\alpha k^2 V V'' - 3a\alpha k^2 \left(V'\right)^2 - 4\left(a\alpha^3 + \beta - (2\alpha c_2 + b_1) V - b_2 V^2\right) V^2 = 0. \tag{10}$$

By the virtue of homogeneous balance rule between the terms VV'' and V^4, yields $m+(m+2)=4m$, which generates $m=1$.

3. Basic algorithm of the method and application

Undoubtedly, there are dozens of methods in the literature developed for solving different forms of NLPDEs (for instance, variations of the G'/G-expansion methods, the simplest equation method, the unified Riccati equation expansion method, the inverse scattering method, the Hirota bilinear form, Lie symmetry analysis, the Cole-Hopf transformation, the Arnous method, the Bäcklund transformation, the extended hyperbolic function method, mapping methods, the direct algebraic method and its subversions, the Jacobi elliptic method, and many others). Since all of these can be easily identified through a simple literature search, we have not deemed it necessary to cite them individually.

Among the methods that can be added to this list are the Kudryashov-based methods, introduced to the literature in 2012 by the distinguished scientist Kudryashov in his work titled "One method for finding exact solutions of nonlinear differential equations", and subsequently further developed (e.g., the generalized Kudryashov method, the extended Kudryashov method, the improved Kudryashov method, the modified Kudryashov method, the new Kudryashov method, etc.). Of these, the "new Kudryashov method" [17], which has been introduced in recent years, stands out due to its widespread use by many reputable researchers. Its main features include the ability to generate fundamental soliton forms such as bright, dark, and kink solitons without excessive algebraic computation, as well as its ease of application.

In this section, the basic algorithm of the new Kudryashov scheme [17] approach is presented. Suppose that the following form is the solution of Eq. (10):

$$V(\xi)=\sum_{i=0}^{m} A_i R^i(\xi),\ A_m \neq 0, \tag{11}$$

where $A_0,\ A_1,\ \ldots,\ A_m$ are the real coefficients to be determined. By the virtue of $m=1$, Eq. (11) falls into:

$$V(\xi)=A_0+A_1 R(\xi), \tag{12}$$

where $R(\xi)$ is the solution of the following differential formula:

$$\frac{dR(\xi)}{d\xi}-\sqrt{\delta^2 R^2(\xi)\left(1-\chi R^2(\xi)\right)}=0, \tag{13}$$

where $\delta,\ \chi$ are non-zero arbitrary real numbers and Eq. (13) permits us to write:

$$R(\xi)=\frac{4L}{4L^2 e^{\delta\xi}+\chi e^{-\delta\xi}}, \tag{14}$$

in which L is non-zero free real numbers.

One can organize Eq. (14) in hyperbolic structure as follows:

$$R(\xi)=\frac{4L}{(4L^2+\chi)\,cosh(\delta\xi)+(4L^2-\chi)\,sinh(\delta\xi)}. \tag{15}$$

Equations (14), (15) represent the bright-singular straddled optical solitons. The special cases emerge as follows: For $\chi = +\,4L^2$, Eqs. (14), (15) depict the bright soliton:

$$R(\xi) = \frac{1}{2L} sech(\delta\xi). \tag{16}$$

For $\chi = -4L^2$, Eqs. (14), (15) stick out the singular solution:

$$R(\xi) = \frac{1}{2L} csch(\delta\xi). \tag{17}$$

Inserting Eq. (12) into Eq. (10), by considering the Eqs. (8), (13), gives us to reach the following system of algebraic equations which comes from the coefficients of $R^j(\xi),\ 0 \le j \le 4$, respectively:

$$\begin{aligned}
&\text{Coeff. of } R^0(\xi): -b_2A_0^2 + (-2ac_2 - b_1)A_0 + a\alpha^3 + \beta = 0,\\
&\text{Coeff. of } R^1(\xi): -3a\alpha\delta^2k^2 + 4a\alpha^3 - 12\alpha A_0c_2 - 8b_2A_0^2 - 6A_0b_1 + 4\beta = 0,\\
&\text{Coeff. of } R^2(\xi): -4a\alpha^3 + 3\left(a\delta^2k^2 + 8A_0c_2\right)\alpha + 24b_2A_0^2 + 12A_0b_1 - 4\beta = 0,\\
&\text{Coeff. of } R^3(\xi): -3a\alpha\chi\delta^2k^2A_0 + A_1^2(2\alpha c_2 + 4A_0b_2 + b_1) = 0,\\
&\text{Coeff. of } R^4(\xi): -9a\alpha\chi\delta^2k^2 + 4A_1^2b_2 = 0.
\end{aligned} \tag{18}$$

