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Research Article

Energy Exchange Between the Polarization Components of an Optical Pulse Under the Influence of Degenerate Four-Photon Parametric Processes

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Abstract: The present paper investigates the process of energy exchange between the components of an optical pulse that is propagating in a nonlinear dispersive medium under the influence of degenerate four-photon parametric processes. The effects of self-phase modulation (SPM) and cross-phase modulation (XPM) have been taken into account. The obtained results demonstrate that no energy exchange takes place between the optical components of the pulse when the initial polarization is linear or circular. In the presence of initial elliptical polarization, energy exchange can be observed. The intensity of the energy transmission and its period can be determined using the values of the initial energies and the initial phase difference between the optical components of the pulse.

Keywords: degenerate four-photon parametric processes, energy exchange, self-phase modulation, cross-phase modulation, polarization components

1. Introduction

Nowadays the four-photon parametric process is actively studied and of great interest to the researchers. The origin of this effect is the nonlinear response of the medium, under the influence of a high-intensity electromagnetic field. To observe the mentioned process, it is of crucial importance for the phase matching condition to be fulfilled. That requires a special selection of the initial frequencies of the waves. Parametric processes in optical fibers, particularly four-wave mixing, represent a highly relevant and actively researched topic, as evidenced by numerous publications from various authors in recent years [1-3]. The four-photon parametric mixing can be divided into two types: non-degenerate and degenerate [4]. The degenerate four-photon processes (FPP) were investigated in detail within a discrete vector model by the authors in [5, 6]. On the other hand, in [7, 8] evidence of coherent energy exchange between the polarization components of vector solitons in optical fibers, as a result of the action of four-wave mixing (FWM), has been presented. It is shown that the cause of polarization rotation is the degenerate four-photon parametric processes and the energy exchange between the elliptically polarized components of an optical pulse. There is a single-soliton solution of the system of equations that does not show energy exchange and is valid for both linear and circular polarization

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of the components of the vector field. The stability of these solutions has been demonstrated by performing numerical simulations, which prove that for linearly and circularly polarized optical pulses, FPP processes simulate the effect of the influence of cross-phase modulation. A similar problem is considered by Boyd [9] but in an approximation for a fixed intensity. In this case, in the numerical simulations, a rotation of the polarization ellipse is observed.

In the present paper, it is studied the energy exchange between the components of an optical pulse propagating in a nonlinear dispersive medium under the influence of degenerate four-photon parametric processes by taking into account the effects of self-phase modulation (SPM) and cross-phase modulation (XPM). The current investigation provides an important contribution to the understanding of energy exchange mechanisms between the two components of one optical pulse, considering the effects of SPM, XPM, and FPP.

Firstly, the introduction outlines the basics of the four-photon parametric processes and gives some information about similar research done by other authors. The difference between the existing theoretical models and the current one is pointed out.

Secondly, the basic equations which are used to describe the process of propagation of the two components, are presented. This is followed by solving the system under specific initial conditions, allowing further investigation.

Lastly, the energy exchange between the laser pulse components is explored, using the obtained solutions.

2. Basic equations

It is assumed that the vector amplitude function describing the pulse envelope has the following components $\vec{A} = \left(A'_x,\ A'_y,\ 0\right)$. The system of nonlinear equations characterizing the propagation of the two components of the laser pulse, written in dimensionless form, is of the kind:

$$i\frac{\partial A_x}{\partial z} + \frac{1}{2}\frac{\partial^2 A_x}{\partial t^2} + \gamma\left[\left(\left|A_x\right|^2 + \frac{2}{3}\left|A_y\right|^2\right)A_x + \frac{1}{3}A_x^* A_y^2\right] = 0,$$

$$i\frac{\partial A_y}{\partial z} + \frac{1}{2}\frac{\partial^2 A_y}{\partial t^2} + \gamma\left[\left(\left|A_y\right|^2 + \frac{2}{3}\left|A_x\right|^2\right)A_y + \frac{1}{3}A_y^* A_x^2\right] = 0, \tag{1}$$

where:

$$\gamma = \frac{z_{dis}}{z_{nl}},\ z_{dis} = \frac{T_0^2}{\left|k''\right|},\ z_{nl} = \frac{2}{k_0 n_2 I_0},$$

$$z = \frac{z'}{z_{dis}},\ t = \frac{t'}{T_0},\ A_x = \frac{A'_x}{\sqrt{I_0}}, A_y = \frac{A'_y}{\sqrt{I_0}} \tag{2}$$

In the equations above $A_x(z)$ and $A_y(z)$ are the normalized complex amplitude functions of the pulse's components, T_0 is its initial time duration. The parameters n_2, k'', k_0, I_0 characterize respectively the nonlinear refractive index of the medium, the second order of dispersion, the wavenumber, and the initial intensity. With z_{dis} and z_{nl} are represented the dispersive and nonlinear lengths.

