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Research Article

On the Theory of Digital Modulation of Light Transmitted Through a Step-Index Optical Fiber

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Abstract: Analog modulation of wireless micro/radio waves as well as optical light waves by multiplexing. An input signal is well established in practice due to technological breakthroughs. The basic methods of modulation of a carrier wave are amplitude modulation Amplitude Shifted Keying (ASK), frequency modulation Frequency Shifted Keying (FSK), and phase modulation Phase Shifted Keying (PSK). Quadrature Amplitude Modulation (QAM) is a combination of amplitude and phase modulation. Among these methods, the simple amplitude modulation is suitable and of practical use in fiber-optic communication as extraneous errors are prevented are prevented by the jacket of the cable containing the fiber-a scenario not applicable in wireless communication. The wave-guide action in a cylindrical optical fiber enables the development of a theoretical model of light propagation which has undergone amplitude modulation. This theory is presented in this paper for a step-index fiber consisting of a homogeneous core with a cladding of slightly less refractive index. The input signal is assumed to be a batch of N bits, which is multiplexed to the carrier light wave, and a general formula is developed for the field intensity measured by the axial component of the Hertz vector of the associated electromagnetic wave at a receiving point. The expression of that field is found in the form of a Fourier integral, which is computed by Fast Fourier Transform (FFT) for simple numerical cases. The theory covers both single mode fibers which can transmit only the fundamental mode of propagation, as well as multimode fibers of greater diameters. The theory provides the character of the wave at a distant receiving station, when the input is in bits for which demodulation techniques are available. In addition the theory provides a method of serial transmission of a number of bits in a batch, provided suitable demodulation technique is available at the receiver end. A simple maximum field strength measure is suggested in the paper.

Keywords: optical fiber, amplitude modulation ASK, Quadrature Amplitude Modulation (QAM), serial batch digital input, axial Hertz vector component, Electromagnetic (EM) waves

1. Introduction

The transmission of digital data through fiber-optic cables is accomplished by modulation of the analog carrier light wave propagating through the core of the cable. A carrier wave has three basic attributes: its amplitude, frequency and phase. A modulation technique is a change in any of the three attributes or a combination of them in accordance with the digital bit rate of the input signal. The basic three modulation techniques are known as Amplitude Shifted Keying

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(ASK), Frequency Shifted Keying (FSK) and Phase Shifted Keying (PSK), depending on the modulation of amplitude, frequency and phase respectively [1–3]. Of the three techniques amplitude modulation is most effected by external noise. However, as the jacket covering of a fiber-optic cable is an effective barrier to external noise, the simple ASK technique is often employed over a single mode fiber-optic cable for data haul over long distances. In this scheme when the bit 0 is transmitted, there is no transmission of light, while for the transmission of the bit 1, a modulated pulse of light propagates. This off-on character of transmission makes the demodulation of streaming data quite simple to implement. Incorporating phase modulation in ASK, Quadrature Amplitude Modulation (QAM) has also been developed to transmit 2, 4, 16, 64, and even 256 bits simultaneously through a single fiber and demodulate the signal effectively [4–6]. Wireless communication on the other hand is more prone to errors due to external noise. In this regard FSK, in conjunction with phase shifting, is more suitable for transmission of data, and communication techniques have been developed with better quality of transmission, low probability of error, higher security and bandwidth services [4, 7].

The studies reported in the above citations are generally based on simulation techniques using Matrix Laboratory (MATLAB) and Simulink to study the modulation characteristics of the carrier wave and evaluate the probability of error in transmitting a bit of data over the carrier [8–10]. The MATLAB function is however based on the quantum mechanical Nonlinear Shrödinger equation appropriate more for nonlinear propagation characteristics. In this respect some commercial softwares are also available that depend on MATLAB or other computational approaches for nonlinear phenomena. Here in this paper, taking advantage of axial propagation of light along the linear, homogeneous thin cylindrical core of a given refractive index of a fiber-optic cable satisfying the Maxwell equations of electromagnetism, the modulation of a stream of bits multiplexed with a given carrier wave is analysed to investigate the nature of the signal at a distant receiver. The propagation of the carrier wave through the core is however known to be dispersive [11–13] with the possibility of multiple frequencies of propagation for a fixed value of the wave number. The governing dispersion equation involving Bessel functions is numerically solvable, and for the purpose of theoretical development, the fundamental gravest frequency curve is computed and approximated by polynomials over designated subintervals of the frequency. It transpires, as shown by a numerical example, that the higher frequency dispersion curves have much less contribution to the propagating modulated wave. The theoretical formulation begins in a general manner, expressing the solution of the Maxwell equations [14] in terms of the axial component Π_3 of the Hertz vector for a propagating light wave, using cylindrical polar coordinates (r, θ, z). The component Π_3 satisfies a light wave equation, whose solution is represented by modal decomposition $m = 0, 1, 2, \cdots$. Multiplexing this carrier wave with a stream of N bits at the source $z = 0$, the ASK modulated wave is shown to be in the form of a Fourier integral which can only be evaluated numerically by the Fast Fourier Transform (FFT) [15] at a distant receiver position where it is assumed that the field can be measured. As an illustration therefore, a toy numerical example is first considered, which can sustain the fundamental mode $m = 0$ of propagation. The computations yield the wave form at a distant destination when the bit 1 is transmitted, and no signal when the bit 0 is transmitted. typically as expected in ASK. The general formulation opens up the possibility of transmitting a batch of N bits in series with a suitable demodulation technique. In the example treated only $N = 2$ is treated and a method of demodulation is suggested which is based on the maximum field intensity at the destination. Next, it is shown that propagation of QAM for two bits is also possible by the formulation, carrying out the computation for the multiplexing of two bits 11. Finally, multimode transmission is studied in brief for the next mode $m = 1$, and the wave form of light reaching the destination is obtained when the bit 1 is multiplexed. Though the illustrative examples are very simple, the formulation is general enough to treat any specified batch of N bits, in any number of modes of propagation in a fiber, provided cross-talk is precluded in actual implementation and the respective fields are measurable.