By solving the Eq. (18), we get the following possible solution sets:

$$\begin{aligned}
&Set_{1,\,2} = \left\{\begin{array}{l} a = \dfrac{-60\alpha^2c_2^2 - 60\alpha b_1c_2 - 64\beta b_2 - 15b_1^2}{64b_2\alpha^3},\ A_0 = \dfrac{-6\alpha c_2 - 3b_1}{8b_2},\\[2ex] \delta = -\dfrac{2(2\alpha c_2 + b_1)\alpha}{\sqrt{-60\alpha^2c_2^2 - 60\alpha b_1c_2 - 64\beta b_2 - 15b_1^2 k}},\ A_1 = \mp\dfrac{3\sqrt{\chi}(2\alpha c_2 + b_1)}{8b_2} \end{array}\right\},\\[2ex]
&Set_{3,\,4} = \left\{\begin{array}{l} a = \dfrac{-60\alpha^2c_2^2 - 60\alpha b_1c_2 - 64\beta b_2 - 15b_1^2}{64b_2\alpha^3},\ A_0 = \dfrac{-6\alpha c_2 - 3b_1}{8b_2},\\[2ex] \delta = \dfrac{2(2\alpha c_2 + b_1)\alpha}{\sqrt{-60\alpha^2c_2^2 - 60\alpha b_1c_2 - 64\beta b_2 - 15b_1^2 k}},\ A_1 = \mp\dfrac{3\sqrt{\chi}(2\alpha c_2 + b_1)}{8b_2} \end{array}\right\},
\end{aligned} \tag{19}$$

with the constraints $-60\alpha^2c_2^2 - 60\alpha b_1c_2 - 64\beta b_2 - 15b_1^2 > 0,\ b_2 \neq 0,\ k \neq 0,\ \alpha \neq 0,\ 2\alpha c_2 + b_1 \neq 0,\ \chi > 0$. Combination of Eq. (12) with Eqs. (2), (8), (9) and (15) gives the following solution:

$$q(x,\ t) = \left(A_0 + \frac{4A_1L}{(4L^2+\chi)\,cosh(\delta(kx-vt)) + (4L^2-\chi)\,sinh(\delta(kx-vt))} \right)^{1/2} e^{i(-ax+\beta t+\xi_0)}. \tag{20}$$

For $\chi = +\ 4L^2$, Eq. (20) gives the bright soliton:

$$q(x,\ t) = \left(A_0 + \frac{A_1}{2L} sech(\delta(kx-vt)) \right)^{1/2} e^{i(-\alpha x+\beta t+\varphi_0)}. \tag{21}$$

For $\chi = -\ 4L^2$, Eq. (20) represents a singular solution:

$$q(x,\ t) = \left(A_0 + \frac{A_1}{2L} csch(\delta(kx-vt)) \right)^{1/2} e^{i(-\alpha x+\beta t+\varphi_0)}. \tag{22}$$

The parameters $A_0,\ A_1,\ a,\ \delta$ in Eqs. (20), (21) and (22) will be shaped according to the selections to be made from the sets given in Eq. (19).