The second terms in the system of equations (1) describe the influence of the dispersion of the medium; the third, fourth, and last terms respectively correspond to self-phase modulation, cross-phase modulation, and degenerate four-photon parametric mixing (DFPPM).

In [10] it is presented a mathematical model of finding exact analytical solutions of the system of differential equations (1) for arbitrary initial conditions. A similar method was applied in this paper. It is assumed that the amplitude functions of the components of the optical pulse are of the type:

$$A_x(z,\ t) = a_x(z)T(t)\exp i\varphi_x(z,\ t),$$

$$A_y(z,\ t) = a_y(z)T(t)\exp i\varphi_y(z,\ t), \tag{3}$$

where $a_x(z)$ and $a_y(z)$ are unknown functions only of the variable z. The phases of the both components are:

$$\varphi_x(z,\ t) = Ct + \Phi_x(z),\ \ \varphi_y(z,\ t) = Ct + \Phi_y(z),\ \ C = \text{const}. \tag{4}$$

The function $T(t)$, which describes the time-dependence t for both components of the pulse is:

$$T(t) = \frac{1}{ch(t)} = \text{sech}(t) \tag{5}$$

This means that the two components are considered to have equal time durations, but different amplitudes and phases, depending on the normalized dimensionless coordinate z.

Since the energy exchange between the components of the pulse is studied, the system of differential equations is written by their energies. It is shown that the energies of the two components depend on the squares of the amplitude functions $a_x(z)$ and $a_y(z)$ [10]:

$$E_x = \int_{-\infty}^{+\infty} \left|A_x\right|^2 dt = a_x^2 \int_{-\infty}^{+\infty} T^2 dt = 2a_x^2,$$

$$E_y = \int_{-\infty}^{+\infty} \left|A_y\right|^2 dt = a_y^2 \int_{-\infty}^{+\infty} T^2 dt = 2a_y^2, \tag{6}$$

where $\int_{-\infty}^{+\infty} T^2 dt = \int_{-\infty}^{+\infty} \left(\frac{1}{cht}\right)^2 dt = 2.$

In order to characterize the difference between the energies of the two components of the pulse in the process of energy exchange, is used the function [10]:

$$\Delta(z) = E_y - E_x. \tag{7}$$

A correlation between Δ, E_x and E_y can be found:

$$E_x = \frac{C - \Delta}{2},\ E_y = \frac{C + \Delta}{2} \tag{8}$$

As a next step, it is obtained an equation which describes the energy exchange between the two components:

$$\frac{d\Delta}{dz} = \frac{\gamma}{9}\left(C^2 - \Delta^2\right)\sin\psi. \tag{9}$$

A system of differential equations is formed:

$$\frac{d\psi}{dz} = 2\left(\Phi_y' - \Phi_x'\right) = \frac{2\gamma}{9}\Delta(1 - \cos\psi)$$

$$\frac{d\Delta}{dz} = \frac{\gamma}{9}\left(C^2 - \Delta^2\right)\sin\psi. \tag{10}$$

We divide the equations from the system (10) by terms, and after the transformations, we obtain the following linear ordinary first-order differential equation for the unknown function $\cos\psi$ under an independent variable Δ:

$$\frac{d\cos\psi}{d\Delta} = \frac{2\Delta}{c^2 - \Delta^2}(\cos\psi - 1) \tag{11}$$

The solution of the above listed equation is as follows:

$$\cos\psi = 1 - \frac{B_0}{C^2 - \Delta^2} \tag{12}$$

where B_0 is a constant that is determined by the initial conditions:

$$B_0 = \left(C^2 - \Delta_0^2\right)\left(1 - \cos\psi_0\right). \tag{13}$$

The expression (13) is substituted in (9) and the following expression is obtained:

$$\frac{d\Delta}{dz} = \frac{\gamma}{9}\sqrt{2B_0}\sqrt{\left(C^2 - \frac{B_0}{2}\right) - \Delta^2}. \tag{14}$$

After a number of transformations, the following solution was found:

$$\Delta(\mathrm{z}) = \sqrt{\left(C^2 - \frac{B_0}{2}\right)}\sin\left(\frac{\gamma}{9}\sqrt{2B_0}z\right), \tag{15}$$

In expression (9) the constant C is the sum of the energies of the two components of the pulse ($E_x + E_y = C$) and ψ_0 is the initial generalized phase.