2. Formulation of the problem

Let the refractive indices of the core and the cladding material of the optical fiber be respectively n_1 and n_2, where $n_1 > n_2$ differing by a small amount. Assuming that it is laid along a straight line, say the z-axis, a transmitted signal of light undergoes little loss of intensity due to the wave guide action of the core. The propagation of light is governed by the Maxwell equations of electromagnetism [14], for the electric and magnetic intensity components E and H. The field can however be expressed in terms of a single Hertz vector Π [11], which in the present case has a single component Π_3 along the z-axis [16] satisfying the wave equation

$$\nabla^2\Pi_3 - \frac{1}{c^2}\ddot{\Pi}_3 = 0 \tag{1}$$

where c represents the velocity of light and the dots represent differentiation with respect to time t. In terms of the primitive dielectric permittivity ϵ and magnetic permeability μ, $c = 1/\sqrt{\epsilon\mu}$ and is related to the refractive index n of the medium by the relation $c = c_0/n$, where c_0 is the velocity of light in vacuum. Employing cylindrical polar coordinates $(r,\,\theta,\,z)$, it is shown in [16] that the components of E and H can be derived from the relations

$$E_r = \frac{\partial^2\Pi_3}{\partial r\partial z}, \quad E_\theta = \frac{1}{r}\frac{\partial^2\Pi_3}{\partial\theta\partial z}, \quad E_z = \frac{\partial^2\Pi_3}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2\Pi_3}{\partial t^2} \tag{2}$$

$$H_r = \frac{\varepsilon}{r}\frac{\partial^2\Pi_3}{\partial\theta\partial t}, \quad H_\theta = -\varepsilon\frac{\partial^2\Pi_3}{\partial r\partial t}, \quad H_z = 0 \tag{3}$$

The propagation of light in different modes, is expressed by the modal solutions of Eq. (1). Assuming that the radius of the core is a, the modal solutions in the two domains of the core and the cladding represented by superscripts (1) and (2) respectively can be expressed as

$$\Pi_3^{(1)} = A_m J_m\left(u\frac{r}{a}\right)e^{im\theta}\,e^{i(kz-\omega t)}, \quad 0 \le r \le a \tag{4}$$

$$\Pi_3^{(2)} = C_m K_m\left(v\frac{r}{a}\right)e^{im\theta}\,e^{i(kz-\omega t)}, \quad r > a \tag{5}$$

where A_m, C_m are constant amplitudes and $J_m(\cdot)$, $K_m(\cdot)$ for $m = 0,\,1,\,2,\,\cdots$ are Bessel and modified Bessel functions respectively. The quantities k and ω are respectively the wave number and the angular frequency of the propagating wave. The two parameters u and v are given by

$$u = ak\sqrt{\frac{c_p^2}{c_1^2} - 1}, \quad v = ak\sqrt{1 - \frac{c_p^2}{c_2^2}} \tag{6}$$

in which c_p is the phase velocity of propagation ω/k. Evidently, $c_1 < c_p < c_2$ for real values of u and v, which is consistent with the condition $n_1 > n_2$. The solutions (4) and (5) respectively represent oscillatory and evanescent waves in the core and the cladding regions. The boundary conditions holding at the interface of the two media are $E_z^{(1)} = E_z^{(2)}$ and $H_\theta^{(1)} = H_\theta^{(2)}$. Using Eqs. (2) and (3), the two conditions yield

$$u^2 J_m(u)A_m = -v^2 K_m(v)C_m \tag{7}$$

$$uJ'_m(u)A_m = vK'_m(v)C_m \tag{8}$$

as $\epsilon_1 \approx \epsilon_2$ for the two media. There is a third boundary condition $E_\theta^{(1)} = E_\theta^{(2)}$, which is exactly satisfied for the important case $m = 0$ (single mode transmission) and very nearly satisfied for other values of m as $r \to 0$ for the thin fibers. Eqs. (7) and (8) yield the the dispersion equation as

$$\frac{1}{u}\frac{J_m'(u)}{J_m(u)} + \frac{1}{v}\frac{K_m'(v)}{K_m(v)} = 0 \tag{9}$$

or using the recurrence relations of Bessel functions ([17], p.361, 375)

$$\frac{1}{u}\frac{J_{m+1}(u)}{J_m(u)} + \frac{1}{v}\frac{K_{m+1}(v)}{K_m(v)} - m\left(\frac{1}{u^2} + \frac{1}{v^2}\right) = 0 \tag{10}$$