4. Result and discussion

This section is dedicated to the graphical representations of the solution functions derived from Eqs. (20)-(22) to support them from a graphical perspective. The dark soliton form presented in Figure 1a was obtained as a 3D visualization of the solution function derived from Eq. (21) by employing the parameter choices of Set_1 in Eq. (19), specifically $L = 1,\ \chi = 4,\ b_1 = 1,\ b_2 = 1,\ c_2 = -1.20,\ \alpha = 1,\ \beta = -1.25,\ k = 0.25,\ \varphi_0 = 0.5,\ v = 0.25$. The periodic form of the solution, which is expected according to Euler's formula, is visualized through its real part in Figure 1b. Figure 1c illustrates the contour plot, whereas Figure 1d includes 2D representations of $Re(q(x,\ t)),\ Im(q(x,\ t)),\ \left|q\left(x,\ t_f\right)\right|^2$. In Figure 1d, the green line represents $\left|q\left(x,\ t_f\right)\right|^2,\ t_f = 1,\ 2,\ 3$, which reflects the traveling wave property of the dark soliton. The red and black lines correspond to the periodic representations of $Re(q(x,\ 1))$ and $Im(q(x,\ 1))$, respectively, for $t = 1$. Similarly, Figure 2 represents the graphical results derived from Eq. (21), using the parameter choices of Set_3 from Eq. (19), specifically $L = 1,\ \chi = 4,\ b_1 = 1,\ b_2 = 1,\ c_2 = -1.20,\ \alpha = 1,\ \beta = -1.25,\ k = 0.25,\ \varphi_0 = 0.5,\ v = 0.25$. Figure 2a corresponds to the bright soliton form, Figure 2b visualizes the periodic form over the real part, Figure 2c provides the contour view, and finally, Figure 2d includes the 2D views of the bright solution. From both Figure 1 and 2, it is observed that the solution derived from Eq. (21) has the capacity to generate bright and dark solitons, which are crucial for nonlinear optics, in the absence of the chromatic dispersion term of the model presented in Eq. (1). Furthermore, the graphical results demonstrate that it is possible to obtain both soliton forms (bright and dark) using the appropriate solution set and parameter choices derived via the new Kudryashov method, indicating the method's effectiveness. Among the solutions represented by Eq. (21), graphical representations of singular soliton solutions were excluded since this form does not hold significant meaning in the context of nonlinear optics.