Thus, for the energies of the two components of the optical pulse were found the following expressions:

$$E_x = \frac{C}{2} - \frac{1}{2}\sqrt{\left(C^2 - \frac{B_0}{2}\right)}\sin\left(\frac{\gamma}{9}\sqrt{2B_0}z\right),$$

$$E_y = \frac{C}{2} + \frac{1}{2}\sqrt{\left(C^2 - \frac{B_0}{2}\right)}\sin\left(\frac{\gamma}{9}\sqrt{2B_0}z\right). \tag{16}$$

3. Exploration of the energy exchange between the laser pulse components

In [10] it is shown that the energy exchange between the two polarization components of the optical pulse (15), propagating in a nonlinear medium, is a periodic function and it is described by sine law [11]. Having in mind the expression (15) it can be written in the following form:

$$\Delta(z) = C\sqrt{1-\frac{1}{2}(1-\cos\psi_0)\left(1-\frac{\Delta_0^2}{C^2}\right)}\sin\left(\frac{\gamma\sqrt{2}}{9}C\sqrt{(1-\cos\psi_0)\left(1-\frac{\Delta_0^2}{C^2}\right)}z\right). \tag{17}$$

The energy exchange between the components of the pulse depends on the initial generalized phase ψ_0, the initial difference Δ_0 and C-the sum of the intensities of the two components.

3.1 *Dependance of energy exchange of the initial phase difference between the pulse components*

It investigated the dependence of the energy exchange between the components of an optical pulse and the initial phase difference [12]. The initial phase difference between the components is given by the expression: $\varphi(0) = \Phi_2(0) - \Phi_1(0)$ and the initial generalized phase is $\psi(0) = 2\varphi(0)$.

- For linear polarization $\varphi(0) = 0$ and $\psi(0) = 0$. In this case $\cos\psi_0 = 1$.

Therefore, from the expression (13) it is determined that $B_0 = 0$. By substituting this value in (13) it is found that $\Delta(z) = 0$. Accordingly, when the polarization is linear, no energy exchange occurs between the two components of the laser pulse.

- For circular polarization $\varphi(0) = \frac{\pi}{2}$ and $\psi(0) = \pi$, $\cos\psi_0 = -1$. In this case $\Delta(0) = 0$.

From (13) it was shown that $B_0 = 2C^2$. Substituting these values in (9) it was obtained again that $\Delta(z) = 0$. Therefore, when the initial polarization of the components of the laser pulse is circular, no energy exchange is observed between them.

- For elliptical polarization $\varphi(0) \neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ and $\psi(0) \neq 0, k\pi$, where $k = 1, 2\ldots$

In this case, the difference in the energies characterizing the energy exchange between the two components of the laser pulse is not zero and in its general form it is represented by equation (17). From this equation it is obtained an expression for the period of energy exchange between the components of the pulse [10]:

$$\overline{Z} = \frac{18\pi}{\gamma\sqrt{2(1-\cos\psi_0)(C^2-\Delta_0^2)}}. \tag{18}$$

It is important to mention that this expression can only be applied in the case of an initial elliptical polarization between the components of the pulse. It is clear that the period depends on the initial phase difference $\psi(0)$, the initial difference Δ_0 and the sum C of the energies.

Let us look at some specific cases of initial elliptical polarization:

On an initial condition $\varphi(0) = \frac{\pi}{4} \to \psi(0) = \frac{\pi}{2}$ and $\cos\psi_0 = 0$. From (9) it follows that $B_0 = C^2 - \Delta_0^2$ and $\Delta \neq 0$. In this particular case, for the energy exchange and for the period are obtained the following expressions:

$$\Delta(z) = \frac{1}{2}\sqrt{2(C^2+\Delta_0^2)}\sin\left(\frac{\gamma}{9}\sqrt{2(C^2-\Delta_0^2)}z\right), \tag{19}$$

$$\overline{Z} = \frac{18\pi}{\gamma\sqrt{2(C^2-\Delta_0^2)}}. \tag{20}$$

If we assume that there is no difference in the initial energies of the components of the pulse ($\Delta_0 = 0$), then the level of energy exchange and its period depend only on the sum of the energies of the two components:

$$\Delta(z) = \frac{C}{\sqrt{2}}\sin\left(\frac{\gamma}{9}\sqrt{2C^2}\,z\right), \tag{21}$$

$$\bar{Z} = \frac{18\pi}{\gamma C\sqrt{2}}. \tag{22}$$

From (21) and (22) it follows that, the greater the value of the initial sum of the energies of the components of the pulses, the shorter the period of energy exchange is.