Eq. (9) is well documented in texts such as that by Keiser [12]. Equation (10) can be expressed in nondimensional form by introducing the nondimensional frequency f (commonly denoted by V in the fiber-optic literature defined by the relation)

$$f = \sqrt{u^2 + v^2} = a\omega\sqrt{\frac{1}{c_1^2} - \frac{1}{c_2^2}} = \frac{a\omega}{c_0}\sqrt{n_1^2 - n_2^2} \tag{11}$$

and the variable b called the normalised propagation constant defined by the relation

$$b = \frac{c_0^2/c_p^2 - n_2^2}{n_1^2 - n_2^2} = \frac{v^2}{f^2} \tag{12}$$

In other words the transformation

$$u = \sqrt{1-b}\,f, \quad \text{and} \quad v = \sqrt{b}\,f \tag{13}$$

renders Eq. (10) in nondimensional form. The transcendental equation so obtained can be treated by numerical methods only.

As a numerical study, the nondimensional dispersion equation is solved for $n_1 = 1.5$ and $n_2 = 1.48515$. Since $0 < b < 1$ and $f > 0$, as is apparent from Eq. (13), the left hand side of Eq. (10) is computed for increasing values of b from 0 to 1 and ω for increasing the range of the value of f. The values of f for a ser value of b are picked up which renders the side very small, of the order of 10^{-5}. It transpires that multiple values of f are found by this process, indicating the existence of several branches of the dispersion curve [13]. However in the transmission process of data, the principal first branch with the lowest value of f is all that matters as is found in the simulation of modulation presented in the following sections, with the rest of the higher frequency branches contributing insignificantly to the field. The first two branches for the fundamental mode of propagation $m = 0$ for single mode fibers and $m = 1$ for multimode fibers are presented in Figures 1 and 2 respectively as examples.

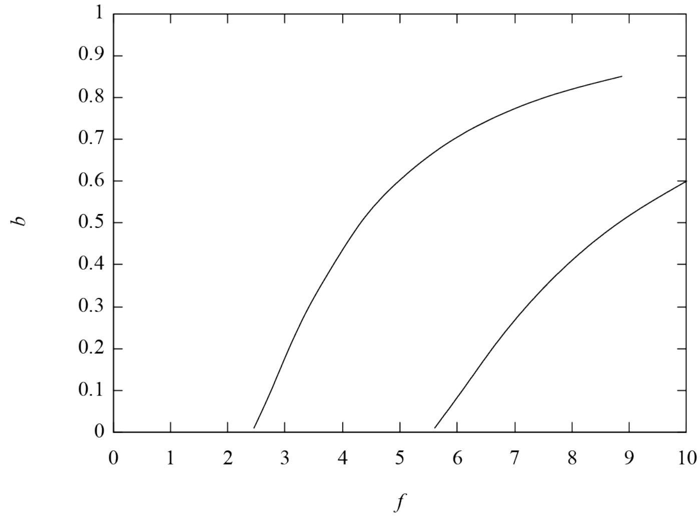

Figure 1. Dispersion curves (mode $m = 0$)

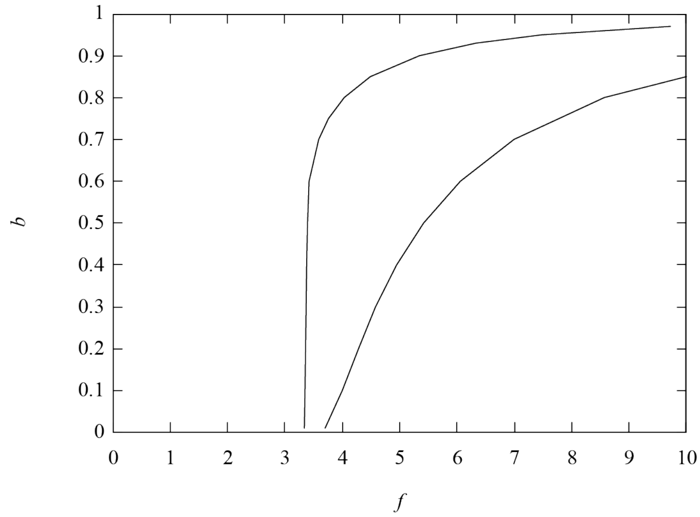

Figure 2. Dispersion curves (mode $m = 1$)