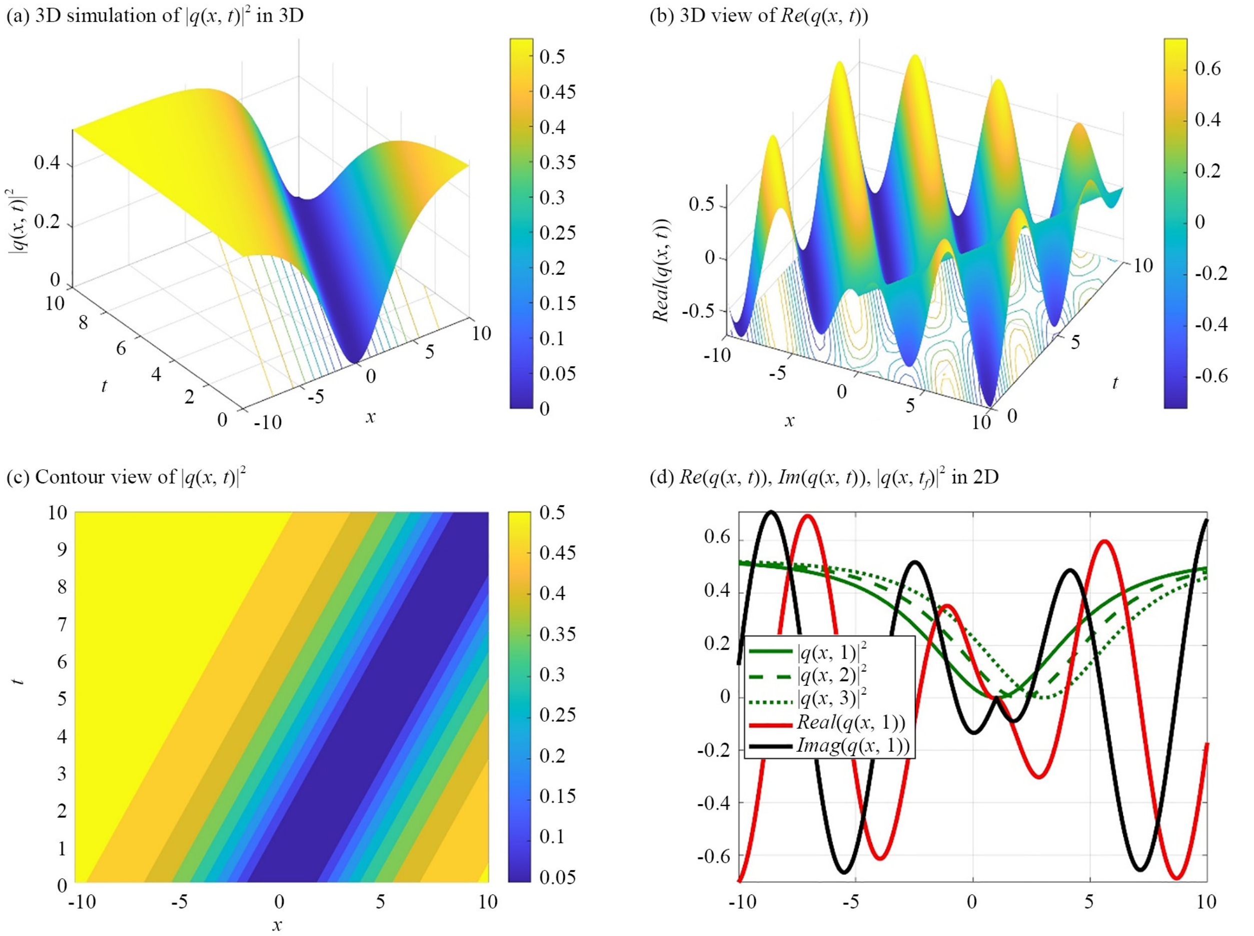
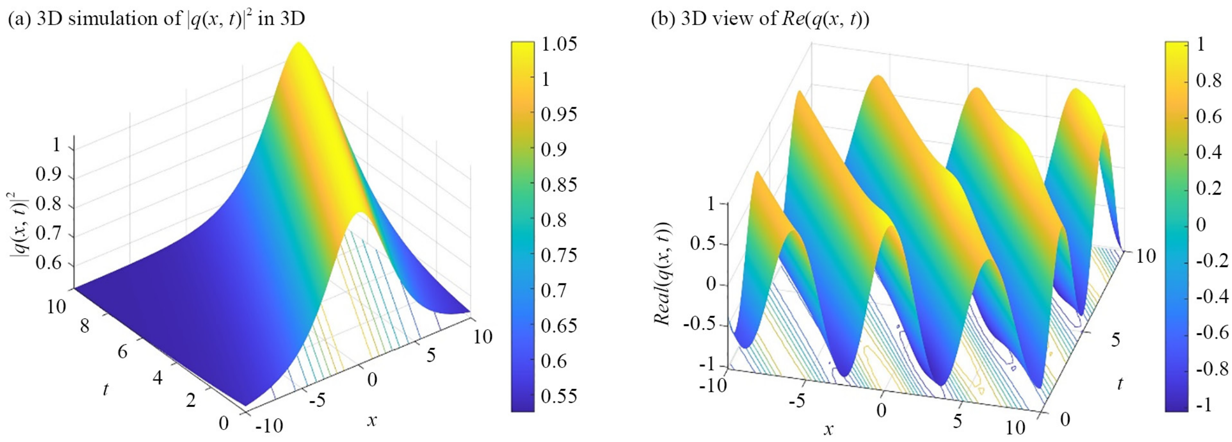


Figure 1. The dark soliton simulation of Eq. (21) with Set_1 in Eq. (19) and $\{L = 1,\ \chi = 4,\ b_1 = 1,\ b_2 = 1,\ c_2 = -1.20,\ \alpha = 1,\ \beta = -1.25,\ k = 0.25,\ \varphi_0 = 0.5,\ v = 0.25\}$