On an initial condition $\varphi(0) = \frac{\pi}{6} \to \psi(0) = \frac{\pi}{3}$ and $\cos\psi_0 = 1/2$. From (9) follows that $B_0 = \frac{1}{2}(C^2 - \Delta_0^2)$ and $\Delta \neq 0$. In this case, for the energy exchange and its period, the following expressions are obtained:

$$\Delta(z) = \frac{1}{2}\sqrt{3C^2 + \Delta_0^2}\,\sin\left(\frac{\gamma}{9}\sqrt{\left(C^2 - \Delta_0^2\right)}\,z\right), \tag{23}$$

$$\bar{Z} = \frac{18\pi}{\gamma\sqrt{\left(C^2 - \Delta_0^2\right)}}. \tag{24}$$

On an initial condition $\varphi(0) = \frac{\pi}{3} \to \psi(0) = \frac{2\pi}{3}$ and $\cos\psi_0 = -1/2$. From (13) follows that $B_0 = \frac{3}{2}(C^2 - \Delta_0^2)$ and $\Delta \neq 0$. In this case, for the energy exchange and for its period, the following expressions are obtained:

$$\Delta(z) = \frac{1}{2}\sqrt{C^2 + 3\Delta_0^2}\,\sin\left(\frac{\gamma}{9}\sqrt{3\left(C^2 - \Delta_0^2\right)}z\right), \tag{25}$$

$$\bar{Z} = \frac{18\pi}{\gamma\sqrt{3\left(C^2 - \Delta_0^2\right)}}. \tag{26}$$

Based on the obtained solutions, numerical simulations have been conducted to analyze the energy exchange between the two components of the optical pulse.

Figures 1 and 2 illustrate the energy transfer dynamics when the initial sum of the energies is set to $C = 1$, with initial differences $\Delta_0 = 0.1$ and $\Delta_0 = 0.5$ respectively.

Figures 3 and 4 present the corresponding results for $C = 1.5$ while maintaining the same initial differences $\Delta_0 = 0.1$ and $\Delta_0 = 0.5$.

The energy exchange is shown for three different initial phase differences: $\frac{\pi}{6}$, $\frac{\pi}{4}$ and $\frac{\pi}{3}$, represented in blue, red and green, respectively.

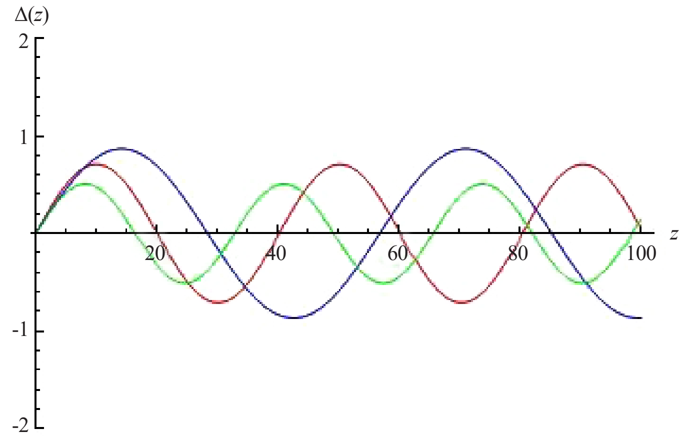

Figure 1. Energy exchange between the components of the optical pulse at initial conditions $C = 1$ and $\Delta_0 = 0.1$ and initial phase differences $\frac{\pi}{6}$ (blue), $\frac{\pi}{4}$ (red) and $\frac{\pi}{3}$ (green)