The equations of the trendline of the dispersion curves are next determined by polynomial interpolation of the $(f,\ b)$ points of the computed zeroes of Eq. (10). The trendlines for the least frequency branch for the two modes $m = 0,\ 1$ under consideration are

$$b = \begin{cases} 0, & 0 < f \leq 2.458 \\ -1.3399 + 0.7659\, f - 0.1041\, f^2 + 0.0064\, f^3 - 0.0001\, f^4, & 2.458 < f \leq 8.869 \\ 0.4824 + 0.0567\, f - 0.0017\, f^2, & 8.869 < f \leq 16.50 \\ 1, & f > 16.50 \end{cases} \tag{14}$$

for mode $m = 0$. In the case of $m = 1$ however, the principal branch of b as a function of f is slightly double valued, which for $f \leq 3.3936$ renders computation difficult for computing the values of b unambiguously from those of f. So a very small approximation is made in the data for that range to obtain single valued functional approximation, as stated below

$$b = \begin{cases} 0, & 0 < f \leq 3.340 \\ -31.2120 + 9.3487 f, & 3.340 < f \leq 3.3936 \\ -5.9334 + 3.1529 f - 0.3661 f^2, & 3.3936 < f \leq 4.4905 \\ 0.5226 + 0.0989 f - 0.0054 f^2, & 4.4905 < f \leq 9.7223 \\ 0.9443 + 0.0026 f, & 9.7223 < f \leq 17.2835 \\ 0.9836 + 0.0003 f, & 17.2835 < f \leq 49.7494 \\ 1, & f > 49.7494 \end{cases} \tag{15}$$

For the above representation of b, far larger values of f were considered than those shown in Figure 2.

The functional approximation of b as a function of f is essential for the modulation of the carrier wave represented by Eq. (4) during the transmission of data.

3. Modulation of the carrier wave

The transmission of the bit streams data packets over long distances, as stated earlier is carried out by Amplitude Shifted Keying (ASK) by multiplexing the progressive wave field by a pulse representing the bits 1 and 0. Accordingly, the modulation theory is developed here analytically. The method being general, elements of PSK and QAM are also covered thereby.

The modulated wave is no longer sinusoidal as represented by Eq. (4) essentially in the core region. Accordingly, let the modulated wave be represented as a separated variable solution

$$\Pi_3^{(1)} = J_m(\lambda r)\, e^{im\theta}\, P(z,\, t) \tag{16}$$

where in cylindrical polar coordinates, Eq. (1) is

$$\frac{\partial^2 \Pi_3^{(1)}}{\partial r^2} + \frac{1}{r}\frac{\partial \Pi_3^{(1)}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Pi_3^{(1)}}{\partial \theta^2} + \frac{\partial^2 \Pi_3^{(1)}}{\partial z^2} - \frac{1}{c_1^2}\frac{\partial^2 \Pi_3^{(1)}}{\partial t^2} = 0 \tag{17}$$

where λ is a parameter to be suitably chosen later on. Substitution of the form (16) in Eq. (17), using Bessel's equation yields the equation

$$\frac{\partial^2 P}{\partial z^2} - \frac{1}{c_1^2}\frac{\partial^2 P}{\partial t^2} - \lambda^2 P = 0 \tag{18}$$

The Eq. (18) is solved by the double Fourier-Laplace transform

$$\bar{P}(p,\ \omega) = \int_{-\infty}^{\infty}\int_{0}^{\infty} P(z,\ t)\, e^{-pz}\, e^{i\omega t}\, dz\, dt \tag{19}$$

Hence, multiplying Eq. (18) by $e^{-pz} \times e^{i\omega t}$ and integrating with respect to z and t, and assuming that the function P as well as $\partial P/\partial t$ both vanish at $t \to \pm\infty$, one obtains by partial integration

$$\left(p^2 + \frac{\omega^2}{c_1^2} - \lambda^2\right) \bar{P}(p,\ \omega) = \int_{-\infty}^{\infty} [P'(0,\ t) + pP(0,\ t)]\, e^{i\omega t}\, dt \tag{20}$$

As the stream of data at the transmitter end $z = 0$, produces a pulse of chromatic light wave, it is assumed that $P'(0, t)$ is proportional to $P(0, t)$, say $P'(0, t) = ikP(0, t)$ where k is a function of ω. Also, choosing $\lambda^2 = \omega^2/c_1^2 - k^2$, that is to say, $\lambda = u/a$ according to Eq. (6), then Eq. (20) simplifies to

$$(p^2 + k^2)\bar{P}(p,\ \omega) = (p + ik) \int_{-\infty}^{\infty} P(0,\ t)\, e^{i\omega t}\, dt \tag{21}$$

Writing $P(0,\ t) = P_0(t)$, the right hand side integral of Eq. (21) is $\bar{P}_0(\omega)$, viz. the Fourier transform of $P_0(t)$. Thus multiplying both sides of the Eq. (21) by $J_m(\lambda r)$ one has

$$J_m(\lambda r)\bar{P}(p,\ \omega) = \bar{P}_0 J_m(\lambda r)\, \frac{p + ik}{p^2 + k^2} = \int_{0}^{\infty} \bar{P}_0(\omega) J_m(\lambda r)\, e^{ikz}\, e^{-pz} dz \tag{22}$$

or, using Eq. (19), taking the inverse Laplace transform of Eq. (22)

$$\int_{-\infty}^{\infty} J_m(\lambda r) P(z,\ t)\, e^{i\omega t} dt = \bar{P}_0(\omega) J_m(\lambda r)\, e^{ikz} \tag{23}$$

and now taking the inverse Fourier transform, one has

$$J_m(\lambda r) P(z,\ t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{P}_0(\omega) J_m(\lambda r)\, e^{ikz}\, e^{-i\omega t} d\omega \tag{24}$$