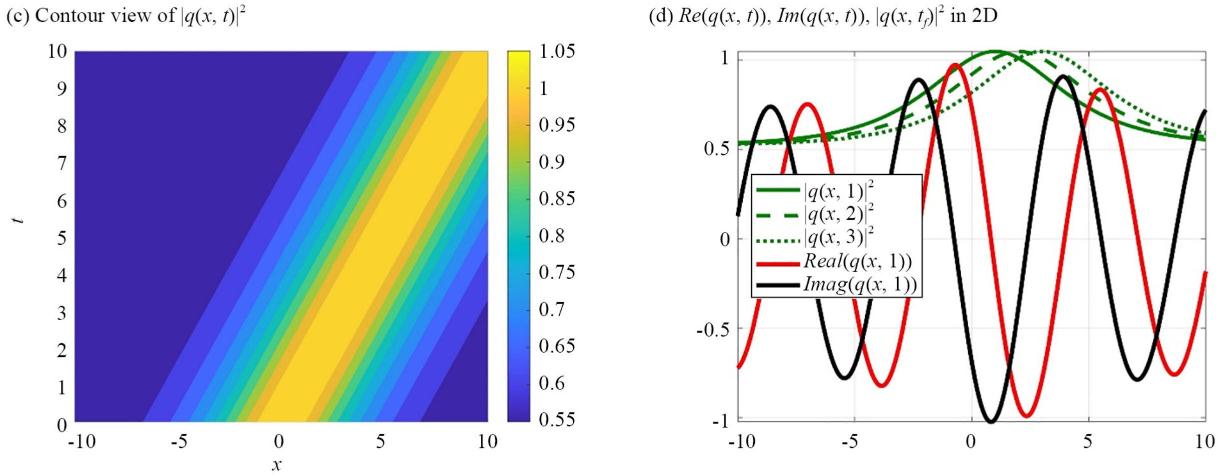


Figure 2. The bright soliton simulation of Eq. (21) with Set_3 in Eq. (19) and $\{L = 1,\ \chi = 4,\ b_1 = 1,\ b_2 = 1,\ c_2 = -1.20,\ \alpha = 1,\ \beta = -1.25,\ k = 0.25,\ \varphi_0 = 0.5,\ v = 0.25\}$

The graphs presented in Figure 3 and Figure 4 focus on examining how the parameters associated with the terms of the cubic law (Kerr law) and quintic law forms in the model referred to as the pure-cubic Schrödinger equation affect the resulting soliton forms. In this context, Figures 3 show the cases where the b_1 parameter representing the cubic law form takes negative and positive values, respectively, while Figures 4 illustrate the influence of the b_2 parameter, which represents the coefficient of the quintic law term. The parameter selections used to produce these graphs are also provided below.

Figure 3a: $\{L = 1,\ \chi = 4,\ b_2 = 1,\ c_2 = 1.20,\ \alpha = 1,\ \beta = -1.25,\ k = 0.25,\ \varphi_0 = 0.5,\ v = 0.25\}$.

Figure 3b: $\{L = 1,\ \chi = 4,\ b_2 = 1,\ c_2 = -1.20,\ \alpha = 1,\ \beta = -1.25,\ k = 0.25,\ \varphi_0 = 0.5,\ v = 0.25\}$.

Figure 4a: $\{L = 1,\ \chi = 4,\ b_1 = 1,\ c_2 = -1.20,\ \alpha = 1,\ \beta = 1.25,\ k = 0.25,\ \varphi_0 = 0.5,\ v = 0.25\}$.

Figure 4b: $\{L = 1,\ \chi = 4,\ b_1 = 1,\ c_2 = -1.20,\ \alpha = 1,\ \beta = -1.25,\ k = 0.25,\ \varphi_0 = 0.5,\ v = 0.25\}$.