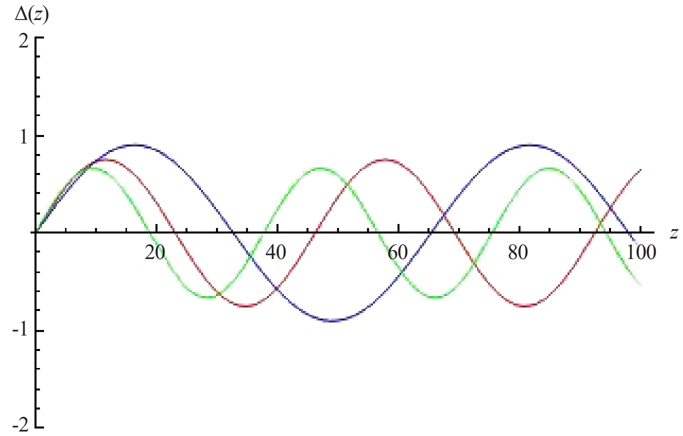

Figure 2. Energy exchange between the components of the optical pulse at initial conditions $C = 1$ and $\Delta_0 = 0.5$ and initial phase differences $\frac{\pi}{6}$ (blue), $\frac{\pi}{4}$ (red) and $\frac{\pi}{3}$ (green)

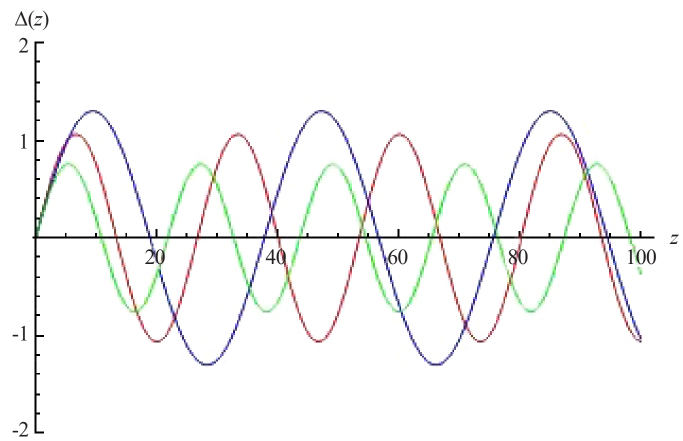

Figure 3. Energy exchange between the components of the optical pulse at initial conditions $C = 1.5$ and $\Delta_0 = 0.1$ and initial phase differences $\frac{\pi}{6}$ (blue), $\frac{\pi}{4}$ (red) and $\frac{\pi}{3}$ (green)

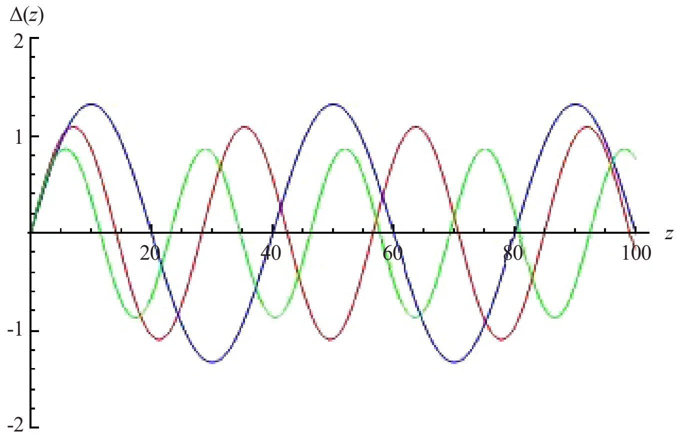

Figure 4. Energy exchange between the components of the optical pulse at initial conditions $C = 1.5$ and $\Delta_0 = 0.5$ and initial phase differences $\frac{\pi}{6}$ (blue), $\frac{\pi}{4}$ (red) and $\frac{\pi}{3}$ (green)

A comparison of the presented results reveals that, in the presence of an initial elliptical polarization between the components of the optical pulse, and under the condition $C^2 > \Delta_0^2$, the longest period of energy exchange occurs when the initial phase difference is smaller $\varphi(0) = \frac{\pi}{6}$ and the initial difference Δ_0 is larger. The most significant energy exchange is observed when the sum of the initial energies is higher.

4. Conclusion

The results obtained in the present paper confirm the statements that in the case of initial linear or circular polarization between the components of an optical pulse, no energy exchange is observed between them. In the presence of initial elliptical polarization, the intensity of the energy transfer process and its period are mainly determined by the values of the initial energies and the initial phase difference between the components of the pulse. It occurs that the smaller the initial phase difference is, the greater are energy exchange and its period.

The obtained analytical expressions for the level of energy exchange (15) and its period (22) make it possible to control the process of energy transfer between the components of the laser pulse. The intensity of the energy exchange and its period are mainly determined by the values of the initial energies E_x and E_y, the initial phase difference ψ_0 between the components of the pulse and the sum of the initial energies of the components of the pulse. By changing the mentioned values of the parameters, we can observe various degrees of intensity of the energy exchange.

Conflict of interest

The author declares no competing financial interest.

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