Thus, the solution (16) takes the form

$$\Pi_3^{(1)} = \frac{1}{2\pi}\, e^{im\theta} \int_{-\infty}^{\infty} \bar{P}_0(\omega) J_m\left(u\frac{r}{a}\right) e^{ikz}\, e^{-i\omega t} d\omega \tag{25}$$

Supposing now that $N + 1$ bits $a_1, a_2, a_3, \cdots, N$ are streamed at equal intervals of time $t = 0, 1, 2, \cdots, N$ in a small total time period of T seconds, then assuming that the field remains momentarily stationary the same as at the time of release, the modulating field at $z = 0$ can be expressed as

$$P_m(t) = P_0\, a_\nu, \quad (\nu - 1)\,\frac{T}{N} \le t < \frac{\nu T}{N} \tag{26}$$

for $\nu = 1, 2, 3, \cdots, N+1$. Hence the Fourier transform $\bar{P}_m(\omega)$ of $P_m(t)$ reduces to a finite integral, which can be easily evaluated as

$$\bar{P}_m(\omega) = \int_0^T P_m(t)\, e^{i\omega t} dt = P_0 \frac{2\sin(\omega T/2N)}{\omega} \sum_{\nu=1}^{N} a_\nu\, e^{i\left[\frac{\omega T}{N}(\nu - 1/2)\right]} \tag{27}$$

Now suppose that the modulation field (27) is carried by a carrier wave of angular frequency ω_c at $z = 0$, then the field of the carrier wave is

$$P_c(t) = e^{-i\omega_c t} \tag{28}$$

whose Fourier transform is

$$\bar{P}_c(\omega) = \int_{-\infty}^{\infty} e^{i(\omega - \omega_c)t} dt = 2\pi\, \delta(\omega - \omega_c) \tag{29}$$

where $\delta(\cdot)$ is the Dirac delta-function. The modulated field at $z = 0$ is then $P_0(t) = P_m(t) \times P_c(t)$, whose Fourier transform by convolution is

$$\bar{P}_0(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{P}_m(\omega - \omega_1\, \bar{P}_c(\omega_1)\, d\omega_1 = P_0 \frac{2\sin(\omega - \omega_c)T/2N}{\omega - \omega_c} \sum_{\nu=1}^{N} a_\nu\, e^{i\left[(\omega - \omega_c)\frac{T}{N}(\nu - 1/2)\right]} \tag{30}$$

Hence, Eq. (25) representing the field of the light wave traveling through the core of the fiber is represented by the equation

$$\Pi_3^{(1)} = \frac{P_0}{\pi}\, e^{im\theta} \int_{-\infty}^{\infty} \sum_{\nu=1}^{N} a_\nu\, J_m\!\left(u\frac{r}{a}\right) \frac{\sin[(\omega - \omega_c)T/2N]}{\omega - \omega_c}\, e^{i\left[(\omega - \omega_c)\frac{T}{N}(\nu - 1/2) + kz\right]}\, e^{-i\omega t}\, d\omega \tag{31}$$

field represented by Eq. (31) propagates along the z-direction but is also distributed over the radial direction $r \le a$. So for computational purpose of the modulated light, the average value of the field over the r values is considered. Thus, setting $r_1 = r/a$, the average value of $J_m(ur/a)$ is

$$\frac{2}{u^2} \int_0^u J_m(r_1)\, r_1\, dr_1 = \sum_{n=0}^{\infty} D_{mn}\, \frac{J_{m+2n+1}(u)}{u} \tag{32}$$

where

$$D_{mn} = \begin{cases} 2\delta_{mn}, & \text{for} \quad m = 0 \\ \\ \dfrac{4m(m+2n+1)}{(m+2n)(m+2n+2),} & \text{for} \quad m = 1, 2, 3, \cdots \end{cases} \tag{33}$$

using the formula 11.1.1, p.480 of Abramowitz and Stegun [17] for the definite inegral of Eq. (32). Thus the average light wave field $\bar{\Pi}_3^{(1)}$ in nondimensional form can be written as

$$\bar{\Pi}_3^{(1)} = \frac{P_0}{\pi} e^{im\theta} \int_{-\infty}^{\infty} \sum_{\nu=1}^{N} a_\nu \sum_{n=0}^{\infty} D_{mn} \frac{J_{m+2n+1}(u)}{u} \times \frac{\sin\left[(\omega'-1)\dfrac{\omega_c T}{2N}\right]}{\omega'-1} \times e^{i\left[(\omega'-1)\frac{\omega_c T}{N}(\nu-1/2)+kz\right]} \times e^{-i(\omega_c t)\omega'} d\omega' \tag{34}$$

for which the substitution $\omega = \omega_c\, \omega'$ is made in the ω-integral of Eq. (31), so that ω; is a nondimensional variable.

The general propagation formula (34) leads to several ways in which data can be transmitted. The characteristic properties of propagation in the two cases $m = 0$ and $m = 1$ are numerically presented in the following sections in some detail.