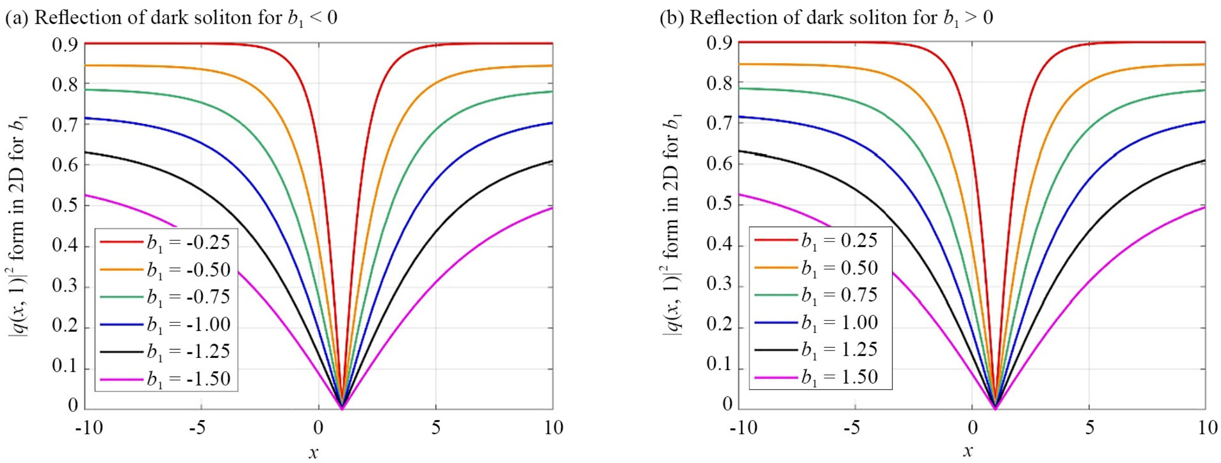


Figure 3. The effect of b_1 on dark soliton form

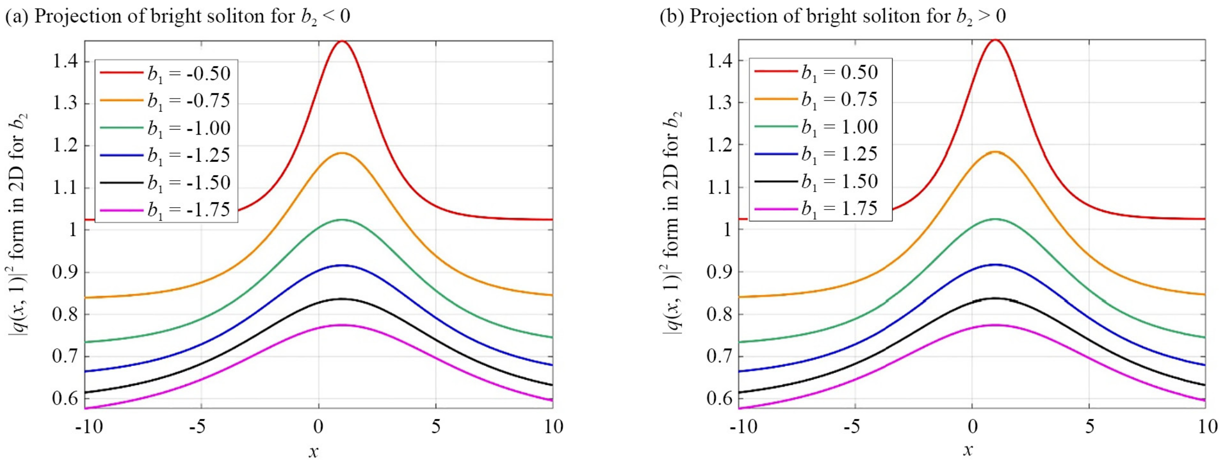


Figure 4. The impact of b_2 on dark soliton form

It is clear from both Figure 3 and Figure 4 that the parameters b_1 and b_2, which belong to the cubic and quintic law nonlinearity terms, respectively, have a noticeable effect on the dark and bright soliton forms. Although the graphs in Figure 3 may appear to share the same shape at first glance, but a closer look at the parameter choices in each case reveals that they are not identical. The same applies to Figure 4. Consequently, this once again emphasizes the importance of considering the mathematical constraints of the proposed model, the applied wave transformation, and the method itself when aiming to obtain the relevant soliton

5. Conclusions

This paper recovered pure-cubic perturbed optical solitons when the chromatic dispersion is replaced by a third-order dispersion term. The perturbation terms are from soliton self-frequency shift and nonlinear dispersion terms, respectively. To achieve the pure-optical soliton solutions the new Kudryashov approach has been applied and we derived bright, dark, and singular optical solitons. Dark and bright solitons were preferred in graphic presentations, but the singular form was not preferred because it was not very meaningful for nonlinear optics. The related graphical illustrations have been served by presenting the 2D and 3D graphics. To emphasize the effect of the cubic and quintic law of SPM on soliton form we also presented the related graphics. The new Kudryashov scheme is of great value and will later be applied to additional forms of self-phase modulation which will all be presented with time. Naturally, for researchers working in this field, models that include fractional, additional perturbation terms, stochastic effects, multi-solitons, chaotic behavior, etc. are just a few of the topics that can be researched. This is just the tip of the iceberg.

Conflict of interest

The authors do not have any competing interests in this research.

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