4. Mode *m* = 0 for single mode fibers

It is assumed here that the core radius is small enough to sustain only the fundamental mode $m = 0$. In practice this mode is important for long range transmission. In this particular case the sum over n in Eq. (34) reduces to a single term in virtue of the definition of D_{mn} for the case given in Eq. (33). The real and imaginary parts of the Fourier integral in Eq. (34) actually yields two fields. If the part Re is considered, then the equation can be written as

$$\bar{\Pi}_3^{(1)} = \frac{P_0}{\pi} Re \int_{-\infty}^{\infty} \sum_{\nu=1}^{N} a_\nu \frac{2J_1(u)}{u} \times \frac{\sin\left[(\omega'-1)\dfrac{\omega_c T}{2N}\right]}{\omega'-1} \times e^{i\left[(\omega'-1)\frac{\omega_c T}{N}(\nu-1/2)+kz\right]} \times e^{-i(\omega_c t)\omega'} d\omega' \tag{35}$$

$$= \frac{2P_0}{\pi} Re \int_{0}^{\infty} \frac{J_1(u)}{u} \times \frac{\sin\left[(\omega'-1)\dfrac{\omega_c T}{2N}\right]}{\omega'-1} \sum_{\nu=1}^{N} a_\nu e^{i\left[(\omega'-1)\frac{\omega_c T}{N}(\nu-1/2)+kz\right]}$$

$$+ \frac{\sin\left[(\omega'+1)\dfrac{\omega_c T}{2N}\right]}{\omega'+1} \sum_{\nu=1}^{N} a_\nu e^{i\left[(\omega'+1)\frac{\omega_c T}{N}(\nu-1/2)+kz\right]} \times e^{-i(\omega_c t)\omega'} d\omega' \tag{36}$$

The integrand of Eq. (36) however is a rapidly oscillating function and can only be numerically computed by the Fast Fourier Transform (FFT) method to some degree of accuracy [15]. For that purpose, typical numerical values of the parameters appearing in the equation are chosen. As before the refractive indices of the core and the cladding material are chosen as $n_1 = 1.5$ and $n_2 = 1.48515$ respectively. The core radius is taken as $a = 4.5 \times 10^{-6}$ m for a typical single mode fiber. In practice, the frequency of the carrier wave varies over a wide band of 1 to 1,000 THz. However infrared light of wave lengths between 850 to 1,550 nm have significantly lower absorption in standard glass fibers, the typical value in use being the upper value of 1,550 nm for long range transmission. The frequency with which these ultra-long infrared waves propagate is nearly 193 THz. Accordingly, the angular frequency is chosen as $\omega_c = 2\pi \times 10^{12} = 1.21266 \times 10^{15}$ Hz. Similarly technology permits data transmission rate to also vary over a long range, usually varying from 10 to 300 Gbits/s that can go up to 1 Tbits/s. The commonly used rate is 100 Gbits/s or 10^{12} bits/s. Thus since in the theoretical formulation N bits are transmitted over T seconds of time, $T/N = 10^{-11}$ s, and hence $\omega_c T/N = 2\pi \times 193 \times 10^{12} \times 10^{-11} = 1{,}2126.55$. The other parameters appearing in Eq. (36) are u and kz. The former is given by Eq. (13) in which f is given by Eq. (11) with $\omega = \omega_c\, \omega'$, and $c_0 = 3 \times 10^8$ m/s. Thus $u = \sqrt{1 - bf}$ where b varies with f according to Eq. (14) with $f = 3.82981\, \omega'$ and so is a variable function of ω'. Finally, for a value of kz, it can be shown from Eqs. (6) and (13) that

$$ak = \sqrt{\frac{n_1^2 v^2 + n_2^2 u^2}{n_1^2 - n_2^2}} = f \sqrt{\frac{(n_1^2 - n_2^2)\, b + n_2^2}{n_1^2 - n_2^2}} = 3.82981 \sqrt{b + 49.75627}\, \omega' \tag{37}$$

Now, suppose that the length of the fiber cable is assumed to be measured in units of $D = 10$ km, $D/a = (20/9) \times 10^9$ and so

$$kz = ak \times \frac{D}{a} \times \frac{z}{D} = 0.51069 \sqrt{b + 49.75627} \times 10^9 \times \frac{z}{D}\, \omega' \tag{38}$$

The value of b in the above equation (38) is obtained from the dispersion equation function given by Eq. (14) for the mode under consideration. The value of the distance z is assumed to be 10 km in the simulations for which $z/D = 1$ in Eq. (38).

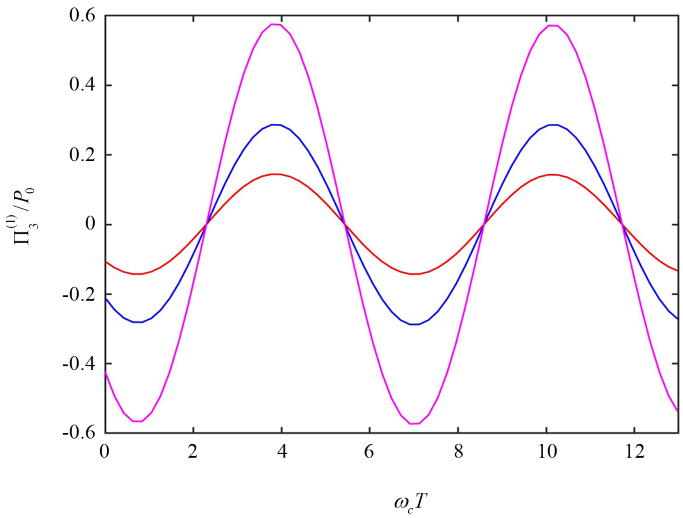

Figure 3. Response at receiver (mode $m = 0$) (bits 01: amber, bits 10: blue, bits 11: pink)

The formula (36) shows that it is possible to transmit N number of bits in a series, and detect the set of bits by a suitable measure. As an example, consider the transmission of a pair of bits in a series. In this case, the possible bit pairs are 01, 10, and 11. In the respective Fourier integrals, as the spectrum frequency is not much different from the carrier frequency, the nondimensional upper limit of the integral Eq. (36) is assumed to be 30, which is divided into windows of 2^{16} subintervals. The time intervals for the computing the values follow the rule of the FFT method [15]. The response at the receiving end in this manner is shown in Figure 3. In order to clearly demarcate the difference in the responses between the first two sets, the response for the first case 01 is multiplied by 1/2, which tantamounts to a phase shift of

$\pi/3$. For the purpose of detection, suppose the maximum of the field is measured, then that maximum can act as a simple detecter of the bit-pair. The full data of response shows that this maximum of the field are respectively 0.11466, 0.33128 and 0.61201 for the three cases, distinguishing the nature of the bit pairs.

The blue curve for the transmission of bit 10 is the standard form response of ASK modulation in which the bit 1 is transmitted followed by a null field corresponding to 0.

In the Quadrature Amplitude Modulation (QAM) for a pair of bits to be transmitted simultaneously, the modulated field is supposed to be the superimposition of the real part for one bit as represented by the real part (Eq. (36)), and the second bit given by the imaginary part of Eq. (34). Thus if the two bits in general are represented by a_1 and a_2, the field due to a_1 is supposed to be $\Pi_3^{(1)}$ given by Eq. (36) with $N = 1$, and the field $\Pi_3^{(1)'}$ due to a_2 is supposed to be given by the imaginary part of the integral Eq. (34). For the latter one has

$$\Pi_3^{(1)'} = \frac{2P_0}{\pi} Im \int_0^\infty \frac{J_1(u)}{u} \times \frac{\sin\left[(\omega'-1)\dfrac{\omega_c T}{2N}\right]}{\omega'-1} \sum_{\nu=1}^{N} a_\nu e^{i\left[(\omega'-1)\frac{\omega_c T}{N}(\nu-1/2)+kz\right]}$$
$$- \frac{\sin\left[(\omega'+1)\dfrac{\omega_c T}{2N}\right]}{\omega'+1} \sum_{\nu=1}^{N} a_\nu e^{i\left[(\omega'+1)\frac{\omega_c T}{N}(\nu-1/2)+kz\right]} \times e^{-i(\omega_c t)\omega'} d\omega' \tag{39}$$

The Total field due to transmission of a_1 and a_2 adds upto $\Pi_3^{(1)} + \Pi_3^{(1)'}$. In Figure 4, this total field response is presented for the transmission of the pair of bits 11. The method can be generalised to higher order QAM's of 16 and 64 bits by incorporating phase shift keying.

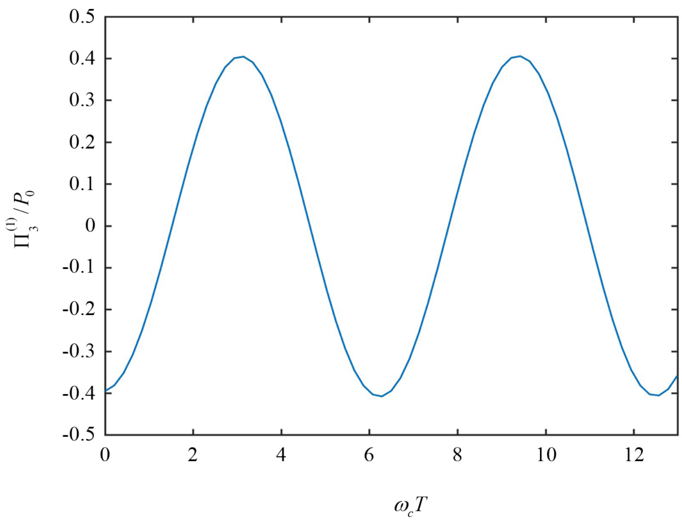

Figure 4. Response at receiver (mode $m = 0$) (QAM for bits 11)

5. Mode $m = 1$ for multimode fibers

As in the case of mode $m = 0$, Eq. (34) for mode $m = 1$ also yields two fields represented by the real and imaginary parts of Eq. (34) with the value of m being 1. Considering only the real part of that equation, the field in question is represented by the expression

$$\bar{\Pi}_3^{(1)} = \frac{2P_0}{\pi}\cos\theta\, Re\int_0^\infty \sum_{n=0}^{\infty} \frac{4(n+1)}{(2n+1)(2n+2)}\frac{J_{2n+2}(u)}{u}$$

$$\times\left[\frac{\sin\left[(\omega'-1)\frac{\omega_c T}{2N}\right]}{\omega'-1}\sum_{\nu=1}^{N} a_\nu e^{i\left[(\omega'-1)\frac{\omega_c T}{N}(\nu-1/2)+kz\right]} - \frac{\sin\left[(\omega'+1)\frac{\omega_c T}{2N}\right]}{\omega'+1}\sum_{\nu=1}^{N} a_\nu e^{i\left[(\omega'+1)\frac{\omega_c T}{N}(\nu-1/2)+kz\right]}\right]$$

$$\times e^{-i(\omega_c t)\omega'}\, d\omega' \tag{40}$$

The rapidly oscillating integral in Eq. (40) is evaluated by FFT as in the case of mode $m = 0$. As an illustration, let a single bit 1 be transmitted through a typical multimode fiber of radius $a = 25\times10^{-6}$ m, with other attributes of transmission rate and refractive properties remaining the same as in the case of mode $m = 0$. This means that $n_1 = 1.5$, $n_2 = 1.48515$, $c_0 = 3\times10^8$ m/s, $\omega_c = 1.21266\times10^{15}$ Hz, $T/N = 10^{-11}$ s, $\omega_c T/N = 1.2126.55$ with $N = 1$ for the single bit transmitted. Due to larger values of a as compared to the previous case, the expression for f however becomes $f = 21.27674\,\omega'$, and

$$ak = 21.27674\sqrt{b+49.75627}\,\omega' \tag{41}$$

With $D = 10$ km as before, $D/a = 4\times10^9$ and so

$$kz = 85.10696\sqrt{b+49.75627}\times10^9\times\frac{z}{D}\,\omega' \tag{42}$$

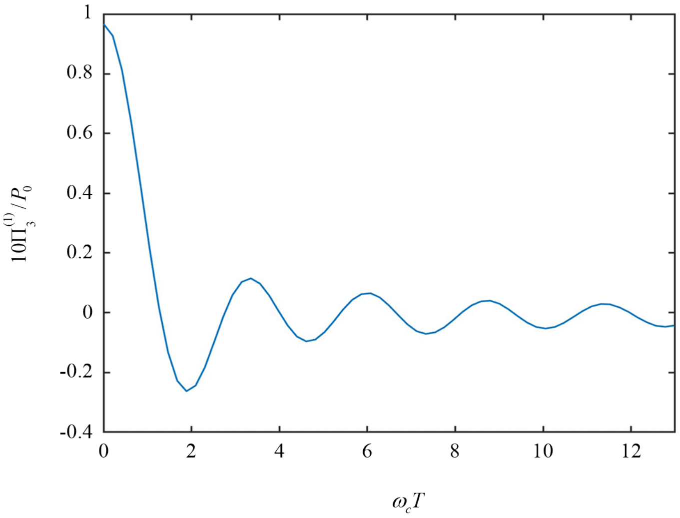

Figure 5. Response at receiver (mode $m = 1$); transmission of bit 1

The values of b in Eq. (42) are obtained from the mode 1 dispersion function given by Eq. (15). The value of z/D is taken to be 1 as before. The computation of the integral in Eq. (40) for these data is presented in Figure 5, for which the factor $\cos\theta$ is suppressed. The phase of the wave is oppositely polarised in the two segments $(-\pi/2, \pi/2)$ and $(\pi/2, 3\pi/2)$. So if the average over the first phase $(-\pi/2, \pi/2)$ is taken, the contribution of the mean over that phase to the computed graph is $2/\pi$ of the values shown in the figure. On the other hand if the average over the other segment $(\pi/2, 3\pi/2)$ is taken, the corresponding factor is $-2/\pi$. It is interesting to note that the amplitude of the wave falls considerably for the long distance traversed.

6. Conclusion

The basic analog modulation methods ASK, FSK, PSK, QAM etc. are well understood and applied in practice whether in wireless digital communication or in fiber-optic transmission of data. As far as open literature is concerned, the propagation characteristics are mostly studied by applying simulation techniques provided provided by Matlab and Simulink. However, as the propagation in fifiber-optic cables is unidirectional, the mechanism of multiplexing a stream of digital data is amenable to an exact formulation based on the Maxwell equations of electromagnetism. Adopting this procedure as in Bose [16], a method is is developed here for determining the received signal at a distant point in the form of a Fourier integral, adopting amplitude modulation (ASK) of the carrier light wave. This modulation technique is in fact of practical use because of little chance of interfering with noise from other sources. From this theoretical development a formula for 2-bit QAM is also obtained. In as much as propagation in optical fibers can take place in several modes, special attention is paid to propagation in single mode fibers. A typical numerical example provides the wave pattern at the receiver by Fast Fourier Transform (FFT) for evaluating the Fourier integrals. The next higher mode is also studied for a multi-mode optical fiber. A numerical example shows a considerable drop in the received wave when a bit is transmitted. As the theory is for a batch of streaming bits, serial batch transmission of data is a possibility to explore. Moreover, the multiplexed transmission of differently polarised waves duly modulated is a distinct possibility for serial batch transmission by ASK, provided the peak values of each polarised wave are measurable adequately.

Declarations

No data were generated or analysed by AI or otherwise in the presented research.

Conflict of interest

There are no conflicts of interest in the research reported in this paper.

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